Improvement of Minimum MSE Performance in LMS-type Adaptive Equalizers Combined with Genetic Algorithm

Nam-Yong Kim

Abstract

In this paper the Individual tap - Least Mean Square(IT-LMS) algorithm is applied to the adaptive multipath channel equalization using hybrid-type Genetic Algorithm(GA) for achieving lower minimum Mean Squared Error(MSE). Owing to the global search performance of GA, LMS-type equalizers combined with it have shown preferable performance in both global and local search but those still have unsatisfying minimum MSE performance. In order to lower the minimum MSE we investigated excess MSE of IT-LMS algorithm and applied it to the hybrid GA equalizer. The high convergence rate and lower minimum MSE of the proposed system give us reason to expect that it will perform well in practical multi-path channel equalization systems.

Key words : Multipath Channel, Equalizer, Minimum MSE, Genetic Algorithm.

Ⅰ. Introduction

Since GA is a powerful optimization technique which is robust and problem-independent[1], it has been applied in many research field. The GA-based methods are expected to be efficient for wireless channel equalization problems because GA is a global optimization technique and it is able to find the global optimum solution without being trapped in local minima. In adaptive signal processing, GA was applied to the weight training of the neural networks[2], the parameter estimation of linear and nonlinear adaptive filters[3] and a maximum likelihood equalizer for multipath channel equalization[4].

But GA has the limitations of poor performance in searching for local solution[1]. To overcome this convergence problem in local solution, the hybrid structure which combines GA and LMS algorithm in system identification was proposed by S. H. Han and et al[5]. Its convergence rate is still not satisfying in multipath channel equalization. Besides still slow convergence, it can not lower its minimum MSE because fluctuations around minimum MSE of LMS algorithm induce big excessive MSE and they may hinder fine searching for optimum local solution.

IT-LMS algorithm introduced in [6] has fast convergence and simple updating mechanism. It can be a preferable candidate for fine local tuning after global search by GA. The reason for investigating this method is that it provides lower excess MSE than LMS algorithm as well as faster convergence. In this work we show that it has lower excess MSE and by computer simulations the proposed hybrid structure has desirable minimum MSE for multi-path channel equalization.

This paper is organized in the following manner. Section 2 presents GA for multi-path equalization. Section 3 introduces IT-LMS algorithm briefly and presents its excess MSE. In section 4 the proposed combined structure is explained and experimental results and discussions are presented.

Ⅱ. Multipath Channel and Equalizers

In Fig. 1 a block diagram of a typical digital communication system is shown. We can consider the whole system between the data source and the receiver

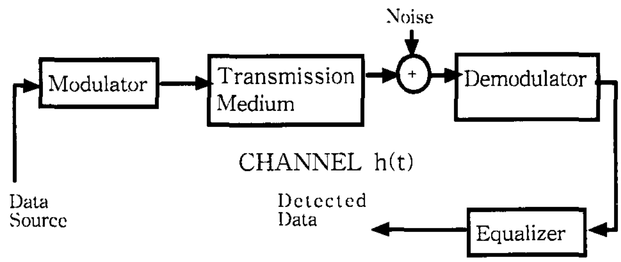

Fig. 1. Typical digital communication system.

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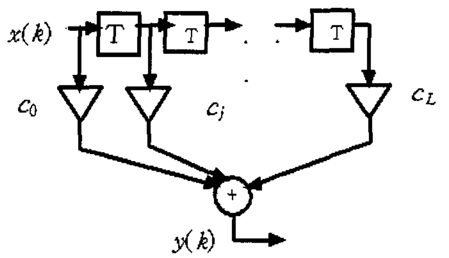

Fig. 2. TDL equalizer structure.

as a discrete channel with additive noise. the channel impulse response is denoted by $h(t)$.

At the receiver the filtered signal, which is distorted by multipath fading and corrupted by additive noise, $w(t)$. The transfer function of the multipath channel with M elements can be written as

$$H(z)=\sum_{i=0}^{M-1} h_i\, z^{-i} \qquad (1)$$

The equalizer input is sampled every T seconds and this sampled signal, $x(k)$, is presented to the TDL (Tapped Delay Line) equalizer. The output, $y(k)$, which is to be a good approximation to the transmitted symbol $d(k)$. The equalizer input is given by

$$x(k)=\sum_i h_i d(k-i)+w(k) \qquad (2)$$

In the equation (2) $w(k)$ is additive white gaussian noise. The output sample $y(k)$ at time k of the TDL equalizer is

$$y(k)=\sum_{i=0}^{L} c_i(k)x(k-i) \;=C(k)^T X(k) \qquad (3)$$

where

$$X(k)=[x(k),x(k-1),\ldots,x(k-L)]^T$$

$$C(k)=[c_0(k),\ldots,c_i(k),\ldots,c_L(k)]^T$$

$c_i(k)$ is the i-th tap coefficient

For simplicity, we assume all the values are real. The error related to this symbol becomes

$$e(k)=d(k)-y(k) \qquad (4)$$

Ⅲ. Genetic Algorithm in TDL Equalizer

GA consists of three processes: fitness evaluation, selection, crossover and mutation emulating the evolutionary behavior of biological systems. It starts with a set of equalizer tap coefficient vectors as parent chromosomes. The number of randomly generated initial coefficient vectors, M, are correspondent to a population of size M. In the fitness evaluation session, a fitness value for j-th chromosome is defined as

$$MSE_j=\frac{1}{Block}\sum_{i=0}^{Block-1}[d(k-i)-y_j(k-i)]^2 \qquad (5)$$

Block is the number of accumulated errors, $d(k)$ is the desired output and $y_j(k)$ is the estimated output for j-th chromosome. In the selection process, the best chromosome having the smallest fitness value becomes the best parent from the current population. Here, the best is copied to the TDL as a current filter coefficient vector $C(k)$.

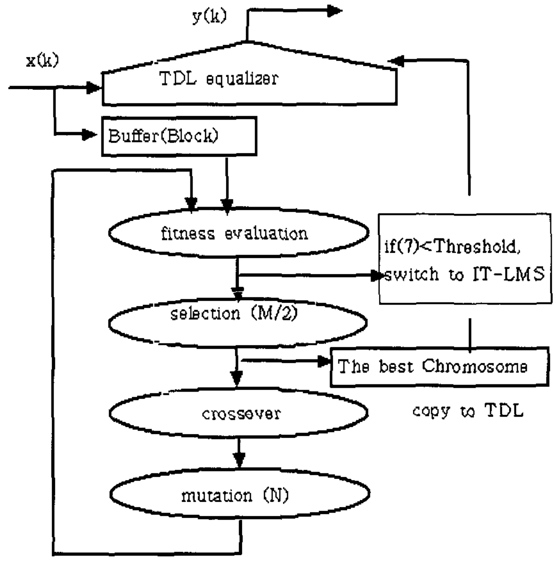

Fig. 3. GA in TDL equalizer.

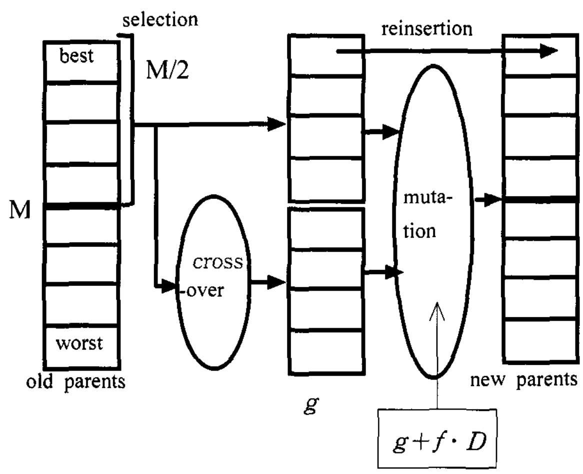

Fig. 4. Crossover and mutation process.

From the M parents which are in order of value from the best to the worst, the first half, M/2 "good" parents are selected as qualified parents for next mutation process but the remaining M/2 "bad" parents are discarded.

The first half, M/2 "good" parents go through crossover process[7] and resulting M/2 parents are placed in the room of M/2 vacant places which were for the M/2 "bad" parents. That is, the selected M/2 good parents and their crossover processed M/2 versions, total M parents, are to undergo the following mutation session.

The intention of the mutation operator is the introduction of some extra variability into the population. A mutation feature is to guard against premature convergence and to ensure that all solutions in the search space can ultimately be reached. We use the random mutation method[8],[9]. With the help of mutation rate, f, and the independent, identically N(0, 1)-distributed random number D, the offspring vector g in Fig. 4 becomes new parents as depicted in (6).

$$new\ parents = g + f \cdot D \tag{6}$$

The mutation process is applied to M-1 chromosomes in stead of M, because the best chromosome is reinserted to the new parents group. In the case that the chromosomes in the current population are completely replaced by the offsprings, one can argue that this strategy may make the best chromosome of the population fail to reproduce offspring in the next generation[10]. So it is usually combined with elitist strategy where one or a few of the best chromosomes are reinserted into the succeeding generation. The strategy may increase the speed of domination of a population by a super chromosome, but on balance it appears to improve the performance[11]. All these processes are performed when an input sample is arrived at the equalizer. The new parents go through fitness and selection process, and the best is copied to the TDL equalizer as an updated filter coefficient vector, $C(k+1)$.

Ⅳ. Transfer from GA to IT-LMS

Using the input past Block samples, the fitness value of the j-th chromosome is evaluated by (5). It is also time averaged MSE of the j-th chromosome. First we use the GA for a global search.. By comparing the best chromosome's fitness value, i.e., error performance,

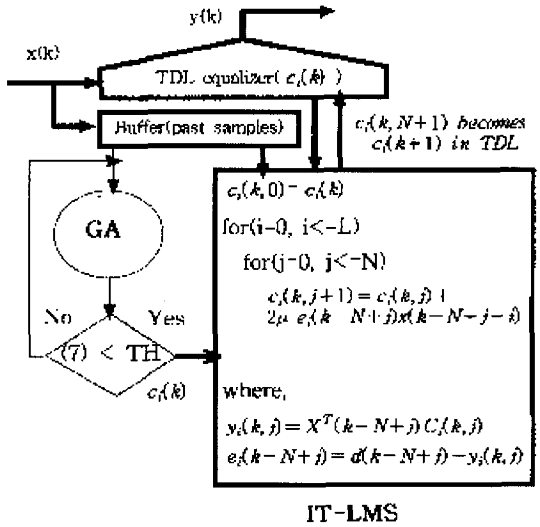

Fig. 5. GA-IT-LMS algorithm.

with a threshold, TH, we decide if we transfer the current coefficient update algorithm, GA, to IT-LMS algorithm for fine tuning.

$$MSE_j = \frac{1}{Block} \sum_{i=0}^{Block-1} [d(k-i) - y_j(k-i)]^2, \quad j = \text{The best chromosome} \tag{7}$$

If current MSE is smaller than a fixed value, then we turn the best chromosome, $C(k)$, obtained by the GA, which will be near the optimum solution, to IT-LMS algorithm as it's initial coefficients for fine search, and IT-LMS gains control of filter coefficient updating. For convenience we will call it GA-IT-LMS.

Letting i-th coefficient distance $v_i(k)$ be $v_i(k) = c_i(k) - c_i^o(k)$ where $c_i^o(k)$ is the i-th optimum coefficient, MSE, E[$e(k)^2$] can be expressed in terms of the coefficient distance vector $V(k)$.

$$MSE = MSE_{\min} + V(k)^T R\, V(k) \tag{8}$$

where $V(k) = [v_0(k), v_1(k), \ldots, v_L(k)]^T$,

$R = E[X(k)X(k)^T]$ is input correlation matrix and E[ ] denotes the ensemble averaging operation. One of its element $r(i-l)$ is defined as $r(i-l) = E[x(k-i)x(k-l)]$. The input correlation matrix R is real, symmetric and positive definite. Arrangement of this equation in terms of $v_i(k)$ gives the following equation.

$$MSE(v_i(k)) = A\, v_i^2(k) + 2B\, v_i(k) + F \tag{9}$$

where $A = r(0)$, $B = \sum_{l=0, l \neq i}^{L} v_l(k) r(l-i)$ and

$$F=\sum_{l=o,l\neq i}^{L}\sum_{j=o,j\neq i}^{L} v_l(k)v_j(k)r(l-j) + MSE_{\min}$$

Equation (9) shows that MSE is a parabolic function of each tap coefficient in one dimensional space; i.e., when one tap changes while all others are kept constant, the locus of MSE becomes a parabolic function in terms of the i-th coefficient. This locus, which is unique for each tap coefficient at each state of the channel, is called the characteristic function of that tap coefficient[12].

Based on these characteristics, an iterative algorithm for adjusting the equalizer coefficients one by one can be developed. The algorithm uses the steepest descent method to update the i-th tap coefficient, holding all other coefficients constant. The process continues for the other tap coefficients at time k and all the L+1 tap coefficients are updated. As the time k increases, the coefficient vector will be approach the Wiener optimum $C^o(k)$.

The gradient of $v_i(k)$ with all others fixed is presented in equation (10),

$$\begin{aligned}\frac{\partial MSE}{\partial v_i} &= 2E[x(k-i)x(k-i)](c_i(k)-c_i^O) \\ &+2\sum_{n=0,n\neq i}^{L}E[x(k-n)x(k-i)](c_n(k)-c_n^O) \\ &=2E[x(k-i)(\sum_{n=0}^{L}x(k-n)c_n(k)-\sum_{n=0}^{L}x(k-n)c_n^o)] \\ &=-2E[x(k-i)e_i(k)] \end{aligned} \quad (10)$$

In updating the i-th tap coefficient while holding all others fixed, the steepest descent using the measured or estimated gradient, $-2x(k-i)e_i(k)$, can be used. This is known as an implementation of the steepest descent using the measured or estimated gradient[13]:

$$new\ c_i(k)=c_i(k)+2\mu\ e_i(k)x(k-i) \quad (11)$$

where $e_i(k)$ is calculated when all other coefficients are fixed. The parameter μ is a step size (convergence factor) that controls stability and rate of adaptation.

Using the samples taken from the TDL and some past samples, the i-th tap coefficient can be updated N+1 times at sample time k. Introducing index j, (11) can become (12).

$$c_i(k,j+1)=c_i(k,j)+2\mu\ e_i(k-N+j)x(k-N+j-i)$$
$$0\le j\le N,\quad c_i(k,N+1)=c_i(k+1) \quad (12)$$

Continuing this process from tap $i=0$ to tap $i=L$, all the tap coefficients are updated one by one and the output sample $y(k)$ is made from the TDL equalizer. For vector representations, a new input vector $X(k-N+j)$ and coefficient vector $C_i(k,j)$ are defined as

$$X(k-N+j)=[x(k-N+j),x(k-N+j-1),\ldots,x(k-N+j-L)]^T \quad (13)$$

$$C_i(k,j)=[c_o(k,j),\ldots,c_i(k,j),\ldots,c_L(k,j)]^T \quad (14)$$

where only $c_i(k,j)$ changes.

Then the temporary output $y_i(k,j)$ and error $e_i(k-N+j)$ during $c_i(k,j)$ adaptation are expressed as

$$y_i(k,j)=X^T(k-N+j)C_i(k,j) \quad (15)$$

$$e_i(k-N+j)=d(k-N+j)-y_i(k,j) \quad (16)$$

Also IT-LMS algorithm (12) can be expressed as the following vector form.

$$C_i(k,j+1)=C_i(k,j)+2\mu(d(k-N+j)-X^T(k-N+j)C_i(k,j))\begin{bmatrix}0\\ \vdots\\ 0\\ x(k-N+j-i)\\ 0\\ \vdots\\ 0\end{bmatrix} \quad (17)$$

The operation of the algorithm at time k can be summarized as equation (12), (15) and (16). This is depicted in Fig. 5.

V. MSE Performance of IT-LMS

The difference of excess MSE between LMS and IT-LMS, Δexcess MSE, can be expressed in equation (18) in terms of eigen value λ_i of input correlation matrix $R=E[X(k)X(k)^T]$ and convergence parameter μ[14].

$$\begin{aligned}\Delta\text{excess MSE} &= \text{excess MSE of LMS} - \text{excess MSE of IT-LMS} \\ &= \mu\ MSE_{\min}\ [\sum_{i=0}^{L}\frac{\lambda_i}{1-\mu\lambda_i}-\sum_{i=0}^{L}\frac{\lambda_i}{1-\mu r_{(0)}}]\end{aligned} \quad (18)$$

Because the sum of eigen values λ_i of R is equal to the sum of diagonal elements $r_{(0)}$ of R, $r_{(0)}$ becomes λ_{av}, averaged value of λ_i. So, equation (18) can be reduced to the following.

$$\Delta\text{excess MSE} = \mu\ MSE_{\min}\ \frac{\mu}{1-\mu\lambda_{av}}\sum_{i=0}^{L}\frac{\lambda_i^2-\lambda_i\lambda_{av}}{1-\mu\lambda_i} \quad (19)$$

Convergence condition for both algorithm, $0 < \mu < 1/\lambda_{max}$, makes $1-\mu\lambda_i$ less than 1. Using this we can acquire the following inequality equation (20).

$$\sum_{i=0}^{L}\frac{\lambda_i^2-\lambda_i\lambda_{av}}{1-\mu\lambda_i} \ge \sum_{i=0}^{L}\lambda_i^2-\lambda_i\lambda_{av} \qquad (20)$$

Also, $$\sum_{i=0}^{L}\lambda_i^2-\lambda_i\lambda_{av}=\sum_{i=0}^{L}(\lambda_i-\lambda_{av})^2 \qquad (21)$$

The right term of the equation (21) is always greater than or equal to zero. This means that the equation (18) is greater than or equal to zero, that is, excess MSE of IT-LMS is always less than or equal to excess MSE of LMS.

Ⅵ. Simulation Results

GA lacks in fine local tuning capabilities but it has the advantage for global search. From our simulation results it has the capability to reach the optimum solution but the convergence is slow. The LMS algorithm compensate it with the advantage for local tuning and it has simple and efficient performance in adaptive equalizer applications. But its convergence speed is not acceptable for multi-path channel equalization either. Instead of LMS algorithm, we propose to use IT-LMS algorithm for local tuning.

Its performance has been investigated in multipath channel equalization applications through computer simulations and compared. For multipath channel environments we used discrete time-dispersive channels which are shown in [13]. The channel impulse response is

$$H_1(z)=0.26+0.93z^{-1}+0.26z^{-2} \qquad (22)$$

$$H_2(z)=0.304+0.903z^{-1}+0.304z^{-2} \qquad (23)$$

The spectral characteristics for the channels possess nulls as typical spectral characteristics of multipath fading channels[13]. The number of coefficient taps, L, for the TDL equalizer is 11 on the channel model. AWGN variance is 0.001 and SNR=30 dB.

The population size M of GA and GA-based algorithms is 30 in this simulation. The window size, Block, for fitness evaluation did not make big differences in performance and it was set to 10. From the results of our research mutation is important in multipath channel equalization applications unlike described in [4]. In the case of no mutation the MSE learning curve of the GA could not converge. In determining the mutation rate, we have considered 5

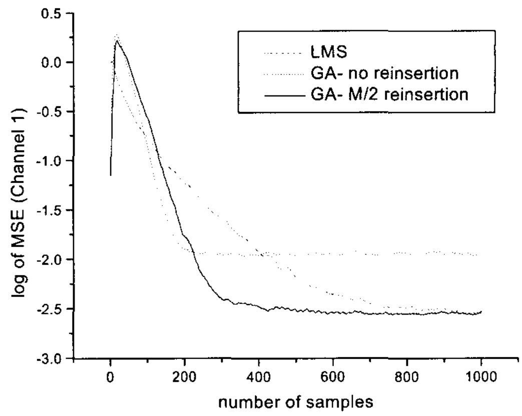

Fig. 6. Convergence characteristics of GA.

cases of mutation rate, D, such as 0(no mutation), 0.01, 0.03, 0.04 and 0.06. As increasing the mutation rate, the minimum MSE was degraded but the convergence speed was increased. The best choice of the mutation rate was 0.04. Though the number of block samples for fitness value evaluation did not give much influence to the performance, the number of reinsertion chromosome have affected performance of the GA. In our simulation results depicted in Fig. 6, it has the capability to reach the optimum solution when the number of reinserted chromosomes is M/2 but its convergence is slow.

If the current MSE is smaller than a fixed value, TH=0.01, then the best chromosome obtained by the GA is turned into IT-LMS algorithm for fine tuning of the equalizer coefficients.

In both simulation environments, for channel 1 and channel 2, the number of block samples N for multiple updating for IT-LMS algorithm is set to 0 in order to compare performances on the fare computational con-

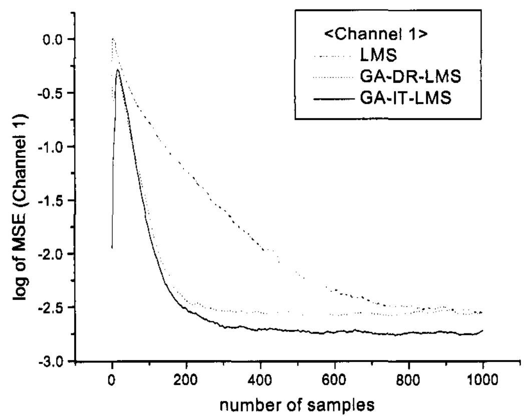

Fig. 7. MSE convergence comparison for $H_1(z)$.

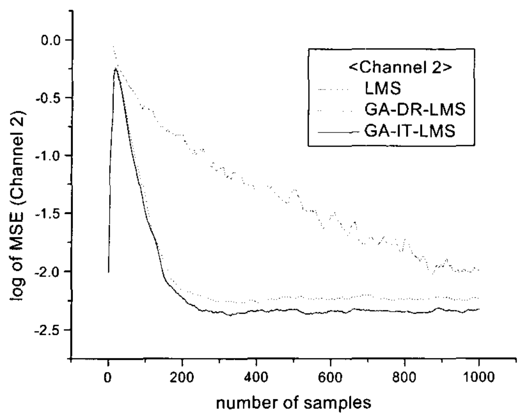

Fig. 8. MSE convergence comparison for $H_2(z)$.

ditions with LMS. In this case IT-LMS algorithm can be described as (11). The convergence parameter μ for the optimum solution of the LMS-type algorithms is set to 0.02.

In Fig. 7 for channel $H_1(z)$, the MSE convergence results for channel model are shown. The results show that the log values of MSE of LMS and GA-hybrid Data-Recycling-LMS (GA-DR-LMS)[15] have reached -2.6. We can see that the GA-hybrid type algorithms have fast convergence speed and when GA-IT-LMS is used, much lower MSE, -2.8 is acquired.

The MSE convergence results for severer channel model $H_2(z)$, are presented in Fig. 8. The log of MSE of LMS has not reached it's minimum MSE but GA-DR-LMS and GA-IT-LMS show fast convergence. Though GA-DR-LMS and GA-IT-LMS show similar convergence speed, GA-IT-LMS gives lower MSE, -2.4 in channel model 2.

Ⅶ. Conclusion

The proposed algorithm combines the advantage of GA for global searches and the advantage of IT-LMS algorithm which is rapid and having low excess MSE for local ones. Its performance has been investigated in multipath channel equalization applications through computer simulations and compared to the method that combines GA and DR-LMS algorithm. Though this method has as fast convergence as the proposed, it could not improve the excess MSE performance. The proposed GA-IT-LMS in which IT-LMS is applied to the combined GA equalizer, could yield much lower MSE.

The capabilities of global search, fast convergence and lower excess MSE of the proposed algorithm make us expect performance improvement when it is applied to multipath channel equalization.

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