A Study of a Nonlinear Viscoelastic Model for Elastomeric Bushing in Radial Mode

Seong Beom Lee[1,#], Jong-Keun Choi[2] and Je-Hong Min[3]

[1] School of Mechanical and Automotive Engineering, Inje University, Kimhae, South Korea
[2] Department of Mechanical Engineering, Gangwon Province University, South Korea
[3] Samsung Electronics Co., Ltd, Suwon, South Korea

ABSTRACT

An elastomeric bushing is a device used in automotive suspension systems to reduce the load transmitted from the wheel to the frame of the vehicle. The relation between the load applied to the shaft or sleeve and the relative displacement of elastomeric bushing is nonlinear and exhibits features of viscoelasticity. A load-displacement relation for elastomeric bushing is important for dynamic numerical simulations. A boundary value problem for the bushing response leads to the load-displacement relation, which requires complex calculations. Therefore, by modifying the constitutive equation for a nonlinear viscoelastic incompressible material developed by Lianis, the data for the elastomeric bushing material was obtained and this data was used to derive the new load-displacement relation for radial response of the bushing. After the load relaxation function for the bushing was obtained from the step displacement control test, Pipkin-Rogers model was developed. Solutions were allowed for comparison between the results of the modified Lianis model and those of the proposed model. It was shown that the proposed Pipkin-Rogers model was in very good agreement with the modified Lianis model.

Key Words: Elastomeric bushing, Pipkin-Rogers model, Nonlinear viscoelastic incompressible material

1. Introduction

Elastomeric bushings are structural elements which are used in automotive suspension systems. A bushing is, in effect, an elastomeric hollow cylinder placed between an outer steel cylindrical sleeve and an inner steel cylindrical rod. The steel sleeve is connected to components of the suspension system and used to transfer forces from the wheel to the chassis. The elastomeric material reduces the shock and vibration in this connection. Bushings exhibit a complicated relation between applied load, geometry of deformation, time, and other factors. Displacements and rotations about several axes are related to their corresponding forces and moments. The bushing material causes a nonlinear and time dependent relation between the corresponding loads and displacements.

The classical linear theory of viscoelasticity was first formulated by Boltzman[1]. His work covered the three-dimensional case for isotropic materials. Gross[2] reformulated the one-dimensional version of the theory in terms of material functions which can be obtained in experiment. Lianis and co-workers[3,4,5,6] carried out an extensive experimental program and completed Lianis constitutive equation for a styrene-butadiene rubber under isothermal conditions. Pipkin and Rogers[7] introduced a constitutive equation for nonlinear viscoelastic response of polymers. Using a modified superposition concept, they constructed a single integral model based on single step relaxation data. They outlined a procedure for improving the accuracy of the model by

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Corresponding Author:
Email: mechlsb@inje.ac.kr
Tell: +82-55-320-3667, Fax: +82-55-324-1723

including multi-step relaxation data. They also extended this model to a full three-dimensional setting.

A standard approach used in applied mechanics to develop a load-displacement relation was used with a three-dimensional constitutive equation for the elastomeric bushing material, which relates the stresses and local deformation under a broad range of deformation and time conditions. This would then be combined with the principles and methods of applied mechanics to solve appropriate boundary value problems which represent the response of the bushing. The modified Lianis model which represents the relation between strain and pressure form of load was proposed and one-dimensional radial step displacement control simulations were considered to obtain pre-data.

Because of the nonlinearity of response, the standard approach in applied mechanics, however, does not result in an explicit mathematical relation between load and displacement. If a load or displacement is specified, the other quantity is determined by solving a boundary value problem with a lengthy time-intensive calculation. Because this process is not satisfactory for dynamic simulation, an explicit relation between load and displacement is needed. The purpose of the proposed research is to study a method for constructing such an explicit relation between load and displacement for elastomeric bushing in radial mode.

2. Constitutive Equation and Nonlinear Viscoelastic Model

2.1 The Modified Lianis Model

Lianis and co-workers[3,4,5,6] carried out an extensive experimental program to determine the specific form of the single integral constitutive equation for styrene-butadiene rubber under isothermal conditions. They made a final form of the constitutive equation by a process as "involving trial and error, cross plotting and curve fitting over a wide range of uniaxial and biaxial relaxation data".

Lee[8,9,10] performed computational simulations with Lianis constitutive equation for elastomeric bushing in axial and torsional mode. From these results, the modified Lianis model, which is the relation between load and strain for elastomeric bushing in radial mode, was proposed and shown as follows:

$$
\begin{aligned}
F(t)=&\left(a+\frac{b}{\left[1+k^2(r,t)\right]^2}+\left[3+k^2(r,t)\right]\left[c+dk^2(r,t)\right]\right)k(r,t)\\
&-\left[c+dk^2(r,t)\right]\left[2+k^2(r,t)\right]k(r,t)\\
&+2k(r,0^+)\left[P_0(t)+k^2(r,t)Q_0(t)\right]\\
&+2\int_{0^-}^{t}\left(P_0(t-s)\frac{\partial k(r,s)}{\partial s}\right)ds\\
&+2k^2(r,t)\int_{0^-}^{t}\left(Q_0(t-s)\frac{\partial k(r,s)}{\partial s}\right)ds\\
&+k(r,0^+)\left(P_1(t)+\frac{Q_1(t)}{\left[1+k^2(r,t)\right]^2}\right)\left[2+k(r,0^+)k(r,t)-k^2(r,t)\right]\\
&+2k(r,t)\int_{0^-}^{t}\left(P_1(t-s)k(r,s)\frac{\partial k(r,s)}{\partial s}\right)ds\\
&+\left[2-k^2(r,t)\right]\int_{0^-}^{t}\left(P_1(t-s)\frac{\partial k(r,s)}{\partial s}\right)ds\\
&+\frac{2k(r,t)}{\left[1+k^2(r,t)\right]^2}\int_{0^-}^{t}\left(Q_1(t-s)k(r,s)\frac{\partial k(r,s)}{\partial s}\right)ds\\
&+\frac{2-k^2(r,t)}{\left[1+k^2(r,t)\right]^2}\int_{0^-}^{t}\left(Q_1(t-s)\frac{\partial k(r,s)}{\partial s}\right)ds \qquad (1)
\end{aligned}
$$

,where, t denotes time, r is radius, $k(r,t)$ is strain, and $F(t)$ is load.

$P_0(t)$, $Q_0(t)$, $P_1(t)$, $Q_1(t)$, a, b, c, and d are material properties from experiments by Lianis and co-workers[6]. They were obtained by using linear extrapoation and interpolation. These material properties for styrene-butadiene rubber determined by Lianis can be used in Eq. (1) and are shown in Fig. 1.

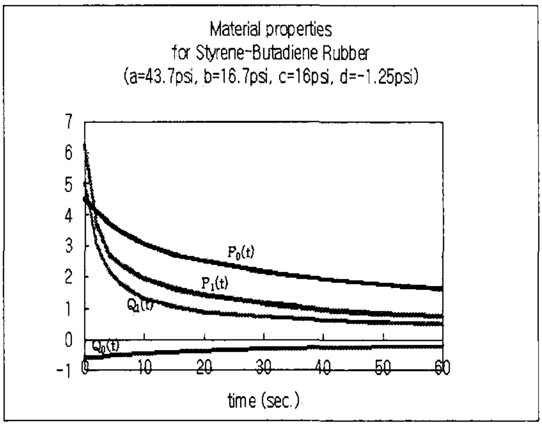

Fig. 1 Material properties for styrene-butadiene rubber

The relation between the radial displacement $u(r,t)$ and strain, $k(r,t)$ is as follows:

$$k(r,t) = \frac{\partial u(r,t)}{\partial r} \tag{2}$$

2.2 Nonlinear Viscoelastic Modeling

In this research, inertia and end effects are ignored and only radial mode deformations are considered. Reference and current configurations in radial mode with cylidrical coodinate system are shown in Fig. 2.

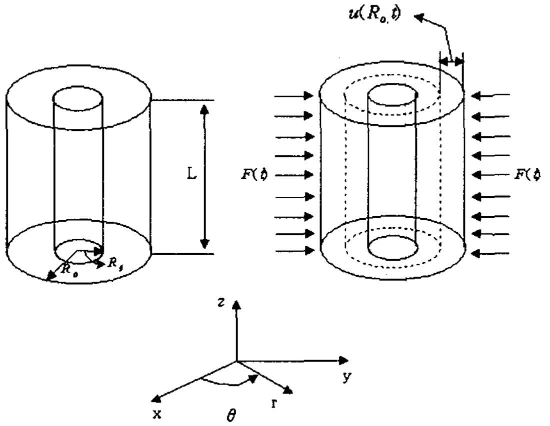

Fig. 2 Bushing in radial mode

The bushing is at rest in its reference configuration for times $t < 0$. For times $t \geq 0$, the outer sleeve is subjected to radial load $F(t)$, and undergoes radial displacement with respect to the inner rod. The nondimensionalized radial displacement at the outer radius is denoted by $u(R_0,t)$ and the cylindrical surface undergoes radial displacement during the deformation. Numerical simulations were carried out to $t = 60$ sec. For time interval, $\Delta t = 0.1$ sec. is used in carrying out the numerical simulations. The outer radius is three times larger than the inner radius, that is, the radius ratio between the inner and outer radius is three.

Given displacement histories, $u(R_0,t) = 0.1$, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0, are shown in Fig. 3 and the load outputs for the step displacment control tests are shown in Fig. 4. Load relaxation phenomena, which are the characteristics of viscoelastic material, are shown in Fig. 4.

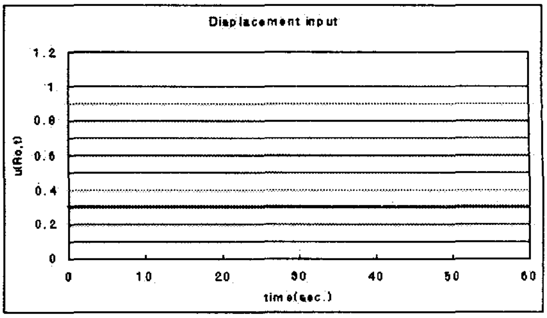

Fig. 3 Displacement input

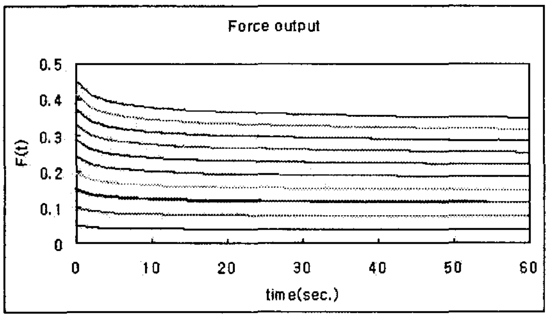

Fig. 4 Load output

Load output $F(t)$ in Fig. 4 can be regarded as load relaxation function $H(u,t)$ and can be expressed as follows:

$$H(u,t) = uG_1(t) + u^3 G_3(t) + u^5 G_5(t) \tag{3}$$

Eq. (3) shows that $H(u,t)$ is expressed explicitly by time and displacement. Load relaxation functions in Fig. 4 were used to get data for finding $G_1(t_a)$, $G_3(t_a)$, $G_5(t_a)$, which were determined to minimize the least-squares error and shown in Fig. 5.

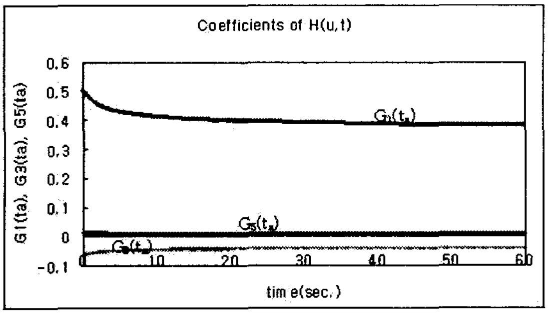

Fig. 5 Coefficients of $H(u,t)$

$G_1(t_a), G_3(t_a), G_5(t_a)$ can be represented by sums of exponential functions and are expressed as follows:

$$G_i(t) = C_{i1} + C_{i2}e^{-t/\tau_{i2}} + C_{i3}e^{-t/\tau_{i3}}, \qquad (4)$$
$$where \qquad i = 1, 3, 5.$$

The parameters, C_{ij} and τ_{ij} (i=1,3,5, j=2,3) were found by using the nonlinear least-squares method and $G_1(t), G_3(t), G_5(t)$ with these C_{ij} and τ_{ij} are shown in Fig. 6.

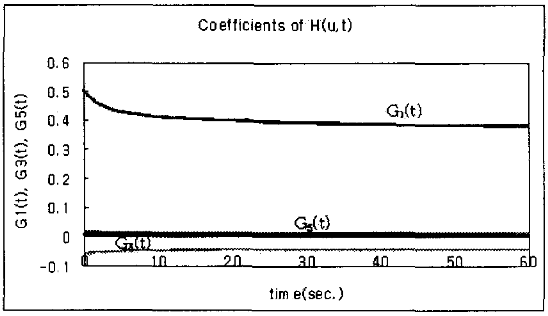

Fig. 6 Coefficients of $H(u,t)$

The relative error E_i is defined as follows:

$$E_i : Error \quad of \quad G_i(t) = \frac{\|G_i(t) - G_i(t_a)\|_2}{\|G_i(t_a)\|_2} \qquad (5)$$
$$\begin{pmatrix} i = 1, 3, 5 \\ a = 1, 2, 3, \cdots, 601 \end{pmatrix}$$

where i=1, 3, 5. From Eq. (5), the relative errors are E_1=0.13%, E_3=1.36%, and E_5=1.36%.

Finally, the complete form of the Pipkin-Rogers model is as follows:

$$F(t) = H(u(R_0,0),t) + \int_0^t \frac{\partial H(u(R_0,s),t-s)}{\partial u(R_0,s)} \frac{du(R_0,s)}{ds} ds \qquad (6)$$

$$H(u(R_0,t),t) = (C_{11} + C_{12}e^{-t/\tau_{12}} + C_{13}e^{-t/\tau_{13}})u(R_0,t) + (C_{31} + C_{32}e^{-t/\tau_{32}} + C_{33}e^{-t/\tau_{33}})\{u(R_0,t)\}^3 + (C_{51} + C_{52}e^{-t/\tau_{52}})\{u(R_0,t)\}^5$$

$C_{11} = 0.3770$, $C_{12} = 0.0536$, $C_{13} = 0.0706$,
$\tau_{12} = 22$, $\tau_{13} = 2.43$,
$C_{31} = -0.0410$, $C_{32} = -0.0169$, $C_{33} = -0.0032$,
$\tau_{32} = 6.2$, $\tau_{33} = 0.03$,
$C_{51} = 0.0093$, $C_{52} = 0.0042$, $\tau_{52} = 5$.

3. Predictive Quality of Pipkin-Rogers Model with the Modified Lianis Model

The complete form of Pipkin-Rogers model in the radial mode was shown in Eq. (6). This model was introduced as an approximation to the modified Lianis model. The purpose of this section is to compare Pipkin-Rogers model with the modified Lianis model. Both the modified Lianis model and Pipkin-Rogers model were used to calculate load response to specified displacement histories. The particular displacement input histories used for the modified Lianis and Pipkin-Rogers models are 'the constant rate displacement and recovery histories' and are expressed as follows:

$$u(R_0,t) \begin{cases} = 0.2t, & 0 \le t \le 5\,\text{sec}. \\ = -0.2(t-10), & 5 \le t \le 10\,\text{sec}. \\ = 0, & 10 \le t \le 60\,\text{sec}. \end{cases} \qquad (7)$$

Fig. 7 shows the load output from the displacement control test by using the modified Lianis model.

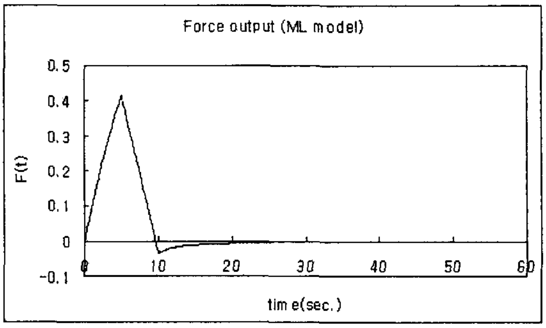

Fig. 7 Load output (ML model)

Fig. 8 shows the load output from the displacement control test by using Pipkin-Rogers model.

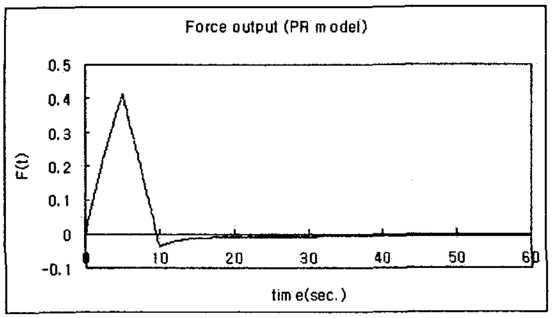

Fig. 8 Load output (PR model)

In Figs. 7 and 8, the relative error E is defined by

$$\frac{\| PR \quad load \quad output \; - ML \quad load \quad output \, \|_2}{\| ML \quad load \quad output \, \|_2}.$$

In Figs. 7 and 8, the relative error is 3.27%.

For another example, the particular displacement input histories used for the modified Lianis and Pipkin-Rogers models are as follows.

$$\begin{aligned} u(R_0,t) &= 0.1t, && 0 \le t \le 10\,\text{sec}. \\ &= -0.1(t-20), && 10 \le t \le 20\,\text{sec}. \\ &= 0, && 20 \le t \le 60\,\text{sec}. \end{aligned} \tag{8}$$

Fig. 9 shows the load output from the displacement control test by using the modified Lianis model.

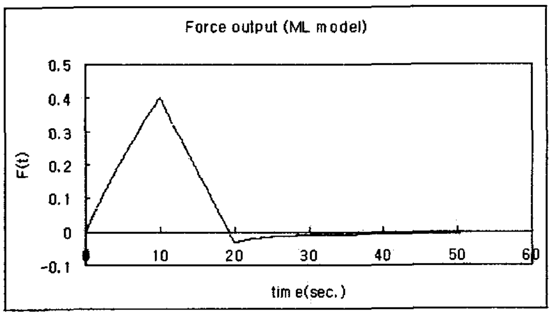

Fig. 9 Load output (ML model)

Fig. 10 shows the load output from the displacement control test by using Pipkin-Rogers model.

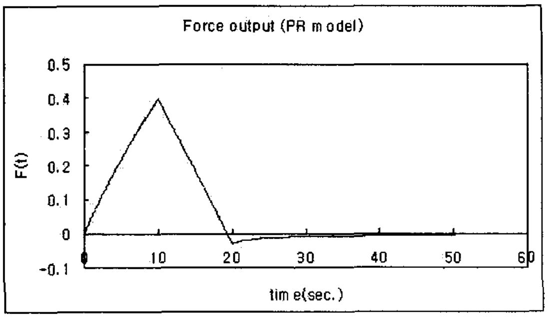

Fig. 10 Load output (PR model)

In Figs. 9 and 10, the relative error is 1.1%. Consequently, the relative errors are below 5%. Hence, Pipkin-Rogers model appears to provide satisfactory agreement with the modified Lianis model.

4. Conclusions

Two different load-displacement relations for radial mode response of an elastomeric bushing have been studied and compared. The first relation is implicit and defined by the equations of a boundary value problem used to simulate the exact bushing response. The second relation is explicit, approximate, and expressed in terms of a load relaxation property determined from the exact model. The results indicate that the proposed explicit load-displacement relation, Pipkin-Rogers model, gives a very good approximation to the exact response, the modified Lianis model.

The present study was carried out for one specific constitutive equation and comparisons were made for only a limited number of displacement histories. Consequently, the results are satisfactory over only a limited time range. This is acceptable for use in multi-body dynamics simulations involving short time intervals, such as when a tire hits a bump or a pot-hole.

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