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FOLDING THEORY OF IMPLICATIVE/FANTASTIC FILTERS IN LATTICE IMPLICATION ALGEBRAS

YOUNG BAE JUN AND SEOK ZUN SONG

ABSTRACT. We discuss the n-fold implicative/fantastic filters in lattice implication algebras, which are extended notions of implicative/fantastic filters. Characterizations of n-fold implicative/fantastic filters are given. Conditions for a filter to be n-fold implicative are provided. Extension property for an n-fold fantastic filter is established.

1. Introduction

Non-classical logic has become a considerable formal tool for computer science and artificial intelligence to deal with fuzzy information and uncertain information. Many-valued logic, a great extension and development of classical logic, has always been a crucial direction in non-classical logic. In order to research the logical system whose propositional value is given in a lattice, Xu [5] proposed the concept of a lattice implication algebra, which is an algebraic structure that is established by combining a lattice and an implication algebra, and discussed some of their properties. For the general development of lattice implication algebras, filter theory plays an important role. Xu and Qin [7] introduced the notion of (implicative) filters in a lattice implication algebra, and investigated their properties. In [4], Jun et al. introduced the concepts of a positive implicative filter and an associative filter in a lattice implication algebra, and gave a relation between a positive implicative filter, an associative filter and an implicative filter. One of the authors (Y. B. Jun) gave an equivalent condition of a filter, and provided some equivalent conditions for a filter to be an implicative filter in a lattice

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implication algebra, and discussed fantastic filters and n-fold implicative filters in a lattice implication algebra (see [1], [2], [3]).

In this paper, we introduce the concept of n-fold implicative/fantastic filters in lattice implication algebras, and investigate some of their properties. We give characterizations of n-fold implicative/fantastic filters, and provide conditions for a filter to be an n-fold implicative filter. We establish an extension property for an n-fold fantastic filter, and give a relation between a weak n-fold fantastic filter and an n-fold fantastic filter.

2. Preliminaries

DEFINITION 2.1. [5] A *lattice implication algebra* is defined to be a bounded lattice $(L; \vee, \wedge, 0, 1)$ with order-reversing involution "$'$" and a binary operation "$\rightarrow$" satisfying the following axioms:

(I1) $x \rightarrow (y \rightarrow z) = y \rightarrow (x \rightarrow z)$,
(I2) $x \rightarrow x = 1$,
(I3) $x \rightarrow y = y' \rightarrow x'$,
(I4) $x \rightarrow y = y \rightarrow x = 1 \Rightarrow x = y$,
(I5) $(x \rightarrow y) \rightarrow y = (y \rightarrow x) \rightarrow x$,
(L1) $(x \vee y) \rightarrow z = (x \rightarrow z) \wedge (y \rightarrow z)$,
(L2) $(x \wedge y) \rightarrow z = (x \rightarrow z) \vee (y \rightarrow z)$,

for all $x, y, z \in L$.

A lattice implication algebra L is called a *lattice H-implication algebra* if it satisfies $x \vee y \vee ((x \wedge y) \rightarrow z) = 1$ for all $x, y, z \in L$. We can define a partial ordering $\leq$ on a lattice implication algebra L by $x \leq y$ if and only if $x \rightarrow y = 1$.

EXAMPLE 2.2. [7] Let $L := \{0, a, b, c, 1\}$. Define the partial order relation on L as $0 < a < b < c < 1$, and define

$$x \wedge y := \min\{x, y\},\ x \vee y := \max\{x, y\}$$

for all $x, y \in L$ and "$'$"and "$\rightarrow$" as follows:

x	x'
0	1
a	c
b	b
c	a
1	0

$\rightarrow$	0	a	b	c	1
0	1	1	1	1	1
a	c	1	1	1	1
b	b	c	1	1	1
c	a	b	c	1	1
1	0	a	b	c	1

Then $(L, \vee, \wedge, \prime, \to)$ is a lattice implication algebra.

In the sequel the binary operation " $\to$ " will be denoted by juxtaposition. In a lattice implication algebra L, the following hold (see [5]):

(p1) $0x = 1$, $1x = x$ and $x1 = 1$.
(p2) $xy \leq (yz)(xz)$.
(p3) $x \leq y$ implies $yz \leq xz$ and $zx \leq zy$.
(p4) $x' = x0$.
(p5) $x \vee y = (xy)y$.
(p6) $((yx)y')' = x \wedge y = ((xy)x')'$.
(p7) $x \leq (xy)y$.

In a lattice H-implication algebra L, the following hold (see [6]).

(p8) $x(xy) = xy$.
(p9) $x(yz) = (xy)(xz)$.

DEFINITION 2.3. [7] A subset F of a lattice implication algebra L is called a *filter* of L if it satisfies

(f1) $1 \in F$,
(f2) $\forall x, y \in L,\ x \in F, xy \in F \ \Rightarrow\ y \in F$.

In particular, if we replace x by a special element $w \in L$ in (f2), then we say that F is a *w-filter* of L.

DEFINITION 2.4. [7] A subset F of a lattice implication algebra L is called an *implicative filter* of L if it satisfies (f1) and

(f3) $\forall x, y, z \in L,\ x(yz) \in F, xy \in F \ \Rightarrow\ xz \in F$.

In particular, if we replace x by a special element $w \in L$ in (f3), then we say that F is an *implicative w-filter* of L.

PROPOSITION 2.5. [1] *Every filter F of a lattice implication algebra L has the following property: $x \leq y$ and $x \in F$ imply $y \in F$.*

DEFINITION 2.6. [2] A nonempty subset F of a lattice implication algebra L is called a *fantastic filter* of L if it satisfies (f1) and

(f4) $\forall x, y, z \in L,\ z(yx) \in F, z \in F \ \Rightarrow\ ((xy)y)x \in F$.

Both an implicative filter and a fantastic filter are a filter (see [2, 7]).

3. n-fold implicative filters

In what follows, let n and L denote a positive integer and a lattice implication algebra, respectively, unless otherwise specified. For any elements x and y of L, let $x^n y$ denote $x(\cdots(x(xy))\cdots)$ in which x occurs n times, and $x^0 y = y$.

DEFINITION 3.1. [3] A nonempty subset F of L is called an *n-fold implicative filter* of L if it satisfies (f1) and

(f5) $\forall x, y, z \in L,\ x^n(yz) \in F,\ x^n y \in F \ \Rightarrow\ x^n z \in F.$

In particular, if we replace x by a special element $w \in L$ in (f5), then we say that F is an *n-fold implicative w-filter* of L.

Note that every n-fold implicative filter of L is a filter of L, but the converse is not true (see [3]). For any $a \in L$ let $A(a)$ denote the set of all elements of L which are greater than or equal to a, i.e.,

$$A(a) := \{x \in L \mid a \leq x\}.$$

Note that $1 \in A(a)$, and $A(a)$ is not a filter of L in general. For example, let $X = \{0, a, b, c, 1\}$ be a lattice implication algebra in Example 2.2. The set $A(b) = \{b, c, 1\}$ is not a filter of L because $ba = c \in A(b)$ but $a \notin A(b)$. Y. Xu and K. Y. Qin gave a condition for the set $A(a)$ to be a filter of L as in the following proposition.

PROPOSITION 3.2. *For any $a \in L$ the set $A(a)$ is a filter of L if and only if $a \leq x$ whenever $a \leq yx$ and $a \leq y$ for all $x, y \in L$.*

THEOREM 3.3. *Let w be an element of L. If $A(w)$ is an n-fold implicative filter of L, then for all $x, y \in L$,*

$$w^{n+1}(xy) = 1,\ w^{n+1}x = 1 \ \Rightarrow\ w^{n+1}y = 1. \tag{1}$$

PROOF. Suppose that $A(w)$ is an n-fold implicative filter of L and let $x, y \in L$ be such that $w^{n+1}(xy) = 1$ and $w^{n+1}x = 1$. Then $w^n(xy) \in A(w)$ and $w^n x \in A(w)$. Since $A(w)$ is an n-fold implicative filter of L, it follows from (f5) that $w^n y \in A(w)$ so that $w \leq w^n y$, that is, $w^{n+1}y = 1$. This completes the proof. □

In the following theorem, we provide a condition for $A(w)$ to be an n-fold implicative w-filter of L.

THEOREM 3.4. *Let w, x and y be elements of L that satisfies the condition* (1). *Then $A(w)$ is an n-fold implicative w-filter of L.*

PROOF. Let $y, z \in L$ be such that $w^n(yz) \in A(w)$ and $w^n y \in A(w)$. Then $w \leq w^n(yz)$ and $w \leq w^n y$, which imply that $w^{n+1}(yz) = 1$ and $w^{n+1}y = 1$. Using the condition (1), we get $w(w^n z) = w^{n+1}z = 1$, i.e., $w \leq w^n z$ and so $w^n z \in A(w)$. Therefore $A(w)$ is an n-fold implicative w-filter of L. □

Since every n-fold implicative w-filter is a w-filter (see [3, Theorem 3.3]), we have the following corollary.

COROLLARY 3.5. *Let w, x and y be elements of L that satisfies the condition* (1). *Then $A(w)$ is a w-filter of L.*

LEMMA 3.6. [3, Theorem 3.4] *Let F be a filter of L. Then the following are equivalent.*

(i) *F is an n-fold implicative filter of L.*
(ii) *$x^{n+1}y \in F$ implies $x^n y \in F$.*
(iii) *$x^n(yz) \in F$ implies $(x^n y)(x^n z) \in F$.*

Note that the trivial filter $\{1\}$ of L is not n-fold implicative (see [3]). We provide a condition for the trivial filter $\{1\}$ to be n-fold implicative.

THEOREM 3.7. *For every $a \in L$, if $A(a)$ is a filter of L then the trivial filter $\{1\}$ is n-fold implicative for every n.*

PROOF. Let $x, y \in L$ be such that $x^{n+1}y = 1$. Then $x \leq x^n y = x(x^{n-1}y)$, and so $x(x^{n-1}y) \in A(x)$. Since $x \in A(x)$, it follows from (f2) that $x^{n-1}y \in A(x)$, i.e., $x \leq x^{n-1}y$ and thus $x^n y = 1$. Hence, by Lemma 3.6, we conclude that $\{1\}$ is an n-fold implicative filter of L. □

COROLLARY 3.8. *If L satisfies the condition* (1) *for all $w, x, y \in L$, then the trivial filter $\{1\}$ of L is n-fold implicative.*

Combining Proposition 3.2 and Theorem 3.7, we have the following corollary.

COROLLARY 3.9. *If L satisfies the condition*

$$a \leq yx,\ a \leq y \Rightarrow a \leq x, \tag{2}$$

then $\{1\}$ is an n-fold implicative filter of L.

DEFINITION 3.10. [3] A lattice implication algebra L is said to be *n-fold implicative* if it satisfies the equality $x^{n+1}y = x^n y$ for all $x, y \in L$.

LEMMA 3.11. [3, Theorem 3.7] *A lattice implication algebra L is n-fold implicative if and only if the trivial filter $\{1\}$ is n-fold implicative.*

Using Theorem 3.7 and Lemma 3.11, we give a condition for a lattice implication algebra to be n-fold implicative.

THEOREM 3.12. (i) *For any $a \in L$, if $A(a)$ is a filter of L then L is n-fold implicative.*

(ii) *If L satisfies the condition* (1) *or* (2), *then L is n-fold implicative.*

LEMMA 3.13. [3, Theorem 3.10] *A lattice implication algebra L is n-fold implicative if and only if every filter is n-fold implicative.*

Combining Theorem 3.12 and Lemma 3.13, we obtain the following corollary.

COROLLARY 3.14. (i) *For any $a \in L$, if $A(a)$ is a filter of L, then every filter of L is n-fold implicative.*

(ii) *If L satisfies the condition* (1) *or* (2), *then every filter of L is n-fold implicative.*

For any nonempty subset F of L and $a \in L$, denote by $F_{a,n}$ the set $\{x \in L \mid a^n x \in F\}$. Note that if F is a filter of L, then $F_{1,n} = F$ and $1 \in F_{a,n}$. In general, $F_{a,n}$ is not a filter of L even if F is a filter of L. For example, let $L := \{0, a, b, c, 1\}$ be a lattice implication algebra in Example 2.2, and consider the trivial filter $\{1\}$ of L. Then $\{1\}_{c,2} = \{b, c, 1\}$ is not a filter of L since $ba = c \in \{1\}_{c,2}$ but $a \notin \{1\}_{c,2}$.

THEOREM 3.15. *If F is a filter of a lattice H implication algebra L, then the set $F_{a,n}$ is a filter of L for all $a \in L$.*

PROOF. Let $x, y \in L$ be such that $x \in F_{a,n}$ and $xy \in F_{a,n}$. Then $a^n x \in F$ and $(a^n x)(a^n y) = a^n(xy) \in F$, which imply that $a^n y \in F$, that is, $y \in F_{a,n}$. Hence $F_{a,n}$ is a filter of L. □

THEOREM 3.16. *Let F be a filter of L. If the set $F_{a,n}$ is a filter of L for all $a \in L$, then F is an n-fold implicative filter of L.*

PROOF. Let $x, y, z \in L$ be such that $x^n(yz) \in F$ and $x^n y \in F$. Then $yz \in F_{x,n}$ and $y \in F_{x,n}$. Since $F_{x,n}$ is a filter of L, it follows that $z \in F_{x,n}$, that is, $x^n z \in F$. Hence F is an n-fold implicative filter of L. □

Combining Theorems 3.15 and 3.16 induced the following corollary.

COROLLARY 3.17. *Every filter of a lattice H implication algebra is n-fold implicative.*

We know from [7, Proposition 8] that if every filter of L is implicative, then L is a lattice H implication algebra. We now pose an open problem.

OPEN PROBLEM 3.18. If there exists k $(\geq 2) \in \mathbb{N}$ such that every filter of L is k-fold implicative, then is L a lattice H implication algebra?

4. n-fold fantastic filters

DEFINITION 4.1. A nonempty subset F of L is called an *n-fold fantastic filter* if it satisfies (f1) and

(f6) $\forall x, y, z \in L,\ z(yx) \in F,\ z \in F \Rightarrow ((x^n y)y)x \in F$.

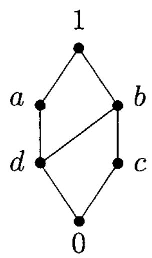
DEFINITION 4.2. A nonempty subset F of L is called a *weak n-fold fantastic filter* of L if it satisfies (f1) and

(f7) $\forall x, y, z \in L,\ z((y^n x)x) \in F,\ z \in F \Rightarrow (xy)y \in F$.

Putting $y = 1$ and $y = x$ in (f6) and (f7), respectively, and using (p1) and (I2), we know that every (weak) n-fold fantastic filter is a filter.

In Theorem 4.10, we will show that every n-fold fantastic filter is a weak n-fold fantastic filter.

EXAMPLE 4.3. Let $L := \{0, a, b, c, d, 1\}$ be a set with the following Hasse diagram as a partial ordering. Define a unary operation "$'$" and a binary operation "$\to$" as follows:



x	x'
0	1
a	c
b	d
c	a
d	b
1	0

$\to$	0	a	b	c	d	1
0	1	1	1	1	1	1
a	c	1	b	c	b	1
b	d	a	1	b	a	1
c	a	a	1	1	a	1
d	b	1	1	b	1	1
1	0	a	b	c	d	1

Define $\vee$- and $\wedge$-operations on L as follows:

$$x \vee y := (xy)y \quad \text{and} \quad x \wedge y := ((x'y')y')'$$

for all $x, y \in L$. Then L is a lattice implication algebra. It is easy to see that $F := \{a, 1\}$ is a 2-fold fantastic filter of L.

THEOREM 4.4. *Let F be a filter of L. Then*

(i) *F is an n-fold fantastic filter of L if and only if $((x^n y)y)x \in F$ for all $x, y \in L$ with $yx \in F$.*

(ii) *F is a weak n-fold fantastic filter of L if and only if $(xy)y \in F$ for all $x, y \in L$ with $(y^n x)x \in F$.*

PROOF. Assume that F is an n-fold fantastic filter of L and let $x, y \in L$ be such that $yx \in F$. Then $1(yx) = yx \in F$ and $1 \in F$. It follows from (f6) that $((x^n y)y)x \in F$. Conversely, let F be a filter of L such that $((x^n y)y)x \in F$ for all $x, y \in L$ with $yx \in F$. Let $x, y, z \in L$ be such that $z(yx) \in F$ and $z \in F$. Then $yx \in F$ by (f2), and hence $((x^n y)y)x \in F$ by assumption. Thus F is an n-fold fantastic filter of L. Similar argument induces the second part. □

THEOREM 4.5. (Extension property for an n-fold fantastic filter) *Let F and G be filters of L such that $F \subseteq G$. If F is n-fold fantastic, then so is G.*

PROOF. Let $x, y \in L$ be such that $yx \in G$. Setting $w = (yx)x$, then $yw = y((yx)x) = (yx)(yx) = 1 \in F$. Since F is n-fold fantastic, it follows from Theorem 4.4(i) and (I1) that

$$(yx)(((w^n y)y)x) = ((w^n y)y)((yx)x) = ((w^n y)y)w \in F \subseteq G,$$

which implies from (f2) that $((w^n y)y)x \in G$. Since $x \leq w$, we have $w^n y \leq x^n y$, and so $((w^n y)y)x \leq ((x^n y)y)x$. Using Proposition 2.5, we know that $((x^n y)y)x \in G$, and hence G is an n-fold fantastic filter of L by Theorem 4.4(i). □

COROLLARY 4.6. *Every filter of L is n-fold fantastic if and only if the filter $\{1\}$ is n-fold fantastic.*

Let F be a filter of L. We define a binary relation "$\sim$" on L as follows: For every $x, y \in L$, $x \sim y$ if and only if $xy \in F$ and $yx \in F$. Then "$\sim$" is a congruence relation on L. Denote $L/F := \{[x] \mid x \in L\}$, where $[x] := \{y \in L \mid x \sim y\}$, and define binary operations "$\sqcup$", "$\sqcap$", "$\rightharpoonup$" and unary operation "$\bullet$" on L/F as follows.

$$[x] \sqcup [y] = [x \vee y],\ [x] \sqcap [y] = [x \wedge y],\ [x] \rightharpoonup [y] = [xy],\ \text{and}\ [x]^\bullet = [x'],$$

respectively. Then $(L/F, \sqcup, \sqcap, \bullet, \rightharpoonup)$ is a lattice implication algebra (see [7]).

THEOREM 4.7. *A filter F of L is n-fold fantastic if and only if every filter of the quotient algebra L/F is n-fold fantastic.*

PROOF. Assume that F is an n-fold fantastic filter of L and let $x, y \in L$ be such that $[x] \rightharpoonup [y] = [1]$. Then $xy \in F$, and so $((y^n x)x)y \in F$ by Theorem 4.4(i). Hence

$$(([y]^n \rightharpoonup [x]) \rightharpoonup [x]) \rightharpoonup [y] = [((y^n x)x)y] = [1],$$

which proves that $\{[1]\}$ is an n-fold fantastic filter of L/F. By Corollary 4.6, every filter of L/F is n-fold fantastic. Conversely, suppose that every filter of L/F is n-fold fantastic and let $x, y \in L$ be such that $yx \in F$. Then $[y] \rightharpoonup [x] = [yx] = [1]$. Since $\{[1]\}$ is an n-fold fantastic filter of L/F, it follows from Theorem 4.4(i) that

$$[((x^n y)y)x] = (([x]^n \rightharpoonup [y]) \rightharpoonup [y]) \rightharpoonup [x] = [1],$$

that is, $((x^n y)y)x \in F$. Hence F is an n-fold fantastic filter of L by Theorem 4.4(i). □

A lattice implication algebra L is said to be *n-fold fantastic* if it satisfies the equality $((x^n y)y)x = yx$ for all $x, y \in L$.

Note that, in an n-fold fantastic lattice implication algebra, the notion of filters, n-fold fantastic filters, and weak n-fold fantastic filters coincide.

THEOREM 4.8. *The following are equivalent.*

(i) *L is n-fold fantastic.*
(ii) $(x^n y)y \le (yx)x$, $\forall x, y \in L$.
(iii) $x^n z \le yz$, $z \le x \Rightarrow y \le x$.
(iv) $x^n z \le yz$, $z \le x, y \Rightarrow y \le x$.
(v) $y \le x \Rightarrow (x^n y)y \le x$.

PROOF. (i) ⇔ (ii) If L is n-fold fantastic, then

$$((x^n y)y)((yx)x) = (yx)(((x^n y)y)x) = (yx)(yx) = 1,$$

that is, $(x^n y)y \le (yx)x$ for all $x, y \in L$. Conversely, assume that the inequality $(x^n y)y \le (yx)x$ holds for all $x, y \in L$. Then

$$(yx)(((x^n y)y)x) = ((x^n y)y)((yx)x) \ge ((x^n y)y)((x^n y)y) = 1,$$

and hence $(yx)(((x^n y)y)x) = 1$, i.e., $yx \le ((x^n y)y)x$. Now we have

$$(((x^n y)y)x)(yx) \ge y((x^n y)y) = (x^n y)(yy) = (x^n y)1 = 1,$$

and so $(((x^n y)y)x)(yx) = 1$, that is, $((x^n y)y)x \le yx$. Hence L is n-fold fantastic.

(ii) ⇒ (iii) Let $x, y, z \in L$ be such that $x^n z \le yz$ and $z \le x$. Using (I1), (ii) and (p3),we have

$$1 = (x^n z)(yz) = y((x^n z)z) \le y((zx)x) = y(1x) = yx,$$

and so $yx = 1$, i.e., $y \le x$.

(iii) ⇒ (iv) is trivial.

(iv) ⇒ (v) Let $x, y \in L$ be such that $y \le x$. Note that $y \le (x^n y)y$ and $x^n y \le ((x^n y)y)y$. It follows from (iv) that $(x^n y)y \le x$.

(v) ⇒ (ii) Since $x \le (yx)x$, we have $((yx)x)^n y \le x^n y$ by the mathematical induction. Since $y \le (yx)x$, it follows from (p3) and (v) that

$$(x^n y)y \le (((yx)x)^n y)y \le (yx)x.$$

This completes the proof. □

THEOREM 4.9. *A lattice implication algebra L is n-fold fantastic if and only if its trivial filter $\{1\}$ is n-fold fantastic.*

PROOF. The necessity is obvious. Assume that $\{1\}$ is an n-fold fantastic filter of L and let $w = (yx)x$ for every $x, y \in L$. Then

$$yw = y((yx)x) = (yx)(yx) = 1 \in \{1\},$$

and hence $((w^n y)y)w = 1$, that is, $(w^n y)y \leq w$ by Theorem 4.4(i). Now $x \leq w$ implies $w^n y \leq x^n y$, and so $(x^n y)y \leq (w^n y)y$. It follows that $1 = ((w^n y)y)w \leq ((x^n y)y)w$ so that $((x^n y)y)((yx)x) = ((x^n y)y)w = 1$, that is, $(x^n y)y \leq (yx)x$. Hence, by Theorem 4.8, L is an n-fold fantastic lattice implication algebra. □

THEOREM 4.10. *Every n-fold fantastic filter is a weak n-fold fantastic filter.*

PROOF. Let F be an n-fold fantastic filter of L. Then L/F is n-fold fantastic. Let $x, y \in L$ be such that $(y^n x)x \in F$. Then

$$\begin{aligned} [(xy)y] &= ([x] \rightharpoonup [y]) \rightharpoonup [y] \\ &\geq ([y]^n \rightharpoonup [x]) \rightharpoonup [x] \qquad \text{by Theorem 4.8} \\ &= [(y^n x)x] = [1], \end{aligned}$$

and so $[(xy)y] = [1]$, i.e., $(xy)y \in F$. It follows from Theorem 4.4(ii) that F is a weak n-fold fantastic filter of L. □

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Young Bae Jun
Department of Mathematics Education
Gyeongsang National University
Chinju (Jinju) 660-701, Korea
E-mail: ybjun@nongae.gsnu.ac.kr

Seok Zun Song
Department of Mathematics
Cheju National University
Cheju 690-756, Korea
E-mail: szsong@cheju.ac.kr