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SPECTRAL THEORY, TENSOR PRODUCTS AND INFINITE DIMENSIONAL HOLOMORPHY

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ABSTRACT. In this expository article we survey some recent developments on the construction of a holomorphic functional calculus for an infinite number of elements in a commutative unital Banach algebra.

1. Joint spectrum of a finite set of elements

Our general references for spectral theory are [2, 13], for tensor products [4] and for infinite dimensional holomorphy [5, 13].

An n-dimensional functional calculus consists in assigning to each n-tuple $\mathbf{a}$ of elements from a complex unital Banach algebra $\mathcal{A}$ a subset $\sigma(\mathbf{a})$ of C^n and constructing an algebra homomorphism $\theta_{\mathbf{a}}$

$$\theta_{\mathbf{a}} : \mathcal{H}(\sigma(\mathbf{a})) \longrightarrow \mathcal{A} \tag{1}$$

such that

$$\sigma(\theta_{\mathbf{a}}(f)) = f(\sigma(\mathbf{a})) \tag{2}$$

for all f in $\mathcal{H}(\sigma(\mathbf{a}))$ and

$$\theta_{\mathbf{a}}(z_i') = a_i \tag{3}$$

where z_i' is the i^{th} coordinate evaluation mapping on $C^n, 1 \leq i \leq n$.

We call $\sigma(\mathbf{a})$ the spectrum or joint spectrum of $\mathbf{a}$ while $\mathcal{H}(\sigma(\mathbf{a}))$ is the space of holomorphic germs on $\sigma(\mathbf{a})$. Many different spectra have been defined over the last sixty years but we confine ourselves in this survey to the classical definition given below and developed in the 1950's by R. Arens and A. P. Calderón [1], G. E. Shilov [15] and L. Waelbroeck [16] for n-tuples in a commutative unital Banach algebra and to generalizations of this theory to infinite sets of elements.

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From now on $\mathcal{A}$ will denote a commutative complex Banach algebra with identity $\mathbf{1}$ and set of non-zero multiplicative linear functionals, $\mathcal{S}(\mathcal{A})$, and $\theta_{\mathbf{a}}$ will denote a homomorphism which implements a functional calculus. Let $\mathbf{a} = (a_1, \ldots, a_n)$ denote an n-tuple of elements from $\mathcal{A}$. We let

$$\sigma(\mathbf{a}) = \{(h(a_1), \ldots, h(a_n)) : h \in \mathcal{S}(\mathcal{A})\} \tag{4}$$

and call $\sigma(\mathbf{a})$ the joint spectrum of $(a_1, \ldots, a_n)$. The set $\sigma(\mathbf{a})$ is a non-empty compact subset of C^n. An alternative equivalent definition using invertibility has the following form. A point $(\lambda_1, \ldots, \lambda_n)$ is not in $\sigma(\mathbf{a})$ if and only if there exists $(b_i)_{i=1}^n \subset \mathcal{A}$ such that

$$\sum_{i=1}^{n} b_i(a_i - \lambda_i \mathbf{1}) = \mathbf{1}. \tag{5}$$

If K is a compact subset of a Banach space X let $\mathcal{H}(K) = \bigcup_{K \subset U} \mathcal{H}(U) / \sim$ where U ranges over all open subsets of X containing K, $\mathcal{H}(U)$ is the space of holomorphic mappings from U into C and $f \sim g$ if there exists an open set W on which both f and g are defined and coincide. Elements of $\mathcal{H}(K)$ are called holomorphic germs on K. We obtain the same set of germs on replacing each $\mathcal{H}(U)$ by the Banach space $\mathcal{H}^\infty(U) := \{f \in \mathcal{H}(U) : \|f\|_U < \infty\}$. For U an open subset of X we let τ_ω denote the Nachbin topology [5] on $\mathcal{H}(U)$. We let $(\mathcal{H}(K), \tau_b) := \lim_{K \subset U}(\mathcal{H}(U), \tau_\omega) = \lim_{K \subset U} \mathcal{H}^\infty(U)$. We may also endow each $\mathcal{H}(U)$ with the compact open topology τ_0 and obtain a further locally convex topology on $\mathcal{H}(K)$, which we also denote by $\tau_0, \lim_{K \subset U}(\mathcal{H}(U), \tau_0)$. We have $\tau_b = \tau_0$ if and only if X is a finite dimensional Banach space.

We now briefly mention some further properties of the homomorphism $\theta_{\mathbf{a}}$. We first remark that $\theta_{\mathbf{a}}$ is a continuous mapping and by (1) and (3) we have for $\mathbf{a} = (a_1, \ldots, a_n)$ and any polynomial

$$\theta_{\mathbf{a}}\Big(\sum_m \alpha_m z_1^{m_1} \cdots z_n^{m_n}\Big) = \sum_m \alpha_m a_1^{m_1} \cdots a_n^{m_n} \tag{6}$$

where $m := (m_1, \ldots, m_n)$ is an n-tuples of non-negative integers and the summation is over a finite set. Since the polynomials may not form a dense subalgebra of $\mathcal{H}(\sigma(\mathbf{a}))$ the Hahn-Banach Theorem cannot be used to show that there is a unique homomorphism satisfying (1), (2) and (3). L. Waelbroeck [16, 17] showed, however, that the correspondence $\mathbf{a} \longrightarrow \theta_{\mathbf{a}}$ is the unique system on all finite tuples from $\mathcal{A}$ satisfying (1), (2), (3) and the following condition: if $T : \mathbf{C^n} \longrightarrow \mathbf{C^m}$ is a linear mapping then

$$\theta_{\mathbf{a}}(f \circ T) = \theta_{T(\mathbf{a})}(f) \tag{7}$$

for all $f \in \mathcal{H}(\sigma(T(\mathbf{a})))$. In 1979 W. B. Zame [19] showed, using results from several complex variables, that there is precisely one homomorphism satisfying (1), (2), and (3).

The mapping $\theta_{\mathbf{a}}$ commutes with each $h \in \mathcal{S}(\mathcal{A})$ in the following sense: if $f \in \mathcal{H}(K)$ and $\mathbf{a} = (a_1, \dots, a_n)$ then

$$h(\theta_{\mathbf{a}}(f)) = f(h(a_1), \dots, h(a_n)). \tag{8}$$

This gives the holomorphic spectral mapping theorem $\sigma(\theta_{\mathbf{a}}(f)) = f(\sigma(\mathbf{a}))$.

2. Waelbroeck's joint spectrum for an infinite set of elements

L. Waelbroeck [17] in 1971 was the first to consider the joint spectrum of an infinite set of elements in a way that led to a functional calculus for norm continuous holomorphic germs. To do so he used the projective tensor product of Banach spaces. The projective or π norm on the tensor product of normed linear spaces X and Y is defined by letting

$$\|\theta\|_\pi = \inf\{\sum_{i=1}^{k} \|x_i\| \cdot \|y_i\|; \theta = \sum_{i=1}^{k} x_i \otimes y_i\}.$$

If X and Y are Banach spaces and $\theta \in X \widehat{\bigotimes_\pi} Y$, the completion of $X \otimes_\pi Y$, then

$$\|\theta\|_\pi = \inf\{\sum_{i=1}^{\infty} \|x_i\| \cdot \|y_i\|; \theta = \sum_{i=1}^{\infty} x_i \otimes y_i\}.$$

If X is a Banach space, $\mathcal{A}$ is a commutative unital Banach algebra and $\mathbf{a} = \sum_{n=1}^{\infty} a_n \otimes x_n \in \mathcal{A} \widehat{\bigotimes_\pi} X$, Waelbroeck let

$$\sigma(\mathbf{a}) = \{\sum_{n=1}^{\infty} h(a_n)x_n; h \in \mathcal{S}(\mathcal{A})\}. \tag{9}$$

To see that this extends the definition in the previous section we note that $\mathcal{A}^n \simeq \mathcal{A} \otimes \mathbf{C^n}$ where the identification is given by

$$\mathbf{a} = (a_1, \ldots, a_n) \longrightarrow \sum_{i=1}^{n} a_i \otimes e_i$$

where $(e_i)_{i=1}^n$ is the standard unit vector basis for $\mathbf{C^n}$. We then identify $(h(a_1), \ldots, h(a_n))$ with $\sum_{i=1}^n h(a_i)e_i$ and obtain the finite dimensional joint spectrum. A more transparent representation, which lends itself to generalization, is obtained by using uniform tensor norms.

If X and Y are normed linear spaces let $\mathcal{L}(X;Y)$ denote the space of continuous linear mappings from X into Y endowed with the norm $\|T\| := \sup\{\|Tx\|_Y : \|x\|_X \leq 1\}$. If $Y = \mathbb{C}$ we write X' in place of $\mathcal{L}(X; \mathbf{C})$.

A *uniform cross – norm* or *tensor norm* γ is a method of assigning to each pair of normed linear spaces a norm $\|\cdot\|_\gamma$ on $X \bigotimes Y$ such that $X \bigotimes_\gamma Y := (X \bigotimes Y, \|\cdot\|_\gamma)$ has the following properties;

(a) $\|x \otimes y\|_\gamma = \|x\| \cdot \|y\|$ for all $x \in X, y \in Y$,

(b) if X_i and $Y_i, i = 1, 2$ are normed linear spaces and $T_i \in \mathcal{L}(X_i, Y_i)$, $i = 1, 2$ then $T_1 \otimes T_2 \in \mathcal{L}(X_1 \bigotimes X_2; Y_1 \bigotimes Y_2)$ and $\|T_1 \otimes T_2\| \leq \|T_1\| \cdot \|T_2\|$ ($T_1 \otimes T_2$ is the unique linear mapping from $X_1 \bigotimes X_2$ into $Y_1 \bigotimes Y_2$ satisfying

$$T_1 \otimes T_2(x_1 \otimes x_2) = T_1(x_1) \otimes T_2(x_2)$$

for all $x_1 \in X_1$ and $x_2 \in X_2$).

Since each $h \in \mathcal{S}(\mathcal{A})$ has norm 1 and the projective norm is uniform we can rewrite (9) as

$$\sigma(\mathbf{a}) = \{(h \otimes 1_X)(\mathbf{a}) : h \in \mathcal{S}(\mathcal{A})\} \tag{10}$$

where 1_X denote the identity mapping on X. Equation (10) shows that (9) is well defined–that is it does not depend on the tensor representation – and we take (10) as our definition of the spectrum of $\mathbf{a}$ for any $\mathbf{a} \in \mathcal{A}\widehat{\bigotimes}_\gamma X$ (the completion of $X \bigotimes_\gamma Y$) where γ is any uniform cross norm (see [6] for details). The spectrum $\sigma(\mathbf{a})$ is a non-empty compact subset of $\{x \in X; \|x\| \leq \|\mathbf{a}\|\}$.

Waelbroeck [17] defined $\theta_{\mathbf{a}}(f)$, $\mathbf{a} \in \mathcal{A}\widehat{\bigotimes}_\pi X$, by considering in turn the cases where f was (a) a polynomial (b) a holomorphic function of bounded type on an open ball (c) a holomorphic function on a product domain and (d) the pullback of a holomorphic function on a product domain. In this way he constructed a τ_b-continuous functional calculus for

$\theta_{\mathbf{a}}, \mathbf{a} \in \mathcal{A}\widehat{\bigotimes_{\pi}}X$, which satisfied and was uniquely determined by (1), (2), (3) and (7). Subsequently his results were extended in various directions by M. Chidami [3], M. C. Matos [10, 11] and J. M. Ortega [14]. J. M. Gale [9] showed that (1), (2) and (3) determined the restriction of $\theta_{\mathbf{a}}$ to the holomorphic germs which are weakly uniformly continuous on a neighbourhood of $\sigma(\mathbf{a})$. In a set of three papers [6, 7, 8] the authors considered the general problem of constructing the holomorphic functional calculus for $\mathbf{a} \in \mathcal{A}\widehat{\bigotimes_{\gamma}}X$ and addressed the problem of τ_0 continuity and uniqueness. In the following section we discuss the main result from [8] and highlight some aspects of this result which the technicalities in [8] tended to hide.

3. Recent extensions of the holomorphic functional calculus

To state the main result in [8] we require some definitions and, at the same time, we discuss an extension property which, for a large class of Banach spaces, turns out to be both necessary and sufficient for the existence of a unique τ_0 continuous holomorphic functional calculus.

For Banach spaces X and Y and n a positive integer let $\mathcal{P}(^nX;Y)$ denote the space of all continuous n-homogeneous polynomials from X into Y [5]. If $P \in \mathcal{P}(^nX;Y)$ we let $\overset{\vee}{P}$ denote the unique symmetric n-linear form associated with P. Given $P \in \mathcal{P}(^nX;Y), X, Y$ Banach spaces and $\mathcal{A}$ a commutative unital Banach algebra consider the $2n$-linear mapping

$$\begin{aligned} L : \mathcal{A}^n \times X^n &\longrightarrow \mathcal{A} \otimes Y \\ L(a_1, \ldots, a_n, x_1, \ldots, x_n) &:= a_1 \cdots a_n \otimes \overset{\vee}{P}(x_1, \ldots, x_n). \end{aligned}$$

The definition of tensor product and associativity imply that there exists a unique symmetric n-mapping $L_1 : (\mathcal{A} \otimes X)^n \longrightarrow \mathcal{A} \otimes Y$ such that $L_1(a_1 \otimes x_1, \ldots, a_n \otimes x_n) = a_1 \cdots a_n \otimes \overset{\vee}{P}(x_1, \ldots, x_n)$ for $a_i \in \mathcal{A}$ and $x_i \in X$ all i. We denote by $P_{\mathcal{A}}$ the n-homogeneous polynomial associated with L_1. Clearly $P_{\mathcal{A}}(a \otimes x) = a^n \otimes P(x)$ where $a \in \mathcal{A}$ and $x \in X$. If we identify X with $\mathbf{1} \otimes X$ then we may regard $P_{\mathcal{A}}$ as an extension of P.

If γ is a uniform cross-norm then $P_{\mathcal{A}}$ admits an extension (which is necessarily unique) to $\mathcal{A} \hat{\otimes}_\gamma X$ to define an element of

$$\mathcal{P}\left(^n(\mathcal{A} \hat{\otimes}_\gamma X); \mathcal{A} \hat{\otimes}_\gamma Y\right)$$

if and only if

$$\sup_{\substack{\mathbf{a}\in\mathcal{A}\otimes X\\ \|\mathbf{a}\|_\gamma\le 1}} \|P_{\mathcal{A}}(\mathbf{a})\| < \infty.$$

A polynomial is a finite sum of homogeneous polynomials and if $P := \sum_{j=0}^{n} P_j \in \mathcal{P}(X;Y)$, (the space of continuous polynomials from X to Y), $P_j \in \mathcal{P}({}^jX;Y)$, $0 \le j \le n$ we let $P_{\mathcal{A}} = \sum_{j=0}^{n} (P_j)_{\mathcal{A}}$. The mapping $P \longrightarrow P_{\mathcal{A}}$ is linear and an algebra homomorphism when the range space Y is a Banach algebra. If X is a Banach space with Schauder basis $(e_i)_i$ and $P \in \mathcal{P}(X)$ then we have the monomial expansions

$$P\left(\sum_{i=1}^{\infty} z_i e_i\right) = \sum_m \alpha_m z^m$$

and

$$P_{\mathcal{A}}\left(\sum_{i=1}^{\infty} a_i \otimes e_i\right) = \sum_m \alpha_m \boldsymbol{a}^m.$$

in the following cases

(a) $X = l_1, \gamma = \pi$ and $\mathcal{A}$ a commutative unital Banach algebra,

(b) $X = c_0, \gamma = \varepsilon$ and $\mathcal{A}$ a quotient of a unital uniform Banach algebra.

In case (b) the series is generally only conditionally convergent and a special ordering is necessary, we refer to [5, 7] for details. In the above expansion $\mathbf{a}^m = {a_1}^{m_1} \cdots {a_t}^{m_t}$ for $m := (m_1, \ldots, m_t) \in \mathbb{N}^t$.

If $\mathcal{A}$ is a commutative unital Banach algebra and γ is a uniform crossnorm we say that the Banach space X has the **$(\mathcal{A}, \gamma)$-extension property** if all $P \in \mathcal{P}(X)$ can be extended to $\mathcal{A}\, \hat{\otimes}_\gamma X$ and there exists $c > 0$ such that

$$\|P_{\mathcal{A}}\| \le c^n \|P\| \tag{11}$$

for all $P \in \mathcal{P}({}^nX)$ and all n.

A positive real number c which satisfies (11) is called a $(\mathcal{A}, \gamma)$-extension constant for X. All extension constants c are greater than or equal to 1 and [7, Example 10] shows that we may require $c > 1$.

Results in [7] show that any Banach space X has the $(\mathcal{A}, \pi)$-extension property for any Banach algebra $\mathcal{A}$ and the $(\mathcal{U}, \varepsilon)$-extension property for any uniform algebra $\mathcal{U}$ (ε as usual denotes the injective tensor norm). The $(\mathcal{A}, \gamma)$-extension property places uniform bounds on the norms of polynomial extensions and may be rephrased as a holomorphic extension.

If U is an open subset of a Banach space X, $f \in \mathcal{H}(U)$ and $x_0 \in U$ then there exists a sequence $(P_n)_{n=0}^{\infty}$ of polynomials, $P_n \in \mathcal{P}({}^nX)$ all n, and $\delta > 0$ such that

$$f(x_0 + y) = \sum_{n=0}^{\infty} P_n(y) \tag{12}$$

for all $y \in X$ for which $\|y\| < \delta$. The sequence $(P_n)_{n=1}^{\infty}$ is uniquely determined by f and x_0 and we use the notation

$$P_n := \frac{\hat{d}^n f(x_0)}{n!}$$

for all n. The expansion (12) is called the Taylor series expansion of f about x_0.

Let $\mathcal{H}_b(U)$ denote the subspace of $\mathcal{H}(U)$ consisting of all holomorphic mappings which are bounded on the bounded subsets of X which lie strictly inside U. Endowed with the topology τ_b of uniform convergence on these sets $\mathcal{H}_b(U)$ is a Fréchet space. If U is an open subset of a finite-dimensional space then $(\mathcal{H}_b(U), \tau_b) = (\mathcal{H}(U), \tau_0)$ is a Fréchet nuclear space.

PROPOSITION 1. *If $\mathcal{A}$ is a commutative unital Banach algebra, γ is a uniform crossnorm, X is a Banach space and each $P \in \mathcal{P}(X)$ can be extended to $\mathcal{A}\,\hat{\otimes}_\gamma X$ then X has the $(\mathcal{A},\gamma)$-extension property if and only if for each $f := \sum_{n=0}^{\infty} P_n \in \mathcal{H}_b(X)$ we have $f_{\mathcal{A}} := \sum_{n=0}^{\infty} (P_n)_{\mathcal{A}} \in \mathcal{H}_b(\mathcal{A}\,\hat{\otimes}_\gamma X)$.*

Proof. Since $f = \sum_{n=0}^{\infty} P_n \in \mathcal{H}_b(X)$ if and only if

$$\limsup_{n\longrightarrow\infty} \|P_n\|^{1/n} = 0$$

it is immediate that $f_{\mathcal{A}} \in \mathcal{H}_b(\mathcal{A}\,\hat{\otimes}_\gamma X)$ when X has the $(\mathcal{A},\gamma)$-extension property.

Conversely suppose $f_{\mathcal{A}} \in \mathcal{H}_b(\mathcal{A}\,\hat{\otimes}_\gamma X)$ whenever $f \in \mathcal{H}_b(X)$. If $\mathbf{a} = \sum_{i=1}^{t} \mathsf{a}_i \otimes x_i \in \mathcal{A} \otimes X$ and $P \in \mathcal{P}({}^mX)$ then

$$P_{\mathcal{A}}(\mathbf{a}) = \sum_{\substack{|m|=n \\ m\in\mathbb{N}^t}} \frac{n!}{m!} \overset{\vee}{P}(w^m)\boldsymbol{a}^m \tag{13}$$

where $\boldsymbol{a}^m = a_1{}^{m_1} \cdots a_t{}^{m_t}$ when $m := (m_1, \ldots, m_t) \in \mathbb{N}^t$. Hence the mapping

$$P \in \mathcal{P}(X) \longrightarrow \|P_{\mathcal{A}}(\mathbf{a})\|$$

defines a continuous semi-norm on $(\mathcal{P}(^nX), \|.\|)$. Since each $P \in \mathcal{P}(^nX)$ can be extended to $\mathcal{A}\,\hat{\otimes}_\gamma X$, $\|P_{\mathcal{A}}\| = \sup_{\substack{\mathbf{a}\in\mathcal{A}\otimes X\\ \|\mathbf{a}\|\leq 1}} \|P_{\mathcal{A}}(\mathbf{a})\| < \infty$, and as $(\mathcal{P}(^nX), \|.\|)$ is a Banach space, $P \longrightarrow \|P_{\mathcal{A}}\|$ defines a continuous semi-norm on $\mathcal{P}(^nX)$. Let $B_r(X) = \{x \in X : \|x\| < r\}$. Since $(\mathcal{H}_b(X), \tau_b)$ is a Fréchet space

$$f \longrightarrow |||f_{\mathcal{A}}|||_\rho := \sum_{n=0}^{\infty} \left\| \frac{\hat{d}^n f_{\mathcal{A}}(0)}{n!} \right\|_{B_\rho(X)}$$

defines a continuous semi-norm on $\mathcal{H}_b(X)$. Hence there exists $M > 0$ and $c > 0$ such that

$$|||f_{\mathcal{A}}|||_1 \leq M|||f|||_c$$

for all $f \in \mathcal{H}_b(X)$. If $f = P \in \mathcal{P}(^nX)$ this implies

$$\|P_{\mathcal{A}}\| \leq M \|P\|_{B_c(X)} = Mc^n \|P\|$$

and X satisfies the $(\mathcal{A}, \gamma)$-extension property. This completes the proof. □

A Banach space X has the bounded (respectively bounded projection) approximation property if there exists a bounded net $(T_\alpha)_\alpha$, $T_\alpha \in \mathcal{L}(X, X)$ of finite rank operators (respectively finite rank projections), that is $T_\alpha(X)$ is finite-dimensional for all α, such that $T_\alpha \longrightarrow I_X$ uniformly on compact subsets of X as $\alpha \longrightarrow \infty$. The bounded projection approximation property is strictly stronger than the bounded approximation property but since a separable Banach space has the bounded approximation property if and only if it is isomorphic to a complemented subspace of a Banach space with a Schauder basis we see that a separable Banach space is a complemented subspace of a Banach space with the bounded approximation property if and only if it is a complemented subspace of a Banach space with the bounded projection approximation property.

We are now in a position to state the main result from [8].

THEOREM 2. *If $\mathcal{A}$ is a commutative unital Banach algebra, γ is a uniform crossnorm and X is a Banach space with the $(\mathcal{A}, \gamma)$-extension property then for each $\mathbf{a} \in \mathcal{A}\,\hat{\otimes}_\gamma X$ there exists a continuous homomorphism*

$$\theta_{\mathbf{a}} : (\mathcal{H}(\sigma(\mathbf{a})), \tau_0) \longrightarrow \mathcal{A}$$

such that for all $h \in \mathcal{S}(\mathcal{A})$, $f \in \mathcal{H}(\sigma(\mathbf{a}))$, $P \in \mathcal{P}(X)$ and $x' \in X'$

$$h(\theta_{\mathbf{a}}(f)) = f([h \otimes I_X](\mathbf{a})), \tag{14}$$

$$\theta_{\mathbf{a}}(P) = P_{\mathcal{A}}(\mathbf{a}) \tag{15}$$

and

$$\theta_{\mathbf{a}}(x') = \left[1_{\mathcal{A}} \otimes x'\right](\mathbf{a}) = x'_{\mathcal{A}}(\mathbf{a}). \tag{16}$$

Moreover, if X is a complemented subspace of a Banach space with the bounded projection approximation property and the $(\mathcal{A},\gamma)$-extension property then

(a) *$\theta_{\mathbf{a}}$ is the unique τ_0-continuous homomorphism from $\mathcal{H}(\sigma(\mathbf{a}))$ into $\mathcal{A}$ satisfying* (14) *and* (16);

(b) *$\theta_{\mathbf{a}}$ is the unique τ_b-continuous homomorphism from $\mathcal{H}(\sigma(\mathbf{a}))$ into $\mathcal{A}$ satisfying* (14) *and* (15).

The existence of a τ_0 continuous functional calculus for arbitrary X is established in four stages and it is necessary to establish uniqueness in each of the first three stages in order to obtain existence at the final stage. However, the stronger results in (a) for the more restrictive class of Banach spaces avoids the most difficult technical step (the final one) in this general programme. We confine ourselves to a discussion of this case. We begin, however, by showing that the $(\mathcal{A},\gamma)$-extension property is necessary in order to obtain a τ_0 continuous functional calculus when the Banach space X has the bounded approximation property. We use the notation of Theorem 2.

PROPOSITION 3. *If X has the bounded approximation property and for all $\mathbf{a} \in \mathcal{A}\,\hat{\otimes}_\gamma X$ there exists a τ_0 continuous homomorphism $R_{\mathbf{a}}$ from $\mathcal{H}(\sigma(\mathbf{a}))$ into $\mathcal{A}$ which satisfies*

$$R_{\mathbf{a}}(x') = \left[1_{\mathcal{A}} \otimes x'\right](\mathbf{a}) = x'_{\mathcal{A}}(\mathbf{a}). \tag{17}$$

then X has the $(\mathcal{A},\gamma)$-extension property.

Proof. If $T : X \longrightarrow X$ is a finite rank operator and $P \in \mathcal{P}(X)$ then $P \circ T$ lies in the algebra generated by all $x' \in X'$. If $R_{\mathbf{a}}$ satisfies (16) then $R_{\mathbf{a}}(P \circ T) = (P \circ T)_{\mathcal{A}}(\mathbf{a}) = P_{\mathcal{A}}(T_{\mathcal{A}}(\mathbf{a}))$ for all $\mathbf{a} \in \mathcal{A}\,\hat{\otimes}_\gamma X$. Let $(T_\alpha)_\alpha$ denote a bounded net of finite rank operators from X to X which converges to the identity on compact subsets of X. Let U denote a bounded neighbourhood of $\sigma(\mathbf{a})$. Since $R_{\mathbf{a}}$ is τ_0-continuous there exists a compact subset K of U and $C > 0$ such that $\|R_{\mathbf{a}}(P)\| \le C\,\|P\|_K$ for all $P \in \mathcal{P}(X)$. Hence

$$\|P_{\mathcal{A}}((T_\alpha)_{\mathcal{A}}(\mathbf{a})) - R_{\mathbf{a}}(P)\| \le C\,\|P \circ T_\alpha - P\|_K \longrightarrow 0$$

as $\alpha \longrightarrow \infty$. Hence the mapping $P^{\mathcal{A}} : \mathbf{a} \in \mathcal{A}\,\hat{\otimes}_\gamma X \longrightarrow R_{\mathbf{a}}(P)$ is the pointwise limit of a bounded net of polynomials of bounded degree on the Banach space $\mathcal{A}\,\hat{\otimes}_\gamma X$ and $P^{\mathcal{A}} \in \mathcal{P}\left(\mathcal{A}\,\hat{\otimes}_\gamma X\right)$.

If $P \in \mathcal{P}(^nX)$ and $\mathbf{a} = \sum_{i=1}^t \mathsf{a}_i \otimes x_i \in \mathcal{A} \otimes X$ then

$$P_{\mathcal{A}}\left((T_\alpha)_{\mathcal{A}}(\mathbf{a})\right) = \sum_{\substack{|m|=n \\ m \in \mathbb{N}^t}} \frac{n!}{m!} \overset{\vee}{P}\left(T_\alpha(x)^m\right) a^m \longrightarrow P_{\mathcal{A}}(\mathbf{a})$$

as $\alpha \longrightarrow \infty$. Hence $P^{\mathcal{A}}(\mathbf{a}) = P_{\mathcal{A}}(\mathbf{a})$ for all $\mathbf{a} \in \mathcal{A} \otimes X$. Hence, each $P \in \mathcal{P}(X)$ can be extended to $\mathcal{A} \hat{\otimes}_\gamma X$, and

$$P^{\mathcal{A}}(\mathbf{a}) = P_{\mathcal{A}}(\mathbf{a}) = R_{\mathbf{a}}(P).$$

This shows that $R_{\mathbf{a}}$ satisfies (15).

We now show that X has the $(\mathcal{A}, \gamma)$-extension property. If $\mathbf{a} \in \mathcal{A} \hat{\otimes}_\gamma X$ then $\sigma(\mathbf{a}) \subset \{x : \|x\| \leq \|\mathbf{a}\|\}$. Since $R_{\mathbf{a}}$ is τ_0-continuous, our analysis so far shows that for all $\mathbf{a} \in \mathcal{A} \hat{\otimes}_\gamma X$ there exists $c(\mathbf{a}) > 0$ such that for every polynomial $P \in \mathcal{P}(X)$

$$\|P_{\mathcal{A}}(\mathbf{a})\| \leq c(\mathbf{a}) \|P\|_{B_{\|\mathbf{a}\|+1}(X)}. \tag{18}$$

Hence for all $\mathbf{a} \in \mathcal{A} \hat{\otimes}_\gamma X$, $\|\mathbf{a}\| = 1$, all n and all $P \in \mathcal{P}(^nX)$

$$\|P_{\mathcal{A}}(\mathbf{a})\| \leq c(\mathbf{a}) 2^n \|P\|. \tag{19}$$

For $c > 0$ let

$$\begin{aligned} V_c &= \big\{\mathbf{a} \in \mathcal{A} \hat{\otimes}_\gamma X : \|P_{\mathcal{A}}(\mathbf{a})\| \leq c^n \|P\| \|\mathbf{a}\|^n \\ &\qquad \text{for all } n \text{ and all } P \in \mathcal{P}(^nX)\big\}. \end{aligned}$$

By (19), $\bigcup_{m=1}^\infty V_m = \mathcal{A} \hat{\otimes}_\gamma X$. If $\mathbf{a}_j \in V_m$ for all j and $\mathbf{a}_j \longrightarrow \mathbf{a}$ as $j \longrightarrow \infty$ then for each $P \in \mathcal{P}(^nX)$

$$\|P_{\mathcal{A}}(\mathbf{a})\| = \lim_{j \longrightarrow \infty} \|P_{\mathcal{A}}(\mathbf{a}_j)\| \leq \lim_{j \longrightarrow \infty} m^n \|P\| \|\mathbf{a}_j\|^n = m^n \|P\| \|\mathbf{a}\|^n$$

and $\mathbf{a} \in V_m$. Hence each V_m is closed. By the Baire category theorem there exists m_0 such that V_{m_0} has non-empty interior. If $\mathbf{a}_0 + B_\delta\left(\mathcal{A} \hat{\otimes}_\gamma X\right) \subset V_{m_0}$ then, since V_{m_0} is balanced, [5, Lemma 1.10 (a)] implies $B_\delta\left(\mathcal{A} \hat{\otimes}_\gamma X\right) \subset V_{m_0}$. If $\|\mathbf{a}\| \leq \delta$ then

$$\|P_{\mathcal{A}}(\mathbf{a})\| \leq m_0^n \|P\| \|\mathbf{a}\|^n$$

and $\|P_{\mathcal{A}}\| \leq m_0^n \|P\|$ for all n and all $P \in \mathcal{P}(^nX)$. Hence X has the $(\mathcal{A}, \gamma)$-extension property. □

We now discuss the construction of $\theta_{\mathbf{a}}$ when $\mathbf{a} \in \mathcal{A} \hat{\otimes}_\gamma X$ and X has the bounded approximation property. If $(P_\alpha)_\alpha$ is a bounded null net in

$(\mathcal{P}(^nX), \|.\|)$ and $\epsilon > 0$ then the $(\mathcal{A}, \gamma)$-extension property and Lemma 1.10(c) in [5], which says

$$|P(x) - P(y)| \le \frac{n^n}{n!}(1+\varepsilon)^n \|x-y\| \|P\|$$

whenever $P \in \mathcal{P}(^nX)$, $\|y\| < \varepsilon$ and $\|x-y\| < 1$, allow us to choose $\mathbf{b} = \sum_{i=1}^t b_i \otimes w_i \in \mathcal{A} \otimes X$ such that for all α

$$\sup_\alpha \|(P_\alpha)_{\mathcal{A}}(\mathbf{a}) - (P_\alpha)_{\mathcal{A}}(\mathbf{b})\| < \varepsilon. \tag{20}$$

By (13)

$$(P_\alpha)_{\mathcal{A}}(\mathbf{b}) = \sum_{\substack{|m|=n \\ m\in\mathbb{N}^t}} \frac{n!}{m!} \overset{\vee}{P}_\alpha(w^m) b^m. \tag{21}$$

Let K denote the closed convex hull of the set $A := \{w_1, \ldots, w_t\}$. Since A is finite K is a compact subset of X. By the Polarization Formula

$$|\overset{\vee}{P}(w^m)| \le \frac{n^n}{n!} \|P_\alpha\|_K \tag{22}$$

for all α and m. Hence $(P_\alpha)_{\mathcal{A}}(\mathbf{b}) \longrightarrow 0$ as $\alpha \longrightarrow \infty$ and combining this with (20) we see that $\theta_{\mathbf{a}}(P_\alpha) = (P_\alpha)_{\mathcal{A}}(\mathbf{a}) \longrightarrow 0$ as $\alpha \longrightarrow \infty$. If c is an $(\mathcal{A}, \gamma)$-extension constant for X and $r > c\|\mathbf{a}\|$ then the above can be combined with Propositions 3.34(c) and 3.36(a) in [5] and the estimates in Proposition 1 to show that for $f := \sum_{n=0}^\infty P_n \in \mathcal{H}_b(B_r(X))$

$$f_{\mathcal{A}}(\mathbf{a}) := \sum_{n=0}^\infty (P_n)_{\mathcal{A}}(\mathbf{a}) \in \mathcal{A}$$

and the mapping $\theta_{\mathbf{a}} : f \in \mathcal{H}_b(B_r(X)) \longrightarrow f_{\mathcal{A}}(\mathbf{a}) \in \mathcal{A}$ is continuous with respect to the Fréchet topology on $\mathcal{H}_b(B_r)$ and with respect to the compact open topology on bounded sets. Since the algebra generated by X' is τ_0 dense in $\mathcal{P}(X)$ and $\mathcal{P}(X)$ is τ_b dense in $\mathcal{H}_b(B_r(X))$ the ideas in the proof of Proposition 3 can be modified to show that $\theta_{\mathbf{a}}$ is the unique homomorphism from $\mathcal{H}_b(B_r(X))$ into $\mathcal{A}$ satisfying $\theta_{\mathbf{a}}(x') = [1_{\mathcal{A}} \otimes x'](\mathbf{a}) = x'_{\mathcal{A}}(\mathbf{a})$ for all $x' \in X'$ with these continuity properties when X has the bounded approximation property.

The next extension of $\theta_{\mathbf{a}}$ is given in the following lemma. We sketch some points in the proof.

LEMMA 4. *Suppose $X = X_1 \oplus X_2$, has the $(\mathcal{A}, \gamma)$ extension property with extension constant c and $\dim(X_1) < \infty$. Let $\mathbf{a} = \mathbf{a}_1 \oplus \mathbf{a}_2$ where $\mathbf{a}_1 \in \mathcal{A} \hat{\otimes}_\gamma X_1$ and $\mathbf{a}_2 \in \mathcal{A} \hat{\otimes}_\gamma X_2$. If U_1 is an open neighbourhood of*

$\sigma(\mathbf{a}_1)$ in X_1 and $U_2 = B_r(X_2)$ where $r > c\|\mathbf{a}_2\|$ then there exists a unique continuous homomorphism

$$\theta_{\mathbf{a}} : \mathcal{H}_b(U_1 \oplus U_2) \longrightarrow \mathcal{A}$$

such that

1. *$h(\theta_{\mathbf{a}}(f)) = f([h \otimes I_X](\mathbf{a}))$ for all $h \in \mathcal{M}(\mathcal{A})$ and $f \in \mathcal{H}_b(U_1 \oplus U_2)$*
2. *$\theta_{\mathbf{a}}$ is continuous on each bounded subset of $\mathcal{H}_b(U_1 \oplus U_2)$ endowed with the compact open topology,*
3. *$\theta_{\mathbf{a}}(x') = [1_{\mathcal{A}} \otimes x'](\mathbf{a}) = x'_{\mathcal{A}}(\mathbf{a})$ for all $x' \in X'$.*

Proof. If U_1 is an open subset of a finite-dimensional Banach space X_1, U_2 is an open subset of a Banach space X_2 then the mapping

$$T : \mathcal{H}(U_1)\,\hat{\otimes}_\pi \mathcal{H}_b(U_2) \longrightarrow \mathcal{H}_b(U_1 \oplus U_2)$$

$[T(f \otimes g)](z,w) = f(z)g(w)$ (for $f \in \mathcal{H}_b(U_1)$, $g \in \mathcal{H}_b(U_2)$, $z \in U_1$ and $w \in U_2$) extends to define a linear and algebraic isomorphism (the product is given on elementary tensors by $(f_1 \otimes g_1)\cdot(f_2 \otimes g_2) := f_1 f_2 \otimes g_1 g_2$).

Let $\theta_{\mathbf{a}_1}$ denote the unique continuous homomorphism : $\mathcal{H}(U_1) \longrightarrow \mathcal{A}$ obtained from the finite dimensional holomorphic functional calculus (see section 1) and let $\theta_{\mathbf{a}_2}$ denote the continuous homomorphism $\theta_{\mathbf{a}_2}$ from $\mathcal{H}_b(U_2)$ into $\mathcal{A}$ constructed above. The homomorphism $\mathbf{a}$ is defined by letting

$$\theta_{\mathbf{a}} = [\theta_{\mathbf{a}_1} \otimes \theta_{\mathbf{a}_2}] \circ T^{-1} : \mathcal{H}_b(U_1 \oplus U_2) \longrightarrow \mathcal{A}.$$

Since $\theta_{\mathbf{a}_1}$, $\theta_{\mathbf{a}_2}$ and T^{-1} are continuous homomorphisms, $\theta_{\mathbf{a}}$ is also a τ_b-continuous homomorphism.

Since $\{\mathcal{H}(U_1)\,\hat{\otimes}_\pi \mathcal{P}(^n X_2)\}_{n=0}^{\infty}$ is an $\mathcal{S}$-absolute decomposition for $\mathcal{H}(U_1)\,\hat{\otimes}_\pi \mathcal{H}_b(U_2)$ [5, 8] and $\mathcal{H}(U_1)\,\hat{\otimes}_\pi \mathcal{P}(^n X_2) = \mathcal{P}(^n X_2; \mathcal{H}(U_1))$ the problem of showing that $\theta_{\mathbf{a}}$ satisfies condition (2) reduces to proving the following: if $(P_\alpha)_\alpha$ is a bounded null net in $\mathcal{H}(U_1)\,\hat{\otimes}_\pi \mathcal{P}(^n X_2)$ then $\theta_{\mathbf{a}}(P_\alpha) \longrightarrow 0$.

Let $\varepsilon > 0$ be arbitrary. Since $\theta_{\mathbf{a}_1}$ and $\theta_{\mathbf{a}_2}$ are continuous there exist K_1 compact in U_1, a positive number $\rho, 0 < \rho < r$ and $C > 0$ such that

$$\|\theta_{\mathbf{a}_1}(f)\| \leq C\|f\|_{K_1} \text{ for all } f \in \mathcal{H}(U_1)$$

and

$$\|\theta_{\mathbf{a}_2}(P)\| \leq C\|P\|_\rho \text{ for all } P \in \mathcal{P}(^n X_2).$$

Since $(P_\alpha)_\alpha$ is bounded the (BB)-property for the projective tensor product of a Fréchet nuclear space and a Banach space implies there exist bounded sets B_1 in $\mathcal{H}(U_1)$ and B_2 in $\mathcal{P}(^nX_2)$ such that for all α we have representations

$$P_\alpha = \sum_{i=1}^{\infty} \lambda_i^\alpha f_i^\alpha \otimes P_i^\alpha$$

where $\sum_{i=1}^{\infty} |\lambda_i^\alpha| \leq 1$, $f_i^\alpha \in B_1$ and $P_i^\alpha \in B_2$. Let $M = \sup_{i,\alpha} \|f_i^\alpha\|_{K_1}$. We can find $\mathbf{b} \in \mathcal{A}\otimes X_2$ such that

$$\|\theta_{\mathbf{a}_2}(P_i^\alpha) - \theta_{\mathbf{b}}(P_i^\alpha)\|_\rho \leq \frac{\varepsilon}{CM}$$

for all i and α. Let $\mathbf{b} = \sum_{i=1}^{t} b_i \otimes w_1$ where $b_i \in \mathcal{A}$ and $w_i \in X_2$. Hence

$$\begin{aligned}\|\theta_{\mathbf{a}}(P_\alpha) - [\theta_{\mathbf{a}_1} \otimes \theta_{\mathbf{b}}](P_\alpha)\| &= \|[\theta_{\mathbf{a}_1} \otimes \theta_{\mathbf{a}_2-\mathbf{b}}](P_\alpha)\| \\ &\leq \left\| \sum_{i=1}^{\infty} \lambda_i^\alpha \theta_{\mathbf{a}_1}(f_i^\alpha)\, \theta_{\mathbf{a}_2-\mathbf{b}}(P_i^\alpha) \right\| \\ &\leq \sum_{i=1}^{\infty} |\lambda_i^\alpha| \cdot C\, \|f_i^\alpha\|_{K_1} \cdot \frac{\varepsilon}{CM} \\ &\leq \varepsilon\end{aligned}$$

for all α. Let K_2 denote the closed convex hull of the set $A := \{w_1, \ldots, w_t\}$. Since A is finite K_2 is a compact subset of X_2. By the Polarization Formula we can find $C' > 0$ such that

$$\|\theta_{\mathbf{b}}(P)\| \leq C' \, \|P\|_{K_2}$$

for all $P \in \mathcal{P}(^nX_2)$. This implies

$$\begin{aligned}\|[\theta_{\mathbf{a}_1} \otimes \theta_{\mathbf{b}}](P_\alpha)\| &\leq \sum_{i=1}^{\infty} |\lambda_i^\alpha| \cdot \|\theta_{\mathbf{a}_1}(f_i^\alpha)\| \cdot \|\theta_{\mathbf{b}}(P_i^\alpha)\| \\ &\leq C' \sum_{i=1}^{\infty} |\lambda_i^\alpha| \cdot C\, \|f_i^\alpha\|_{K_1} \cdot \|P_i^\alpha\|_{K_2} .\end{aligned}$$

Since this holds for any representation nuclearity implies that both $\|[\theta_{\mathbf{a}_1} \otimes \theta_{\mathbf{b}}](P_\alpha)\|$ and $\|\theta_{\mathbf{a}}(P_\alpha)\|$ tend to 0 as $\alpha \longrightarrow \infty$. Hence $\theta_{\mathbf{a}}$ satisfies condition (2). Uniqueness follows from the finite dimensional uniqueness result of Zame [19] and the uniqueness already established for $\mathcal{H}(U_2)$. □

The final step, when X has the bounded projection property, depends on the following lemma.

Lemma 5. *Let X denote a Banach space with open unit ball B. If X has the bounded projection property and the $(\mathcal{A},\gamma)$ extension property with $(\mathcal{A},\gamma)$ and $\mathbf{a} \in \mathcal{A}\ \hat{\otimes}_\gamma X$ then there exists, for every $c > 1, \delta > 0$ such that, for every $\varepsilon > 0$, we have the following decompositions:*

1. $X = X_1(\varepsilon) \oplus X_2(\varepsilon), X_1(\varepsilon)$ *finite dimensional,*
2. $U(\varepsilon) := U_1(\varepsilon) \oplus U_2(\varepsilon), U_1(\varepsilon) \subset X_1(\varepsilon), U_2(\varepsilon) = r(B \cap X_2(\varepsilon))$,
3. $\mathbf{a} = \mathbf{a}_1 \oplus \mathbf{a}_2, \sigma(\mathbf{a}_1) \subset U_1(\varepsilon), c\|\mathbf{a}_2\| < r$,
4. $\sigma(\mathbf{a}) + \varepsilon B \subset U_1(\varepsilon) \oplus U_2(\varepsilon) \subset \sigma(\mathbf{a}) + \varepsilon\delta B$.

By Lemma 5 we can choose a strictly decreasing null sequence of positive numbers $(\varepsilon_n)_n$ such that $U_1(\varepsilon_n) \oplus U_2(\varepsilon_n) \subset U_1(\varepsilon_m) \oplus U_2(\varepsilon_m)$ for $n > m$. This implies

$$(\mathcal{H}(\sigma(\mathbf{a})), \tau_b) = \varinjlim_n \mathcal{H}_b(U_1(\varepsilon_n) \oplus U_2(\varepsilon_n)).$$

The uniqueness in Lemma 4 shows that we can combine the $\theta_{\mathbf{a}}$'s defined on $\varinjlim_n \mathcal{H}_b(U_1(\varepsilon_n) \oplus U_2(\varepsilon_n))$ to define a τ_b-continuous homomorphism $\theta_{\mathbf{a}}$ from $(\mathcal{H}(\sigma(\mathbf{a})))$ into $\mathcal{A}$ which is τ_0 continuous on the bounded subsets of $\mathcal{H}_b(U_1(\varepsilon_n) \oplus U_2(\varepsilon_n))$ for all n. The following deep result of Mujica implies that $\theta_{\mathbf{a}}$ is τ_0 continuous.

Proposition 6 (J. Mujica, [12]). *If K is a compact subset of a Banach space X then*

(a) *the sets $\{f \in \mathcal{H}(K + B_r) : \|f\|_{K+B_r} \leq j\}$, $r > 0$, $j > 0$ form a fundamental system of bounded and compact subsets of $(\mathcal{H}(K),$ $\tau_0)$.*

(b) *$(\mathcal{H}(K), \tau_0)$ is a k-space, that is, mappings from $\mathcal{H}(K)$ into a topological space are continuous if and only if their restrictions to compact sets are continuous.*

Moreover a further application of Proposition 6 and the uniqueness established in Lemma 4 proves the uniqueness in part (a) of Theorem 2.

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