균일단면 선박의 유탄성 수직응답에 대한 해석해

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Exact Solution on the Vertical Hydro-elastic Responses of Ships having Uniform Sectional Properties

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Abstract

Exact solution on the vertical responses of ships having uniform sectional properties in waves is derived. Boundary value problem consisted of Timoshenko beam equation and free-free end condition is solved analytically. The responses are assumed as linear and wave loads are calculated by using strip method. Vertical bending moment, shear force and deflection are calculated. The developed analysis model is used for the benchmark test of the numerical codes in this problem. Also the application on the preliminary design of barge-like ships and VLFS (Very Large Floating Structure) is expected.

1. INTRODUCTION

Recent merchant ships have been evolving rather economic features than the conventional ones. They become bigger in displacement, lighter in light ship weight, wider in breadth

and shallower in draft. Obviously, these trends may promise the better efficiency in cargo transportation. However, they may also cause the new problems. Hull girder flexibility is one of them. It may become more flexible than the older ones with both bending and torsion. Especially, for large container carrier, due to very fast speed and wide hatch opening on main deck, these may cause the fatigue damage due to springing and whipping by coupling of vertical bending

and torsion effects.

In this paper, exact solution on the hydro-elastic response of ships having uniform sectional properties in waves is derived(Korobkin, 2003). Timoshenko beam equation is used as the governing equation. Free-free end conditions at the bow and stern are used as the boundary conditions. The responses are assumed as linear to get the analytic solutions. Vertical bending moment, vertical shear force and vertical deflection are derived for artificial ship. Total hydrodynamic forces are calculated using strip method.

Ship springing is known as the steady resonance response of the ships to incident waves. It could occur over the broad range of low to moderate sea-state, when the encountering wave frequency matches the natural frequency of hull girder. Instead, whipping is transient resonance response of the ship due to impact loads like slamming etc. (박성환 et al. 2000). There are a lot of previous works on this topic. But only a limited number of them are listed here. Linear theory with strip methods is studied by Hoffman and van Hooff (1976). Troesch (1984a, b) studied springing experimentally and theoretically. Jensen and Pedersen (1979, 1981) suggested quadratic strip theory, but only for vertical bending moment. Besides, in domestic, 이호영 et al (2001, 2003) analyzed the ship springing using 3-D pulsating and translating source method. Jung et al. (2003) and 정종진 et al (2004) developed a numerical model using quadratic strip theory.

2. BVP FORMULATION

The vertical response of the advancing ship is described within the Timoshenko beam theory as follows. They are able to account for the shear deformation of the ship-like hull girders

Coordinate system is shown in Fig. 1.

Timoshenko Beam Equation

$$\frac{\partial u_z}{\partial x} = \theta_y(x,t) + \gamma_y(x,t) \tag{1}$$

$$EI(x)\left[1 + \eta_y(x)\frac{\partial}{\partial t}\right]\frac{\partial \theta_y}{\partial x} = M_y(x,t) \tag{2}$$

$$\frac{\partial M_y}{\partial x} = I_y(x)\frac{\partial^2 \theta_y}{\partial t^2} - V_z(x,t) \tag{3}$$

$$\frac{\partial V_z}{\partial x} + F_z(x,t) = \mu(x)\frac{\partial^2 u_z}{\partial t^2} \tag{4}$$

$$k_z(x)A(x)G\left[1 + \alpha_z(x)\frac{\partial}{\partial t}\right]\gamma_y(x,t) = V_z(x,t) \tag{5}$$

In (1)-(5), the notations are;

$u_z(x,t)$ – Vertical displacement of the mass center of the ship section [m]

x – Distance of the ship section from the stern [m]

t – Time [sec]

$\theta_y(x,t)$ – Vertical bending slope

$\gamma_y(x,t)$ – Horizontal shear deformation per unit ship length

E – Modulus of elasticity for the material of the ship [N/m^2]

$I(x)$ – Cross sectional second moment [m^4]

$\eta_y(x)$ – Vertical structural damping coefficient per unit ship length [sec]

$M_y(x,t)$ – Vertical bending moment of the ship girder [Nm]

$V_z(x,t)$ – Vertical shear force in the section [N]

$I_y(x)$ – Inertia moment of the cross section in the vertical bending [kgm]

$\mu(x)$ – Mass of the ship per unit length [kg/m]

$F_z(x,t)$ – Vertical total hydrodynamic force per unit ship length [N/m]

$A(x)$ – Cross sectional steel area of the ship [m^2]

$k_z(x)$ – Vertical shear coefficient per unit ship length

G – Modulus of elasticity in shear for steel [N/m^2]

$\alpha_z(x)$ – Vertical shear structural damping co-

efficient per unit ship length [sec]

The solution of equation (1)–(5) satisfies the free-free beam boundary conditions

$$V_z(0,t) = 0, \quad M_y(0,t) = 0$$
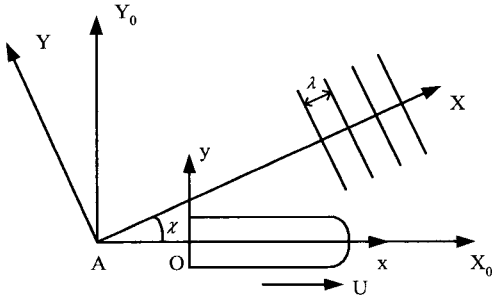$$V_z(L,t) = 0, \quad M_y(L,t) = 0 \qquad (6)$$

Fig. 1 Local and global coordinate systems

3. EXACT SOLUTION – VBM

The solution of the boundary value problem is sought using standard method. The partial differential equation is converted into ordinary differential equation. Within the linear strip theory, the vertical total hydrodynamic force per unit ship length $F_z(x,t)$ is given as;

$$F_z(x,t) = -\text{Re}\left[\frac{D}{Dt}\left(m_z^c(x,\omega_e)\frac{D\overline{Z}_1}{Dt}\right) + \rho g B(x)\overline{Z}_1\right] \quad (7)$$

Where, the instant draught,

$$\overline{z}(x,t) = \text{Re}[\overline{Z}_1(x,t)] = u_z(x,t) - \overline{\varsigma}(x,t) . \qquad (8)$$

$\overline{\varsigma}(x,t) = AE_s(x)\cos[k_c x - \omega_e t]$ is the incident wave elevation and $k_c = k\cos(\chi)$.

E_s; modified Smith correction factor

$$E_s(x) \rightarrow e^{-k\overline{T}} \text{ as } Ak \rightarrow 0$$

$$\frac{D}{Dt} = \frac{\partial}{\partial t} - U\frac{\partial}{\partial x}$$

$$m_z^c(x,\omega_e) = m_{zz}(x,\omega_e) + \frac{i}{\omega_e}N_{zz}(x,\omega_e) \quad , \text{ is the}$$

complex added mass of the section in heave.

For the uniform artificial ship, equation (7)

reduces to;

$$F_z(x,t) = -\text{Re}\left[m_z^c(\omega_e)\frac{D^2\overline{Z}_1}{Dt^2} + \rho g B\overline{Z}_1\right] \qquad (9)$$

The solution of equation (9) is sought in the form of

$$u_z(x,t) = \text{Re}[\widetilde{u}_z(x)e^{-i\omega_e t}] + s.o.t \qquad (10)$$

$$V_z(x,t) = \text{Re}[\widetilde{V}_z(x)e^{-i\omega_e t}] + s.o.t \qquad (11)$$

$$\theta_y(x,t) = \text{Re}[\widetilde{\theta}_y(x)e^{-i\omega_e t}] + s.o.t \qquad (12)$$

$$\gamma_y(x,t) = \text{Re}[\widetilde{\gamma}_y(x)e^{-i\omega_e t}] + s.o.t \qquad (13)$$

$$M_y(x,t) = \text{Re}[\widetilde{M}_y(x)e^{-i\omega_e t}] + s.o.t \qquad (14)$$

Where, s.o.t. stands for the second order terms proportional to the product Ak.

Substituting (8), (9), (10) and (11) into (4) yields;

$$\frac{d\widetilde{V}_z}{dx} + \omega_e^2 m_z^c(\omega_e)\widetilde{u}_z(x) - 2iU\omega_e m_z^c(\omega_e)\frac{d\widetilde{u}_z}{dx}$$
$$-U^2 m_z^c(\omega_e)\frac{d^2\widetilde{u}_z}{dx^2} + \mu\omega_e^2\widetilde{u}_z(x) - \rho g B\widetilde{u}_z(x) = \quad (15)$$
$$= \omega_e^2 m_z^c(\omega_e)\widetilde{\varsigma} - 2iU\omega_e m_z^c(\omega_e)\widetilde{\varsigma}' -$$
$$-U^2 m_z^c(\omega_e)\widetilde{\varsigma}'' - \rho g B\widetilde{\varsigma}$$

where, $\widetilde{\varsigma} = AE_s e^{ik_c x}$, $\widetilde{\varsigma}' = AE_s ik_c e^{ik_c x}$, $\widetilde{\varsigma}'' = -AE_s k_c^2 e^{ik_c x}$

$$R.H.E.(15) = AE_s[\omega_e^2 m_z^c(\omega_e) + 2U\omega_e m_z^c(\omega_e)k_c +$$
$$+ U^2 m_z^c(\omega_e)k_c^2 - \rho g B]e^{ik_c x} =$$
$$= AE_s[m_z^c(\omega_e)(\omega_e^2 + 2U\omega_e k_c + U^2 k_c^2) - \rho g B]e^{ik_c x} =$$
$$= AE_s[m_z^c(\omega_e)(\omega_e + Uk_c)^2 - \rho g B]e^{ik_c x} \qquad (16)$$

Since, $\omega_e + Uk_c = \omega_0$,
R.H.E. (15)= $AE_s[m_z^c\omega_0^2 - \rho g B]e^{ik_c x}$

Therefore, equation (15) takes the form;

$$\frac{d\widetilde{V}_z}{dx} - U^2 m_z^c(\omega_e)\frac{d^2\widetilde{u}_z}{dx^2} - 2iU\omega_e m_z^c(\omega_e)\frac{d\widetilde{u}_z}{dx} + \quad (17)$$
$$[\omega_e^2(m_z^c + \mu) - \rho g B]\widetilde{u}_z(x) = AE_s[m_z^c\omega_0^2 - \rho g B]e^{ik_c x}$$

The complex-value unknown functions $\tilde{u}_z(x), \tilde{\theta}_y(x)$, $\tilde{M}_y(x), \tilde{V}_z(x)$ are only dependent on the longitudinal coordinate x. Therefore, equations (1), (2) and (3) can be reduced to the following ordinary differential equations, respectively;

$$\frac{d\tilde{u}_z}{dx} = \tilde{\theta}_y(x) + \tilde{\gamma}_y(x) \tag{18}$$

$$\frac{d\tilde{\theta}_y}{dx} = \tilde{M}_y(x)/a_1^c(x) \tag{19}$$

$$\frac{d\tilde{M}_y}{dx} = -\omega_e^2 I_y(x)\tilde{\theta}_y(x) - \tilde{V}_z(x) \tag{20}$$

where, $a_1^c(x) = EI(x)[1 - i\omega_e \eta_y(x)]$,

Let define as $a_2^c(x) = k_z(x)A(x)G[1 - i\omega_e\alpha_z(x)]$ and differentiate (18) and (20) with respect to x and combine the results together with (19);

$$\frac{d^2\tilde{u}_z}{dx^2} = \frac{1}{a_1^c}\tilde{M}_y + \frac{1}{a_2^c}\left[-\frac{d^2\tilde{M}_y}{dx^2} - \omega_e^2 I_y \frac{d\tilde{\theta}_y}{dx}\right] =$$

$$= (\frac{1}{a_1^c} - \frac{\omega_e^2 I_y}{a_1^c a_2^c})\tilde{M}_y - \frac{1}{a_2^c}\frac{d^2\tilde{M}_y}{dx^2} \tag{21}$$

Denoting;

$$\frac{1}{a_1^c}(1 - \frac{\omega_e^2 I_y}{a_2^c}) =: a_3^c \tag{22}$$

We arrive at the relation;

$$\frac{d^2\tilde{u}_z}{dx^2} = a_3^c\tilde{M}_y - \frac{1}{a_2^c}\frac{d^2\tilde{M}_y}{dx^2} \tag{23}$$

Now, one needs to differentiate equation (17) twice with respect to x and to use equations (23) and (20)

$$\frac{d^3}{dx^3}\left[-\frac{d\tilde{M}_y}{dx} - \omega_e^2 I_y \tilde{\theta}_y\right] - U^2 m_z^c \frac{d^2}{dx^2}\left[a_3^c\tilde{M}_y - \right.$$

$$\left. -\frac{1}{a_2^c}\frac{d^2\tilde{M}_y}{dx^2}\right] - 2i\omega_e Um_z^c \frac{d}{dx}\left[a_3^c\tilde{M}_y - \frac{1}{a_2^c}\frac{d^2\tilde{M}_y}{dx^2}\right] +$$

$$+ \left[\omega_e^2(m_z^c + \mu) - \rho gB\right]\left[a_3^c\tilde{M}_y - \frac{1}{a_2^c}\frac{d^2\tilde{M}_y}{dx^2}\right] =$$

$$= -AE_s k_c^2\left[m_z^c\omega_0^2 - \rho gB\right]e^{ik_c x} \tag{24}$$

Taking into account equation (19), we transform (24) into ordinary differential equation of the fourth order with respect to the vertical bending moment $\tilde{M}_y(x)$;

$$\left[-1 + \frac{U^2 m_z^c}{a_2^c}\right]\frac{d^4\tilde{M}_y}{dx^4} + \frac{2i\omega_e Um_z^c}{a_2^c}\frac{d^3\tilde{M}_y}{dx_3} -$$

$$-\left[\frac{I_y\omega_e^2}{a_1^c} + \frac{\omega_e^2(m_z^c + \mu) - \rho gB}{a_2^c} + U^2 a_3^c m_z^c\right]\frac{d^2\tilde{M}_y}{dx^2} -$$

$$- 2i\omega_e Um_z^c a_3^c\frac{d\tilde{M}_y}{dx} + a_3^c\left[\omega_e^2(m_z^c + \mu) - \rho gB\right]\tilde{M}_y =$$

$$= -AE_s k_c^2\left[m_z^c\omega_0^2 - \rho gB\right]e^{ik_c x} \tag{25}$$

The equation is presented as;

$$b_5\frac{d^4\tilde{M}_y}{dx^4} + b_4\frac{d^3\tilde{M}_y}{dx^3} + b_3\frac{d^2\tilde{M}_y}{dx^2} +$$

$$+ b_2\frac{d\tilde{M}_y}{dx} + b_1\tilde{M}_y = b_0 e^{ik_c x} \tag{26}$$

General solution of equation (26) has the form;

$$\tilde{M}_y(x) = C_0 e^{ik_c x} + \sum_{j=1}^{4} C_j e^{\lambda_j x} \tag{27}$$

Where,

$$C_0 = b_0/(b_1 + b_2 ik_c - b_3 k_c^2 - b_4 ik_c^3 + b_5 k_c^4) \tag{28}$$

C_j are unknown complex coefficients and $\lambda_1, \lambda_2, \lambda_3, \lambda_4$ are complex roots of the characteristic equation;

$$b_5\lambda^4 + b_4\lambda^3 + b_3\lambda^2 + b_2\lambda + b_1 = 0 \tag{29}$$

Equation (29) is solved numerically by Jenkins – Traub (1972) three-stage algorithm. The boundary conditions (6) give two equations for four unknown coefficients C_1, C_2, C_3, C_4 in expression (27)

$$\tilde{M}_y(0) = 0 \quad \rightarrow \quad \sum_{j=1}^{4} C_j = -C_0 \tag{30}$$

$$\tilde{M}_y(L) = 0 \quad \rightarrow \quad \sum_{j=1}^{4} C_j e^{\lambda_j L} = -C_0 e^{-ik_c L} \tag{31}$$

4. EXACT SOLUTION - VSF

Note that equations (19) and (20) give;

$$\frac{d\widetilde{V}_z}{dx} = \frac{d^2\widetilde{M}_y}{dx^2} - \frac{\omega_e^2 I_y}{a_1^c}\widetilde{M}_y(x) =$$

$$= C_0\left[k_c^2 - \frac{\omega_e^2 I_y}{a_1^c}\right]e^{ik_c x} - \tag{32}$$

$$-\sum_{j=1}^4 C_j\left[\lambda_j^2 + \frac{\omega_e^2 I_y}{a_1^c}\right]e^{\lambda_j x}$$

The vertical shear force $\widetilde{V}_z(x)$ is obtained by integration of equation (32) with the initial condition $\widetilde{V}_z(0) = 0$;

$$\widetilde{V}_z(x) = C_0\left[k_c^2 - \frac{\omega_e^2 I_y}{a_1^c}\right]\frac{e^{ik_c x}-1}{ik_c} -$$

$$-\sum_{j=1}^4 C_j\left[\lambda_j^2 + \frac{\omega_e^2 I_y}{a_1^c}\right]\frac{e^{\lambda_j x}-1}{\lambda_j} \tag{33}$$

Equation (33) provides the distribution of the vertical shear force along the ship once the coefficients C_1, C_2, C_3, C_4, have been determined.

The edge condition $\widetilde{V}_z(L) = 0$, and formula (33) lead to the third equation to evaluate the coefficients;

$$\sum_{j=1}^4 C_j\left[\lambda_j^2 + \frac{\omega_e^2 I_y}{a_1^c}\right]\frac{e^{\lambda_j L}-1}{\lambda_j} =$$

$$= C_0\left[k_c^2 - \frac{\omega_e^2 I_y}{a_1^c}\right]\frac{e^{ik_c L}-1}{ik_c} \tag{34}$$

It is not straightforward to derive the fourth equation to close the system (30), (31) and (33). Note that equation (24), which solution has the form (27), was obtained by double differentiation of the original equation (17). Therefore, solution (27), (33), in general, does not satisfy (17), but satisfies this equation with the right-hand side

$$AE_s[m_z^c\omega_0^2 - \rho gB]e^{ik_c x} + D_1 x + D_0.$$

With respect to the double differentiation in x the latter equation also gives (24).

This implies that we have to calculate, using solution (27) and (33), the coefficients D_1 and D_0 and then, enforce them to be zero. This procedure will provide the fourth equation for the coefficients, C_1, C_2, C_3, C_4, and also the boundary condition to recover the deflection $\widetilde{u}_z(x)$.

In order to follow the described procedure, we derive the deflection $\widetilde{u}_z(x)$ and $\widetilde{\theta}_y(x)$ from equations (18) and (20), where, $\widetilde{M}_y(x)$ and $\widetilde{V}_z(x)$ are given by (27) and (33). We obtain;

$$\widetilde{\theta}_y(x) = -\frac{1}{\omega_e^2 I_y}[\widetilde{V}_z + \frac{d\widetilde{M}_y}{dx}] \tag{35}$$

$$\frac{d\widetilde{u}_z}{dx} = (\frac{1}{a_2^c} - \frac{1}{\omega_e^2 I_y})\widetilde{V}_z - \frac{1}{\omega_e^2 I_y}\frac{d\widetilde{M}_y}{dx} \tag{36}$$

By substituting (27) and (33) into (36) and integrating the result, we arrive at the following expression for the vertical deflection

$$\widetilde{u}_z(x) = u_0 +$$

$$+ (\frac{1}{a_2^c} - \frac{1}{\omega_e^2 I_y})\left\{C_0\left[k_c^2 - \frac{\omega_e^2 I_y}{a_1^c}\right]\frac{e^{ik_c x}-1-ik_c x}{-k_c^2} - \right.$$

$$\left. -\sum_{j=1}^4 C_j\left[\lambda_j^2 + \frac{\omega_e^2 I_y}{a_1^c}\right]\frac{e^{\lambda_j x}-1-\lambda_j x}{\lambda_j^2}\right\} -$$

$$-\frac{1}{\omega_e^2 I_y}\left\{C_0 e^{ik_c x} + \sum_{j=1}^4 C_j e^{\lambda_j x}\right\} \tag{37}$$

Where, $u_0 = \widetilde{u}_z(0)$. Substituting now formulae (32), (23), (36) and (37) into equation (17), we obtain;

$$-\frac{d^2\widetilde{M}_y}{dx^2} - \frac{\omega_e^2 I_y}{a_1^c}\widetilde{M}_y - U^2 m_z^c\left[a_3^c\widetilde{M}_y - \frac{1}{a_2^c}\frac{d^2\widetilde{M}_y}{dx^2}\right] -$$

$$-2i\omega_e U m_z^c\left\{(\frac{1}{a_2^c} - \frac{1}{\omega_e^2 I_y})\widetilde{V}_z - \frac{1}{\omega_e^2 I_y}\frac{d\widetilde{M}_y}{dx}\right\} +$$

$$+\left[\omega_e^2(m_z^c + \mu) - \rho gB\right]u_0 +$$

$$+(\frac{1}{a_2^c} - \frac{1}{\omega_e^2 I_y})\int_0^x \widetilde{V}_z(\xi)d\xi - \frac{1}{\omega_e^2 I_y}\widetilde{M}_y\right\} = \tag{38}$$

$$= AE_s\left[m_z^c\omega_0^2 - \rho gB\right]e^{ik_c x}$$

The left-hand side of equation (38) is the linear combination of the following functions of x:

$$e^{ik_c x}, e^{\lambda_1 x}, e^{\lambda_2 x}, e^{\lambda_3 x}, e^{\lambda_4 x}, x, 1. \qquad (39)$$

Consider the terms with $e^{\lambda_j x}$, $j = 1,2,3,4$.

$$\frac{C_j}{\lambda_j^2}\left\{(-1+\frac{m_z^c U^2}{a_2^c})\lambda_j^4 + \frac{2i\omega_e U m_z^c}{a_2^c}\lambda_j^3 + \right.$$
$$+(-\frac{\omega_e^2 I_y}{a_1^c} - U^2 m_z^c a_3^c - \frac{\omega_e^2(m_z^c+\mu)-\rho g B}{a_2^c})\lambda_j^2 +$$
$$+\frac{2i\omega_e U m_z^c I_y \omega_e^2}{a_1^c}(\frac{1}{a_2^c}-\frac{1}{\omega_e^2 I_y})\lambda_j - \qquad (40)$$
$$\left. -\frac{\omega_e^2 I_y}{a_1^c}(\frac{1}{a_2^c}-\frac{1}{\omega_e^2 I_y})\left[\omega_e^2(m_z^c+\mu)-\rho g B\right]\right\} =$$
$$= \frac{C_j}{\lambda_j^2}\left[b_5\lambda_j^4 + b_4\lambda_j^3 + b_3\lambda_j^2 + b_2\lambda_j + b_1\right] \equiv 0$$

Where b_j, $j = 1,2,3,4,5$ and formula (22) for a_3^c were used.

By simplifying the formula (33) for the vertical shear force;

$$\widetilde{V}_z(x) = C_0\left[k_c^2 - \frac{\omega_e^2 I_y}{a_1^c}\right]\frac{e^{ik_c x}}{ik_c} -$$
$$-\sum_{j=1}^{4}C_j\left[\lambda_j^2 + \frac{\omega_e^2 I_y}{a_1^c}\right]\frac{e^{\lambda_j x}}{\lambda_j} \qquad (41)$$

It is convenient to denote

$$C_0\left[k_c^2 - \frac{\omega_e^2 I_y}{a_1^c}\right]\frac{-1}{ik_c} =: P_0 \qquad (42)$$

$$\left[\lambda_j^2 + \frac{\omega_e^2 I_y}{a_1^c}\right]\frac{1}{\lambda_j} =: P_j, \quad j = 1,2,3,4 \qquad (43)$$

Then the edge conditions provide

$$\widetilde{V}_z(0) = 0 \rightarrow \sum_{j=1}^{4}C_j P_j = P_0 \qquad (44)$$

$$\widetilde{V}_z(L) = 0 \rightarrow \sum_{j=1}^{4}C_j P_j e^{\lambda_j L} = P_0 e^{ik_c L} \qquad (45)$$

The system of algebraic equations (30), (31), (44) and (45) is to evaluate the coefficients C_1, C_2, C_3 and C_4.

5. EXACT SOLUTION – deflection

Collecting the terms in (38), which are independent on x, and taking into account (41), we obtain

$$u_0 = (\frac{1}{a_2^c} - \frac{1}{\omega_e^2 I_y})\left[\frac{P_0}{ik_c} - \sum_{j=1}^{4}\frac{C_j P_j}{\lambda_j}\right] \qquad (46)$$

Substituting (46), (42) and (43) into (37), and by using equation (44), after some lengthy manipulation, we can arrive at the formula for vertical deflection of the ship as;

$$\widetilde{u}_z(x) = -\frac{1}{a_2^c}\left[C_0 e^{ik_c x} + \sum_{j=1}^{4}C_j e^{\lambda_j x}\right] -$$
$$-a_3^c\left[\frac{C_0}{k_c^2}e^{ik_c x} - \sum_{j=1}^{4}\frac{C_j}{\lambda_j^2}e^{\lambda_j x}\right] = \qquad (47)$$
$$= -\frac{1}{a_2^c}\widetilde{M}_y(x) - a_3^c\left[\frac{C_0}{k_c^2}e^{ik_c x} - \sum_{j=1}^{4}\frac{C_j}{\lambda_j^2}e^{\lambda_j x}\right]$$

6. NUMERICAL EXAMPLES

Equations (27), (41) and (47) provide exact solution of the problem under consideration. Derived solutions can be used to calculate the artificial ship which has uniform sectional properties. The example ship is selected as the 281 meters long container ship. Hull and mass data with extensive calculations were provided by the Bishop & Price(1979). Corresponding uniform artificial ship is established by averaging all the sectional properties along the ship length.
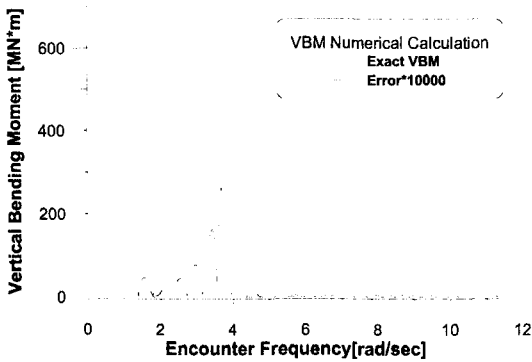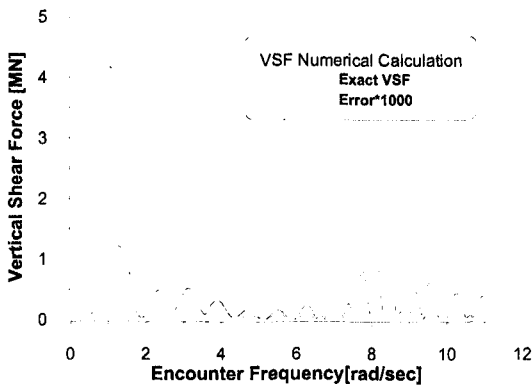
Fig. 2 VBM of uniform ship at 135 Deg. heading

Fig. 3 Vertical shear force of uniform ship

The calculated vertical bending moment and shear force at midship in 135 degrees wave heading are plotted on the Fig. 2 and 3, respectively. Vertical bending moment is plotted along the encounter frequencies of up to 12 rad/sec in Fig. 2. The abscissa is encountered wave frequency in radian/sec while the ordinate is vertical bending moment in MN-m. Springing is observed at encounter frequency around 3.9 radian/sec. The difference between numerical and analytical solution is regarded as the error and presented by dotted line exaggerated by 10000 times for reference. We can observe here the degree of the correlation with an existing

numerical code, for example, by Jung et al (2003). Vertical shear force is also plotted in the Fig. 3. Here, the ordinate is vertical shear force in MN. Again, good correlation is obtained between the exact solution and a numerical one.

7. CONCLUSIONS

From the previous studies, exact solution on the vertical hydro-elastic response of ships is derived. The solutions are used as not only preliminary design of barge-like ships but also the benchmark test of the existing numerical codes. Further application on the beam-like body such as VLFS and risers can be expected.

Reference

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