Tolerance Optimization with Markov Chain Process

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마르코프 과정을 이용한 공차 최적화

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Abstract

This paper deals with a new approach to tolerance optimization problems. Optimal tolerance allotment problems can be formulated as stochastic optimization problems. Most schemes to solve the stochastic optimization problems have been found to exhibit difficulties in multivariate integration of the probability density function. As a typical example of stochastic optimization, the optimal tolerance allotment problem has the same difficulties. In this stochastic model, manufacturing system is represented by Gauss-Markov stochastic process and the manufacturing unit availability is characterized for realistic optimization modeling. The new algorithm performed robustly for a large deviation approximation. A significant reduction in computation time was observed compared to the results obtained in previous studies.

Key Words : Tolerance Optimization, Stochastic Process, Yield, Gauss-Markov Process

1. INTRODUCTION

In tolerance optimization research, two problem types are typically recognized: tolerance allotment problems and design centering problems. The allotment of tolerances is closely tied to the overall quality and cost of a product[1]. If the tolerances are too loose, the probability for an assembly to function acceptably (yield) will be low. On the other hand, if the tolerance is too tight, the manufacturing cost will become high. Thus tolerance allotment becomes an optimization problem to determine the optimal allotment of the tolerances under the constraints of the function requirements and acceptance probability(spec yield). In the design centering problem, nominal dimensions are changed in order to find the maximum yield with fixed tolerances. Therefore the design variables are the nominal dimensions in the design centering problem. Typical numerical

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optimization methods require analysis, perturbation, and reanalysis in an organized algorithmic fashion. In tolerance optimization, there are two main phases in the solution procedure: tolerance analysis and variable synthesis.

Tolerance analysis is to determine whether the yield with the given variable assignment meets the minimum acceptable probability of satisfactory performance. The solution requires the integration of the probability density function over the acceptable region. When the dimensionality of the problem is high (i.e. there are many design variables) and the constraints on the stack-up conditions are complicated, the estimation of yield is performed by simulation techniques. In these cases, both the tolerance allotment problem and the design centering problem are defined as stochastic optimization problems because of the stochastic nature of the yield estimation. Variable synthesis is to determine acceptable individual tolerances or center dimensions to meet the minimum acceptable performance standards while minimizing manufacturing cost.

2. Tolerance Optimization Problems

2.1 Problem modeling

In many tolerancing problems, individual dimensions are assumed to have normal distributions. This assumption provides a method to relate the tolerances to the degree of satisfaction. Therefore the dimension vector **x** is assigned to have the multivariate normal distribution. The mean value μ_j of a dimension x_j is given as nominal dimension in the design process. However the standard deviation σ_j is selected according to the degree of precision achieved with each manufacturing process. As a consequence, the standard deviations σ_j are a function of the tolerances t_j. In many mass production environments, the tolerance of a single dimension is considered as sufficiently large if 99.73% of the dimensions are in spec. For a normally distributed random variables, there is a 99.73% probability that the variable will take on a value within $\pm 3\sigma$ of the nominal value. Therefore, in common cases, σ_j is set to $t_j/6$[2,3]. As a result, the characteristics of the distributions are determined if t_j is given.

In the assembly process, some dimensions interact with other dimensions and have effects on the function of the assembly. Those dimensions are called sum dimensions.

The yield increases as the tolerances become tighter. However manufacturing costs also increase as the tolerances become tighter. The effect of individual tolerance improvement on the total manufacturing cost increase is different for each tolerance. Therefore, the allotment of individual tolerances which minimizes the cost under the design functions and minimum acceptable yield constraints becomes an optimization problem.

It minimizes the cost by reducing the tolerances while fixing the distribution centers on the nominal dimensions. The fact that the tolerance can be changed means that the precisions of manufacturing processes can be changed in order to meet tolerance requirements. In the design centering problem, the tolerance is fixed under the assumption that the manufacturing processes and the precision of the machines are determined already. In a more realistic manufacturing situation, both the precisions of manufacturing processes and the fixture settings could be changed simultaneously to increase the yield and minimize the manufacturing cost. The problem can be described as:

$$\text{Minimize} \quad C(\mathbf{t}) \tag{1}$$

subject to $\Pr(\mathbf{R_R}) > Y_{spec}$
where Y_{spec} represents spec yield,
$\mathbf{R_R}$ represents reliable region,
$\Pr(\mathbf{R_R})$ represents estimated yield.

In constrained optimization problems, a simple way to deal with the constraints is the penalty function method[4,5]. The constraints can be replaced by penalties which increase dramatically as the degree of constraint violations increases. As a consequence, a constrained optimization problem can be replaced by optimization of an unconstrained functional. In real manufacturing processes, the failures in assembly and manufacturing facilities cause additional cost increases. Therefore the optimization problem utilizing real manufacturing cost, Creal, is expressed as

$$\text{Minimize} \quad Creal = C(\sigma) + r < -\left(\int_{x \in R_R} \phi(x)dx - Y_{spec}\right) >^2$$

$$where < a >= \begin{cases} a, & x \geq 0 \\ 0, & a < 0 \end{cases} \quad (2)$$

r is cost coefficient and positive

If the integration of the probability density function is performed by a simulation scheme, the optimal tolerance allotment problem becomes a stochastic optimization problem.

2.2 Assumptions

In order to compute the manufacturing cost, it is necessary to evaluate the convolution of probability distributions corresponding to the forced outage rates of the generating units and that of the load. It is well known in the context of manufacturing system reliability evaluation how time-consuming the exact computations for this purpose is. The large deviation method has been found to be particularly effective in providing accurate approximations for the power generation reliability indices[6] as well as for aiding computations in evaluating composite reliability and production costing indices[7]. Most of the computations for the manufacturing reliability indices have assumed that the load is a deterministic quantity. However, when predictions are made for a future time period for the marginal costs, it will be incorrect to treat load as free of any uncertainty. It was known that when the load is regressed, the residuals can be approximated by a Gauss-Markov process.

It was assumed that the total cost is calculated for a manufacturing system consisting of N generating units. The following additional assumptions are made:

a) The machines in a fixed, pre-assigned loading order, which depends only on the load and the availability of the cutting the units.
b) The ith unit in the cutting order has a capacity c_i, maintenance cost d_i, and a forced outage rate, q_i, i=1,2,...,n.
c) After adjusting for the variations, the load at process t, u(t), Gauss-Markov process[8] with $E[u(t)] = \mu_t$ and $Var[u(t)] = \sigma_t^2$ where μ_t and σ_t are assumed to be known.
d) The operating state of each generating unit i follows a two-state continuous-time alternating renewal process, $Y_i(t)=\{0,1\}$. The stochastic processes describing the up and down times of the manufacturing units are in steady state, and for any two unit i and j, they are statistically independent.

2.3 Calculating marginal cost

Under the above assumptions the manufacturing cost of the system at a specific process t, denoted by $d_{J(t)}$, is determined by the operating cost of the last unit. The last such unit in the loading order is called the marginal unit and denoted by J(t).

The mean of the marginal cost is obtained from the following formulas:

$$E[d_{J(t)}] = \sum_{j=1}^{N} d_j \Pr[J(t) = j] \quad (3)$$

$$E[d_{J(t)}^2] = \sum_{j=1}^{N} d_j^2 \Pr[J(t) = j]$$

Writing $\Pr[J(t) = j]$ as:
$\Pr[J(t) = j] = \Pr[J(t) > j-1] - \Pr[J(t) = j]$

and observing that the events $J(t) > j$ and $u(t) - \sum_{i=1}^{j} c_i Y_i(t) \geq 0$ are equivalent, we obtain:

$$\Pr[J(t) > j] = \Pr[u(t) - \sum_{i=1}^{j} c_i Y_i(t) \geq 0] \quad (4)$$

Therefore, to compute the mean of the marginal cost at process t we need to determine the probability that

$u(t) - \sum_{i=1}^{j} c_i Y_i(t)$ is greater than zero for all values of j. This probability could be calculated by conditioning on the values of $Y_i(t)$. However, the computation time will depend on the many values that the expression $\sum_{i=1}^{j} c_i Y_i(t)$ can take, which in the worst case is 2^N (when j=N). Thus, the computational time increases exponentially as N increases. In order to apply the large deviation approximation to evaluating (2), we define the random variable $X_j(t) = u(t) - \sum_{i=1}^{j} c_i Y_i(t)$. Thereore we can write (4) as

$$\Pr[J(t) > j] = \Pr[X_j(t) \geq 0]$$

Denote the cumulative distribution function of $X_j(t)$ by F(x;j,t) and let the corresponding exponentially tilted distribution function be for a given s be:

$$dF^s(x;j,t) = e^{sx-K(s;j,t)} dF(x;j,t)$$

where $K(s;j,t) = \ln E[e^{sX_j(t)}]$

Then $\Pr[X_j(t) \geq 0] = \int_0^\infty dF(x;j,t)$ can be expressed as a function of the distribution function $F^s(x;j,t)$ by:

$$\Pr[X_j(t) \geq 0] = e^{K(s;j,t)} \int_0^\infty e^{-sx} dF^s(x;j,t) \quad (5)$$

The central limit theorem is used to approximate $F^s(x;j,t)$ by a normal distribution function $\Phi(x, \mu, \sigma^2(s,j,t))$ with appropriately determined mean μ and variance $\sigma^2(s,j,t)$.

$$\Pr[X(t) \geq 0] \cong e^{K(s;j,t)} \int_0^\infty e^{-sx} d\Phi(x;\mu,\sigma^2(s,j,t)) \quad (6)$$

The constant s is next chosen such that the lower limit of the integral is the expected value of the random variable $X_j^s(t)$ whose distribution function is $F^s(x;j,t)$. The equation (5) is valid for any value of s, so we choose s such that $E[X_j^s(t)] = \mu = 0$. This reduces to the following equation:

$$\mu_t + \sigma_t^2 s - \sum_{i=1}^{j} \frac{p_i c_i e^{-sc_i}}{p_i e^{-sc_i} + q_i} = 0 \quad (7)$$

Let s_0 denote the unique root of (7). Then, equation (6) can be rewritten as

$$\Pr[X_j(t) \geq 0] \cong e^{K(s_0;j,t)} \int_0^\infty e^{-s_0 x} d\Phi(x;0,\sigma^2(s_0,j,t)) \quad (8)$$

The normal approximation to F^s is not likely to be very accurate in the tails. If s_0 is positive, the error of the normal approximation in the tails is reduced by the multiplier $e^{-s_0 x}$ After ordering terms and completing squares we obtain

$$\Pr[X_j(t) \geq 0] \cong e^{K(s_0;j,t)+\frac{1}{2}s_0^2\sigma^2(s_0,j,t)} \int_0^\infty d\Phi(x;-s_0\sigma(s_0,j,t),\sigma^2(s_0,j,t))$$

$$\cong e^{K(s_0;j,t)+\frac{1}{2}s_0^2\sigma^2(s_0,j,t)} \int_{s_0\sigma(s_0,j,t)}^\infty d\Phi(z;0,1)$$

Evaluating the integral we have:

$$\Pr[X_j(t) \geq 0] \cong e^{K(s_0;j,t)+\frac{1}{2}s_0^2\sigma^2(s_0,j,t)} [1 - \Phi(s_0\sigma(s_0,j,t);0,1)]$$

Therefore

$$\sigma^2(j,t) = \sigma_t^2 + \sum_{i=1}^{j} \frac{p_i q_i c_i^2 e^{-s_0 c_i}}{[e^{-s_0 c_i} p_i + q_i]^2}$$

The following procedure states the steps involved in computing the marginal cost at process t.

<u>Procedure</u>

Define Pr[J(t) > 0] = 0, Ed = 0, and Ed2 = 0
For j=1,2,...,N

Find s_0 by solving $\mu_t + \sigma_t^2 s_0 - \sum_{i=1}^{j} \frac{p_i c_i e^{-s_0 c_i}}{p_i e^{s_0 c_i} + q_i} = 0$

$$\sigma^2(s_0, j, t) = \sum_{i=1}^{j} \frac{p_i q_i c_i^2 e^{s_0 c_i}}{[e^{-s_0 c_i} p_i + q_i]^2} + \sigma_t^2$$

$$K(s_0; j, t) = \mu_t s_0 - \frac{1}{2}\sigma_t^2 s_0^2 - \sum_{i=1}^{j} Ln(e^{-s_0 c_i} p_i + q_i)$$

$\Pr[J(t) > j]$

$$\cong e^{K(s_0; j,t) + \frac{1}{2} s_0^2 \sigma^2(s_0, j, t)} [1 - \Phi(s_0 \sigma(s_0, j, t); 0, 1)]$$

$\Pr[J(t) = j] = \Pr[J(t) > j-1] - \Pr[J(t) > j]$

$Ed = Ed + d_j \Pr[J(t) = j]$

$Ed2 = Ed2 + d_j^2 \Pr[J(t) = j]$

Endfor

$E[d_{J(t)}] = Ed$

3. Results

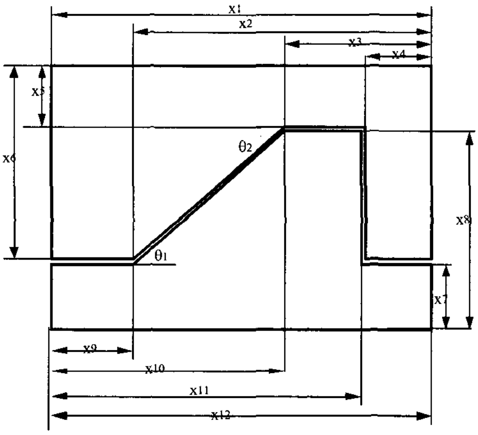
A nonlinear constraint problem posed by Lee[3] was tried. The shape of the assembly and corresponding design functions are shown in Figure 1. The first two design functions are the vertical and the horizontal clearance conditions of the two parts. The third and the fourth design functions restrict the difference between angles θ_1 and θ_2 to be within $\pm\pi/180$ radians (one degree) for successful assembly. The last two conditions require the size difference between two parts to be within ± 0.01. The nominal dimensions are given as $\mathbf{x}^T$ = (50.0, 40.00125, 20.05, 9.9985, 9.9985, 30.0, 10.0, 30.0, 10.05, 30.0, 40.0, 50.0). The modified cost function is used where each cost function $C_i(\sigma_i)$ was used to define total manufacturing cost as

$$C_i(\sigma_i) = \frac{a_i \times 10^{-3}}{(6\sigma_i)^{b_i}} \quad (9)$$

The coefficients in equation (9) were set by Lee as: a_1 = 0.2, a_2 = 1.0 , $a_3 = a_4$ = 0.015, a_5 = 0.008, a_6 = 0.009, a_7 = 0.008, a_8 = 0.006, a_9 = 1.0, a_{10} = 0.01, a_{11} = 0.015 and a_{12} = 0.2; and $b_1 = b_2 = \ldots = b_{12}$ = 2.0.

We consider a manufacturing system patterned after the generation mix of an actual cutting facility. The cost and the reliability parameters of the units belonging to this system are given in Table 1. Reduces gradient method is used as an optimization tool.

The results are shown in the Table 2 For the test run, the number of generations was 300 and the number of



$F1(\mathbf{x}) = (x_6 - x_5) - (x_8 - x_7)$

$F2(\mathbf{x}) = (x_3 - x_4) - (x_{11} - x_{10})$

$F3(\mathbf{x}) = (x_8 - x_7)*(x_2 - x_3) - (x_6 - x_5)*(x_{10} - x_9) + \tan(\pi/180)*\{(x_{10} - x_9)*(x_2 - x_3) - (x_8 - x_7)*(x_6 - x_5)\}$

$F4(\mathbf{x}) = (x_6 - x_5)*(x_{10} - x_9) - (x_8 - x_7)*(x_2 - x_3) + \tan(\pi/180)*\{(x_{10} - x_9)*(x_2 - x_3) - (x_8 - x_7)*(x_6 - x_5)\}$

$F5(\mathbf{x}) = -x_1 + x_{12} + 0.01$

$F6(\mathbf{x}) = x_1 - x_{12} + 0.01$

Fig. 1 Assembly feature

Table 1 A 17-manufacturing units

Units	Xi	Capacity	1/(failure rate)	1/(repair rate)	cost
1-2	X1	400	1100	150	6.00
3	X2, X3	350	1150	100	11.40
4-7	X4, X5	150	960	40	11.40
8-9	X6	150	1960	40	14.40
10-12	X7, X8	200	950	50	22.08
13-15	X9, X10	100	1200	50	23.00
16	X11	50	2940	60	27.60
17	X12	100	450	50	43.50

Table 2 The Results of Nonlinear Problem

dimensions	tolerance
x_1	0.02874
x_2	0.21827
x_3	0.06352
x_4	0.07251
x_5	0.00195
x_6	0.00315
x_7	0.00287
x_8	0.00325
x_9	0.69523
x_{10}	0.04852
x_{11}	0.03652
x_{12}	0.02536
Cost	11.87
Yield	94.91%

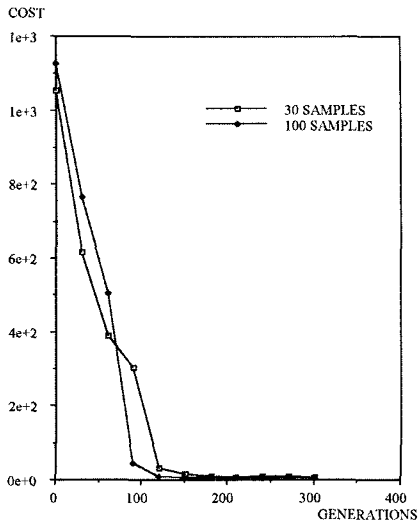


Fig. 2 Iterations

sampling points was 30. The results show that the cost was reduced by about 20% compared to the cost obtained by Lee's algorithm and yield is slightly improved.

In order to examine the relation between the computation effort and the approximation of simulation, several sampling numbers were used for the Monte Carlo simulation. Figure 2 shows the convergence of the algorithm for 30 sampling points and 100 sampling points. This figure shows that the algorithm converges faster when smaller number of sampling points are used even though it takes more iterations.

4. Conclusion

Optimal tolerance allotment problems were solved using the new algorithm. Two problems were successfully solved with a large deviation approximation. Despite the coarse resolution and low precision of the evaluation, the algorithm converged to a spec yield of sufficient precision.

For mathematical programming, the time complexity may increase geometrically with the increase of dimensionality. However, in the method used in this paper, the time complexity increases linearly as the dimension increases. Therefore we believe that the new algorithm can offer significant advantages for high dimension problems.

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Reference

(1) Bjørke, O., 1989, *Computer-Aided Tolerancing*, ASME Press, New York, pp. 86~87.

(2) Evans, D. H., 1974, "Statistical Tolerancing: The State of the Art Part I. Background", *Journal of Quality Technology*, Vol. 6, pp. 188~195.

(3) Lee, W. J., 1989, *Tolerancing: computations on geometric uncertainties*, Doctoral Dissertation, University of Michigan.

(4) Siddall, J. N., 1983, *Probabilities Engineering Design; Principles and Applications*, Marcel Dekker, Inc., New York and Basel, pp. 151~160.

(5) Fiacco, A. V., and McCormick G. P., 1968, *Nonlinear Programming: Sequential Unconstrained Minimization Techniques,* John Wiley, New York.
(6) Iyengar, S., and Mazumdar, M., 1998, "A Saddle Point Approximation for Certain Multivariate Tail Probabilities", *SIAM Journal on Scientific Computing*, Vol. 19, pp. 1234～1244.
(7) Breipohl, A. M., Lee, F. N., Zhai, D., and Adapa, R., 1992, "A Gauss Markov Load Model for Application in Risk Evaluation and Production Simulation", *IEEE Trans. Power Syst.*, Vol. 7, No. 4, pp. 1493～1499.
(8) Duran, H., 1986, "On Improving the Convergence and Accuracy of the Cumulant Method of Calculating Reliability and Production Cost", *IEEE Trans. on Power Syst.*, Vol. 1, No. 3, pp. 121～126.