

Nonlinear Attitude Control for a Rigid Spacecraft by Feedback Linearization

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Attitude control law design for spacecraft large angle maneuvers is investigated in this paper. The feedback linearization technique is applied to the design of a nonlinear tracking control law. The output function to be tracked is the quaternion attitude parameter. The designed control law turns out to be a combination of attitude and attitude rate tracking commands. The attitude-only output function, therefore, leads to a stable closed-loop system following the given reference trajectory. The principal advantage of the proposed method is that it is relatively easy to produce reference trajectories and associated controller.

Key Words : Spacecraft Attitude Control, Feedback Linearization, Quaternion Parameter, Output Tracking Control, Singularity Avoidance Strategy, Sliding Mode Control

1. Introduction

Significant research effort has been devoted to attitude controllers design for spacecraft large angle maneuvers (Wie, 1985 ; 1989). Both closed-loop feedback and optimal open-loop designs are under extensive study. Recent trend is toward various robust feedback control approaches accounting for external disturbances, model uncertainty, and actuator saturation limits. Due to the nonlinear nature of the problem, control laws are dominated by nonlinear designs (Vadali, 1986). The fixed gain feedback controller using body angular rate and quaternion error is regarded as one of the most distinct features of the controllers (Wie, 1989). Also variable structure control or sliding mode control has been investigated by Vadali (1986) resulting in robust maneuver performance. Nonlinear predictive control is based upon a pre-determined reference trajectory which

is followed by the actual system under control (Crassidis, 1986).

One potential nonlinear control design approach which has attracted significant attention recently is the feedback linearization (Lin, 1994 ; Schumacher, 1998). The feedback linearization is generally able to handle nonlinearity more efficiently than conventional linearization techniques. A reference output function is defined a priori, and a controller is designed so that the actual output tracks the reference output. Therefore, the resultant control command consists of the reference and actual states of the system. The controller basically linearizes the system in the sense that stable closed-loop dynamics are achieved for the output function of the original nonlinear systems. The input-output linearization technique attempted in this study is basically different from previous nonlinear control laws dominated by Lyapunov stability theory. The principal objective of this study is to apply the feedback linearization technique to the large angle attitude maneuver of a rigid spacecraft model. The output function is a quaternion vector set which comprises three independent elements. The output function is then used to derive a feedback control law based upon the reference

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trajectory. The reference trajectory is generated in such a manner to satisfy terminal constraints at the initial time and final steady state. The resultant controller leads to smoothed output tracking performance. The key advantage derived from the proposed methodology is that the reference trajectory can be easily constructed using three quaternion elements only. In other words, the kinematics between angular velocity and quaternion do not have to be taken into account in the reference trajectory generation. The corresponding angular velocity profile is implicitly determined from the reference output history. Asymptotic output tracking and sliding mode controls are investigated to compare the performance.

2. Basic Dynamics and Control Theory

2.1 Attitude dynamics and kinematics

First, review on attitude dynamics and kinematics for a generic rigid spacecraft model is presented. The spacecraft model is shown in Fig. 1. For simplicity of analysis, the spacecraft is assumed to be a perfectly rigid body. The basic three-axis attitude dynamics of a rigid spacecraft are written in the form (Wie, 1995)

$$\mathbf{J}\dot{\boldsymbol{\omega}} + \boldsymbol{\omega} \times \mathbf{J}\boldsymbol{\omega} = \mathbf{u} \quad (1)$$

where \mathbf{J} is the spacecraft inertia matrix consisting of the principal moment of inertia, $\boldsymbol{\omega}$ is the body angular velocity vector, and \mathbf{u} is the control input vector.

For attitude representation, the quaternion parameter is introduced as

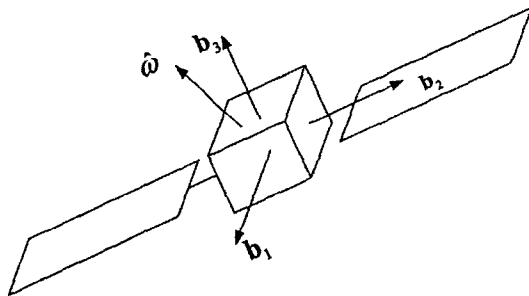


Fig. 1 Rigid spacecraft model

$$\mathbf{q} = \begin{bmatrix} \mathbf{q}_{13} \\ q_4 \end{bmatrix} \quad (2)$$

where each element satisfies (Wie, 1989)

$$\mathbf{q}_{13} = \begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix} = \mathbf{l} \sin \frac{\phi}{2}, \quad q_4 = \cos \frac{\phi}{2} \quad (3)$$

and \mathbf{l} is the Euler principal axis vector, and ϕ represents corresponding principal angle. The time derivative of the quaternion satisfies

$$\dot{\mathbf{q}} = \frac{1}{2} \boldsymbol{\Omega}(\boldsymbol{\omega}) \mathbf{q} = \frac{1}{2} \boldsymbol{\Xi}(\mathbf{q}) \boldsymbol{\omega} \quad (4)$$

for which

$$\boldsymbol{\Omega}(\boldsymbol{\omega}) = \begin{bmatrix} -[\tilde{\boldsymbol{\omega}}] & \boldsymbol{\omega} \\ -\boldsymbol{\omega}^T & 0 \end{bmatrix}, \quad \boldsymbol{\Xi}(\mathbf{q}) = \begin{bmatrix} q_4 \mathbf{I}_{3 \times 3} + [\tilde{\mathbf{q}}_{13}] \\ -\mathbf{q}_{13}^T \end{bmatrix} \quad (5)$$

The notation $[\tilde{\boldsymbol{\omega}}]$ represents

$$[\tilde{\boldsymbol{\omega}}] = \begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{bmatrix} \quad (6)$$

and similar definition is applied to $[\tilde{\mathbf{q}}_{13}]$. The quaternion is also subject to a constraint equation from Eq. (3) as

$$\mathbf{q}^T \mathbf{q} = \mathbf{q}_{13}^T \mathbf{q}_{13} + q_4^2 = 1 \quad (7)$$

Hence only three elements are independent, and other remaining element is determined automatically to satisfy the constraint. From a controller design view point, only three elements are needed for independent control action about three body axes of the spacecraft.

2.2 Feedback linearization

Principal idea of the feedback linearization is briefly presented herein. Majority portion of the material in this section is taken from Lin (1994). The feedback linearization technique is based upon linearized relationship between input and output. For a multi-input and multi-output nonlinear system of the form (Lin, 1994)

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}) + \mathbf{g}(\mathbf{x}) \mathbf{u} \quad (8)$$

the output function is defined by

$$\mathbf{y}(t) = \mathbf{h}(\mathbf{x}(t)) \quad (9)$$

where $\mathbf{f}(\mathbf{x})$ and $\mathbf{g}(\mathbf{x})$ are functions of vector fields in R^n , and the control and output vectors $\mathbf{u}=[u_1, \dots, u_m]^T$, $\mathbf{y}=[y_1, \dots, y_m]$, and $\mathbf{h}=[h_1, \dots, h_m]^T$ are corresponding multiple input and output functions.

The feedback linearization is to find integers $\rho_1, \rho_2, \dots, \rho_m$ and a feedback control of the form

$$\mathbf{u}=\alpha(\mathbf{x})+\beta(\mathbf{x})\mathbf{v} \quad (10)$$

where α and β are smooth vector functions defined in a neighborhood of some point $\mathbf{x}_0 \in R^n$ and $\det \beta(\mathbf{x}_0) \neq 0$ such that the closed-loop system by the control input

$$\begin{aligned} \dot{\mathbf{x}} &= \mathbf{f}(\mathbf{x}) + \mathbf{g}(\mathbf{x})\alpha(\mathbf{x}) + \mathbf{g}(\mathbf{x})\beta(\mathbf{x})\mathbf{v} \\ \mathbf{y} &= \mathbf{h}(\mathbf{x}) \end{aligned} \quad (11)$$

has the property that the ρ th-order derivative of the output satisfies

$$\mathbf{Y}^{(\rho)}(t) = \begin{bmatrix} y_1^{(\rho_1)} \\ y_2^{(\rho_2)} \\ \vdots \\ y_m^{(\rho_m)} \end{bmatrix} = \begin{bmatrix} v_1(t) \\ v_2(t) \\ \vdots \\ v_m(t) \end{bmatrix} = \mathbf{v}(t) \quad (12)$$

for which $\mathbf{v}(t)$ is an arbitrary function to be decided by a control objective. For each output (y_i) the relative degree ρ_i can be introduced as follows

$$\begin{aligned} y_i^{(1)} &= L_f h_i(x) \\ y_i^{(2)} &= L_f^2 h_i(x) \\ &\vdots \\ y_i^{(\rho_i)} &= L_f^{\rho_i} h_i(x) + \sum_{j=1}^m L_{g_j} L_f^{\rho_i-1} h_i(x) u_j \end{aligned} \quad (13)$$

where $L_f^k h(x)$ is called the *Lie derivative* of $L_f^{k-1} h(x)$ along the vector field f , and it is assumed that for at least one j , $1 \leq j \leq m$, $L_{g_j} L_f^{\rho_i-1} h_i(x) \neq 0$ holds. Now by introducing the notations

$$\mathbf{s}=[s_1, \dots, s_m]^T$$

and

$$\mathbf{F}(x) = \begin{bmatrix} L_f^{\rho_1} h_1(x) \\ L_f^{\rho_2} h_2(x) \\ \vdots \\ L_f^{\rho_m} h_m(x) \end{bmatrix}, \quad \mathbf{Y}^{(\rho)} = \begin{bmatrix} y_1^{(\rho_1)} \\ y_2^{(\rho_2)} \\ \vdots \\ y_m^{(\rho_m)} \end{bmatrix} \quad (14)$$

the following relationship in vector notations can be established

$$\mathbf{Y}^{(\rho)} = \mathbf{F}(\mathbf{x}) + \mathbf{G}(\mathbf{x})\mathbf{u} \quad (15)$$

where

$$\mathbf{G}(\mathbf{x}) = \begin{bmatrix} L_{g_1} L_f^{\rho_1-1} h_1(x) & \dots & L_{g_m} L_f^{\rho_1-1} h_1(x) \\ L_{g_1} L_f^{\rho_2-1} h_2(x) & \dots & L_{g_m} L_f^{\rho_2-1} h_2(x) \\ \dots & \dots & \dots \\ L_{g_1} L_f^{\rho_m-1} h_m(x) & \dots & L_{g_m} L_f^{\rho_m-1} h_m(x) \end{bmatrix}$$

If $\mathbf{G}(\mathbf{x})$ is invertible for a give point \mathbf{x}_0 , then the control input can be written as

$$\mathbf{u}(t) = \mathbf{G}^{-1}(\mathbf{x}) (-\mathbf{F}(\mathbf{x}) + \mathbf{v}) \quad (16)$$

so that

$$\mathbf{Y}^{(\rho)}(t) = \mathbf{v}(t), \quad t \in [0, \infty] \quad (17)$$

In the above derivation, on the condition that $\mathbf{G}(\mathbf{x})^{-1}$ exists, $\rho_1, \rho_2, \dots, \rho_m$ are *relative degrees* for multi-input and multi-output systems. Now for the feedback control law, asymptotic tracking of the output signal is realized if the error signal is defined as

$$e_i(t) = y_i(t) - r_i(t), \quad i=1, 2, \dots, m \quad (18)$$

where r_i is the i th reference trajectory for the corresponding output function. The control law is designed by selecting $\mathbf{v} = v_i (i=1, 2, \dots, m)$ as follows

$$v_i(t) = r_i^{(\rho_i)} - c_{i(\rho_i-1)} e_i^{(\rho_i-1)} - \dots - c_{i1} e_i^{(1)} - c_{i0} e_i \quad (19)$$

so that the closed-loop output dynamics result in

$$e_i^{(\rho_i)} + c_{i(\rho_i-1)} e_i^{(\rho_i-1)} + \dots + c_{i1} e_i^{(1)} + c_{i0} e_i = 0 \quad (20)$$

where $i=1, 2, \dots, m$ and c_{ij} are constants for a stable closed-loop system. Obviously, the asymptotic tracking control law is more practical than the exact tracking law handling initial error between the reference and actual system in general cases.

In the similar context, the controller can be extended into a sliding mode control. For a system with vector *relative degrees* $\rho_1, \rho_2, \dots, \rho_m$, the sliding surfaces are defined

$$\begin{aligned} s_i(t) &= e_i^{(\rho_i-1)} + c_{i(\rho_i-1)} e_i^{(\rho_i-2)} + \dots + c_{i1} e_i + c_{i0} \int e_i dt \\ &= 0 \end{aligned} \quad (21)$$

where $i=1, 2, \dots, m$. The stable sliding condition is dictated as follows

$$\frac{1}{2} \frac{d^2 s_i}{dt^2} \leq -\eta_i |s_i|, \eta_i > 0 \quad (i=1, 2, \dots, m) \quad (22)$$

After some algebra, the final form of the sliding mode controller which enforces the sliding condition in Eq. (22) is presented in the form (Lin, 1994)

$$\mathbf{u} = \mathbf{G}^{-1}(-\dot{\mathbf{s}} + Y^{(\rho)} - \mathbf{F} - \mathbf{k} \operatorname{sgn}(\mathbf{s})) \quad (23)$$

where $\mathbf{k} = \operatorname{diag}[k_1, \dots, k_m]$ is a gain matrix to be decided, and $\operatorname{sgn}(\mathbf{s}) = [\operatorname{sgn}(s_1), \dots, \operatorname{sgn}(s_m)]^T$ represents the vector signum function.

3. Tracking Controller Using Quaternion Output Only

3.1 Output control function

As a choice for the output function in feedback linearization, the attitude quaternion introduced in Eq. (3) is selected as $(\mathbf{y} = [y_1, y_2, y_3]^T \in \mathbb{R}^3)$

$$\mathbf{y} = \mathbf{q}_{13} \quad (24)$$

Note that another quaternion element q_4 is automatically determined from the constraint equation (Eq. (7)). Also, the angular velocity is implicitly prescribed from the kinematics in Eq. (4). Once the output function is defined, time derivative of the output is taken as

$$\dot{\mathbf{y}} = \dot{\mathbf{q}}_{13} = -\frac{1}{2} \boldsymbol{\omega} \times \mathbf{q}_{13} + \frac{1}{2} q_4 \boldsymbol{\omega} \quad (25)$$

Furthermore, by taking the derivative one more time, we arrive at

$$\begin{aligned} \ddot{\mathbf{y}} &= \frac{1}{4} \boldsymbol{\omega} \times (\boldsymbol{\omega} + \mathbf{q}_{13}) - \frac{1}{4} \boldsymbol{\omega}^T \mathbf{q}_{13} \boldsymbol{\omega} \\ &+ \frac{1}{2} ([\ddot{q}_{13}] + q_4 \mathbf{I}_{3 \times 3}) (J^{-1} \boldsymbol{\omega}) \times (J \boldsymbol{\omega}) \\ &+ \frac{1}{2} ([\ddot{q}_{13}] + q_4 \mathbf{I}_{3 \times 3}) J^{-1} \mathbf{u} \\ &\equiv \boldsymbol{\alpha}(\boldsymbol{\omega}, \mathbf{q}) + \boldsymbol{\beta}(\boldsymbol{\omega}, \mathbf{q}) \mathbf{u} \end{aligned} \quad (26)$$

It turns out that the second derivative of the output shows explicit dependency on the control input. Therefore by assuming that $\boldsymbol{\beta}^{-1}$ exists, the relative degrees are all equal to two, i.e., $\rho_i = 2$ ($i=1, 2, 3$), and input-output linearization is possible at this stage.

Meanwhile, the reference trajectory of the attitude quaternion should be constructed. For this purpose, it is assumed that initial (\mathbf{q}_{13}^0) and steady

state quaternion (\mathbf{q}_{13}^f) are prescribed as

$$q_{13} = q_{13}^0 \text{ at } t=0, q_{13}(t_f) = q_{13}^f \text{ at } t=\infty$$

A candidate reference trajectory suitable for the above condition is generated as

$$\mathbf{r}(t) = \mathbf{q}_{13}^0 + (\mathbf{q}_{13}^f - \mathbf{q}_{13}^0) (1 - e^{-t/\tau}) \quad (27)$$

where $\mathbf{r} = [r_1, r_2, r_3]^T \in \mathbb{R}^3$ and $\tau (> 0)$ is a time constant to be selected. The reference trajectory satisfies boundary conditions, and it can be seen that high order derivatives of the reference trajectory exist. The time derivatives of the reference signal are

$$\dot{\mathbf{r}}(t) = (\mathbf{q}_{13}^f - \mathbf{q}_{13}^0) (1/\tau) e^{-t/\tau} \quad (28)$$

and

$$\ddot{\mathbf{r}}(t) = -(\mathbf{q}_{13}^f - \mathbf{q}_{13}^0) (1/\tau^2) e^{-t/\tau} \quad (29)$$

The reference trajectory is ready for the tracking control law design. The reference trajectory is generated as a function of time from the initial and final attitude quaternions. This approach is simpler than developing a reference trajectory by solving the dynamic equations directly. It is easy to see that the design parameter (τ) can be used in shaping the dynamic response of the reference trajectory.

3.2 Controller design

The time derivatives of the output and reference trajectories derived in the previous part can be combined together to build an asymptotic tracking control law. From the generalized form of the tracking control law in Eqs. (16) and (19), a tracking controller is proposed as

$$\mathbf{u} = \boldsymbol{\beta}^{-1} [\ddot{\mathbf{r}}(t) - \boldsymbol{\alpha}(t) - \mathbf{c}_1(\dot{\mathbf{y}}(t) - \dot{\mathbf{r}}(t)) - \mathbf{c}_0(\mathbf{y}(t) - \mathbf{r}(t))] \quad (30)$$

where $\boldsymbol{\alpha}$, are $\boldsymbol{\beta}$ parameters in Eq. (26), \mathbf{c}_1 are \mathbf{c}_0 constant diagonal matrices, and $\boldsymbol{\beta}^{-1}$ is assumed to exist. It can be easily shown that

$$\boldsymbol{\beta}^{-1} = \frac{2J}{q_4} \begin{bmatrix} q_4^2 + q_1^2 & q_3 q_4 + q_2 q_1 & q_3 q_1 - q_2 q_4 \\ -q_3 q_4 + q_2 q_1 & q_4^2 + q_2^2 & q_4 q_1 + q_3 q_2 \\ q_3 q_1 + q_2 q_4 & -q_4 q_1 + q_3 q_2 & q_4^2 + q_3^2 \end{bmatrix} \quad (31)$$

Note that the parameter $\boldsymbol{\beta}$, from the definition in Eq. (26), is subject to singularity on the condition that

$$q_4 = \cos \frac{\phi}{2} = 0 \quad (32)$$

where q_4 turns out to be the determinant of the matrix $q_4 I_{3 \times 3} + [\tilde{q}_{13}]$. Namely, the principal angle rotation more than 180 degrees is subject to singularity problem. A singularity avoidance strategy needs to be implemented in such a case. For instance, a series of small maneuvers may be employed in lieu of a single large-angle maneuver.

If the closed-loop system reaches steady state in such a way that

$$\lim_{t \rightarrow \infty} \mathbf{y}(t) = \mathbf{r}(t), \quad \lim_{t \rightarrow \infty} \dot{\mathbf{y}}(t) = \dot{\mathbf{r}}(t) \quad (33)$$

then the relationship in Eq. (26) still leads to a meaningful solution with

$$\alpha(\omega, \mathbf{q}) + \beta(\omega, \mathbf{q}) \mathbf{u} = \ddot{\mathbf{r}} \quad (34)$$

In general cases, once singularity condition takes place, which causes numerical difficulty in evaluating β^{-1} , then the state q_4 can be replaced with.

$$q_4' = q_4 + \delta \quad (35)$$

where the parameter δ is a small number employed to prevent numerical singularity problem.

For further analysis of the control, the following Lyapunov function is introduced.

$$2U = (\mathbf{q}_{13} - \mathbf{r})^T \mathbf{Q} (\mathbf{q}_{13} - \mathbf{r}) + (\dot{\mathbf{q}}_{13} - \dot{\mathbf{r}})^T \mathbf{R} (\dot{\mathbf{q}}_{13} - \dot{\mathbf{r}}) \quad (36)$$

where \mathbf{Q} and \mathbf{R} are positive definite symmetric weighting matrices. Therefore, the Lyapunov function is positive definite with the state variables $(\mathbf{q}_{13}, \dot{\mathbf{q}}_{13})$. The positive definite or positive semi-definite \dot{U} guarantees stability of the closed-loop system. Time derivative of the Lyapunov function becomes

$$\dot{U} = (\mathbf{q}_{13} - \mathbf{r})^T \mathbf{Q} (\dot{\mathbf{q}}_{13} - \dot{\mathbf{r}}) + (\dot{\mathbf{q}}_{13} - \dot{\mathbf{r}})^T \mathbf{R} (\ddot{\mathbf{q}}_{13} - \ddot{\mathbf{r}}) \quad (37)$$

Re-arranging some terms and using the results in Eq. (26), it follows as

$$\begin{aligned} \dot{U} &= (\dot{\mathbf{q}}_{13} - \dot{\mathbf{r}})^T [\mathbf{Q} (\mathbf{q}_{13} - \mathbf{r}) + \mathbf{R} (\ddot{\mathbf{q}}_{13} - \ddot{\mathbf{r}})] \\ &= (\dot{\mathbf{q}}_{13} - \dot{\mathbf{r}})^T [\mathbf{Q} (\mathbf{q}_{13} - \mathbf{r}) + \mathbf{R} (\alpha(\omega, \mathbf{q}) + \beta(\omega, \mathbf{q}) \mathbf{u} - \ddot{\mathbf{r}})] \end{aligned} \quad (38)$$

For stability in the Lyapunov sense, the following equation should hold

$$\begin{aligned} &[\mathbf{Q} (\mathbf{q}_{13} - \mathbf{r}) + \mathbf{R} (\alpha(\omega, \mathbf{q}) + \beta(\omega, \mathbf{q}) \mathbf{u} - \ddot{\mathbf{r}})] \\ &= -\mathbf{W} (\dot{\mathbf{q}}_{13} - \dot{\mathbf{r}}) \end{aligned} \quad (39)$$

where \mathbf{W} is a positive definite weighting matrix. Hence the stabilizing control law is derived as

$$\mathbf{u} = \beta^{-1}(\omega, \mathbf{q}) [-\alpha(\omega, \mathbf{q}) + \ddot{\mathbf{r}} - \mathbf{G}_1 (\dot{\mathbf{q}}_{13} - \dot{\mathbf{r}}) - \mathbf{G}_2 (\mathbf{q}_{13} - \mathbf{r})] \quad (40)$$

where the gain matrices satisfy

$$\mathbf{G}_1 = \mathbf{R}^{-1} \mathbf{W}, \quad \mathbf{G}_2 = \mathbf{R}^{-1} \mathbf{Q} \quad (41)$$

Hence, the control law in Eq. (30) is basically equivalent to that of Eq. (40). The control law by feedback linearization is therefore a stable tracking control law as proved by the Lyapunov function.

Application of the above tracking controller produces a stable output response governed by

$$\ddot{e}_i(t) + c_{i1} \dot{e}_i(t) + c_{i0} e_i(t) = 0 \quad (42)$$

where $e_i(t) = y_i(t) - r_i(t)$ ($i=1, 2, 3$) represents error between the actual and reference outputs. The design parameters c_{ij} ($i=1, 2, 3, j=0, 1$) represent parameters of the closed-loop characteristic equations for each output variable.

In general feedback linearization, stability analysis on the zero dynamics should be examined. However, in this study the zero dynamics analysis has not been conducted since the control command and control variables are essentially collocated. Thus, from a physical sense, the zero dynamics are expected to be stable.

The feedback linearization tracking control law is based upon the condition that exact modeling data must be identified. In particular, the parameters $\alpha(\omega, \mathbf{q})$ and $\beta(\omega, \mathbf{q})$ are functions of the system inertia matrix J . If there is a modeling error in the system inertia matrix such as $J' = (1 + \kappa)J$ where κ is a parameter representing the model uncertainty, then the performance of the control law shall be degraded. Analysis on the stability of the closed-loop system due to the uncertain J should be followed in the future study.

Furthermore, the sliding mode controller can be designed from a vector sliding surface equation of the form

$$\mathbf{s} = \dot{\mathbf{e}} + \mathbf{c}_1 \mathbf{e} + \mathbf{c}_0 \int \mathbf{e} dt = 0 \quad (43)$$

Note that $\mathbf{e} = [e_1, e_2, e_3]^T$ denotes the error vector, $\mathbf{s} = [s_1, s_2, s_3]^T$, and the sliding mode controller is

designed on the condition that $\dot{\mathbf{s}}=0$ which consequently leads to

$$\mathbf{u} = \beta^{-1} [\ddot{\mathbf{r}} - \alpha - \mathbf{c}_1(\dot{\mathbf{y}} - \dot{\mathbf{r}}) - \mathbf{c}_0(\mathbf{y} - \mathbf{r}) - \mathbf{k} \operatorname{sgn}(\mathbf{s})] \quad (44)$$

where \mathbf{k} is a 3 by 3 diagonal gain matrix, and sgn represents the signum function. The signum function is replaced by a saturation function to minimize potential chattering phenomenon. In other words,

$$\operatorname{sgn}(f) = \begin{cases} \operatorname{sgn}(f) & \text{for } |f| > \varepsilon \\ f/\varepsilon & \text{for } |f| < \varepsilon \end{cases} \quad (45)$$

where ε is a dead-band of the sliding surface which represents a minimum band for which the control command is disabled to minimize chattering problem.

4. Simulation and Analysis

Simulation by the proposed control approach has been conducted. The mass moment of inertia of the model spacecraft are assumed to be $J = \operatorname{diag}[300, 320, 250]$ (kg-m²). Initial quaternion elements for simulation purpose are taken as $\mathbf{q}_{13}^0 = [0.2, 0.4, 0.5]^T$, $q_4^0 = 0.742$ while the target quaternion components are given by $\mathbf{q}_{13}^f = [0.6, -0.2, -0.4]^T$, $q_4^f = 0.663$ respectively. The controller gains \mathbf{c}_1 , \mathbf{c}_0 , and \mathbf{k} are set to be diagonal matrices, for which each matrix consist of identical numbers of 0.2, 0.1, and 0.05. The time constant (τ) selected for the reference output is 10 seconds. Both asymptotic tracking control and sliding mode control scenarios are simulated separately. The simulation results are plotted in Fig. 2. Body angular velocity and quaternion responses are plotted. As it can be shown the quaternion parameter follows the final target \mathbf{q}_{13}^f asymptotically. The parameter q_4 is not plotted here for simplicity. As a whole, satisfactory tracking performance is achieved. The angular velocity responses are dictated by the quaternion history, so that the convergence to zeros for the angular velocity is evident from Eq. (4).

Next additional set of initial and final attitude are investigated with $\mathbf{q}_{13}^0 = [0.2, 0.4, 0.5]^T$, $q_4^0 = -0.742$ and $\mathbf{q}_{13}^f = [0.6, -0.2, -0.632]^T$, $q_4^f = 0.447$. In this case the state q_4 crosses the singular

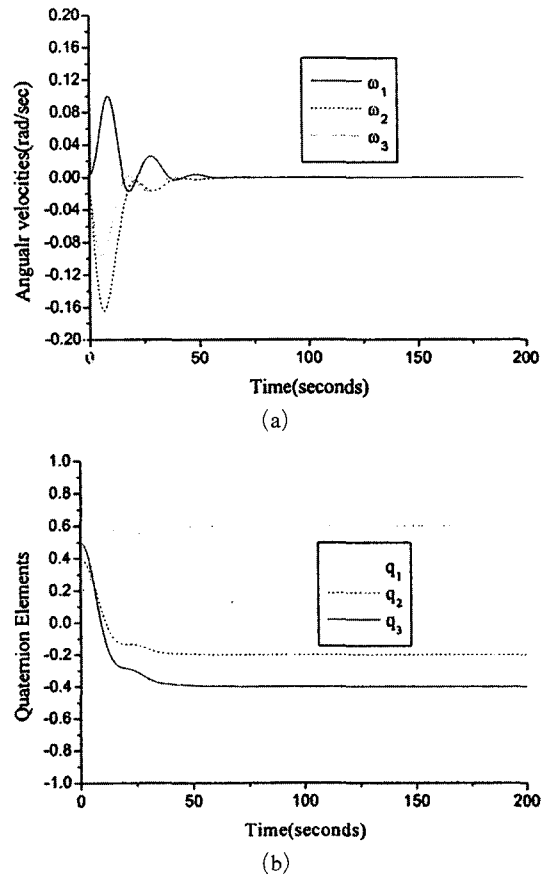


Fig. 2 Simulation results with the tracking control law

point. As discussed early, there may exist a singular condition. Time response of q_4 is displayed in Fig. 3. As one can see, q_4 reaches the singular point less than 10 seconds. The control gain parameter (β) has been modified by the singularity avoidance strategy as Eq. (35). In the actual implementation of the control law, q_4' replaces q_4 when q_4 reaches a small number across the zero crossing line. Further analysis may be needed on the stability analysis for the closed-loop system with respect to the size of δ . The parameter δ was provided with the same sign as q_4 . It also needs to be examined whether the control command has a different sign with q_4' when q_4 crosses zero. In the simulation results, there was no sign change observed in the control command.

From Fig. 3, it is hard to see visual difference between the two cases: one with original gain

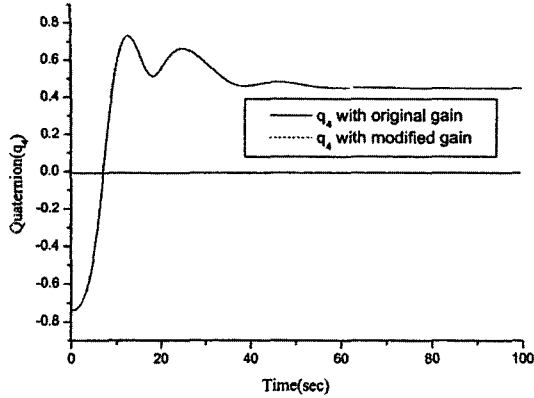


Fig. 3 Time response of the parameter q_4

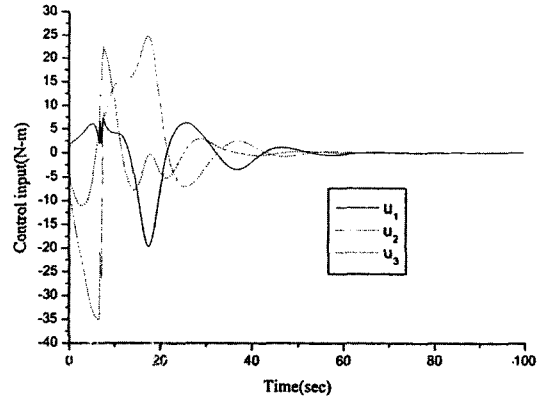


Fig. 5 Time responses of the control command

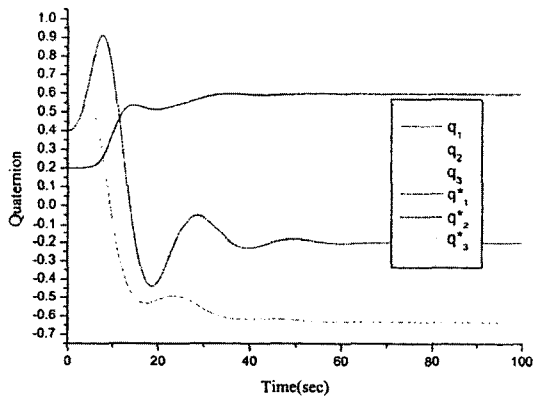


Fig. 4 Time responses of the parameters q_1, q_2, q_3

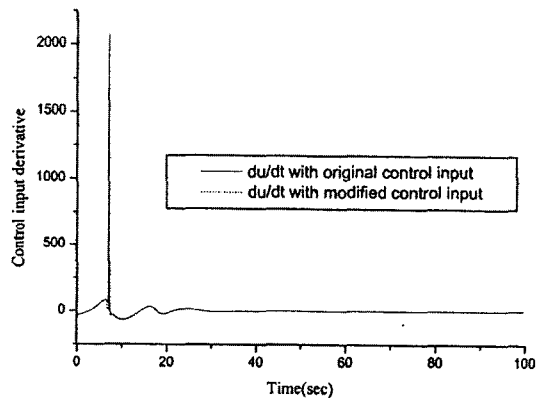


Fig. 6 Time rate of change of the control command

and the other with modified gain. Also, other quaternion parameters (q_1, q_2, q_3) are plotted in Fig. 4. They still show satisfactory tracking performance despite the singularity problem. It turns out that the singularity did not raise significant difficulty in the process of numerical simulation because a finite time step was taken for numerical integration. The exact singularity condition was not easy to realize since it is also sensitive to the closed-loop dynamics.

The control command trends are provided in Fig. 5. Some sharp rise in the control command, in particular, around the singularity point is observed. The near-singular condition causes the control input to change with large values in derivative.

In order to verify the effect of the singularity avoidance technique, the time derivate of the control command is plotted in Fig. 6. There is a

sharp peak around the singular point. Such a sharp change in the control command may cause a saturation problem in the actual hardware systems. The perturbation parameter (δ) used in the simulation is equal to 0.1. The maximum value for the control input derivative (du/dt) was about 2073 when q_4 was in the control command, while it was as low as about 570 with q_4' . This illustrates the useful advantage of the singularity avoidance idea proposed in this study.

The singularity avoidance idea is illustrated just by the simulation result. It is a case for which the control command is regularized from a sharp rise by an intuitive approach. For assurance of guaranteed avoidance of the singularity, further analysis by including the q_4' in the closed-loop dynamics may be required.

Next simulation with the sliding mode control law in Eq. (46) is conducted with the results

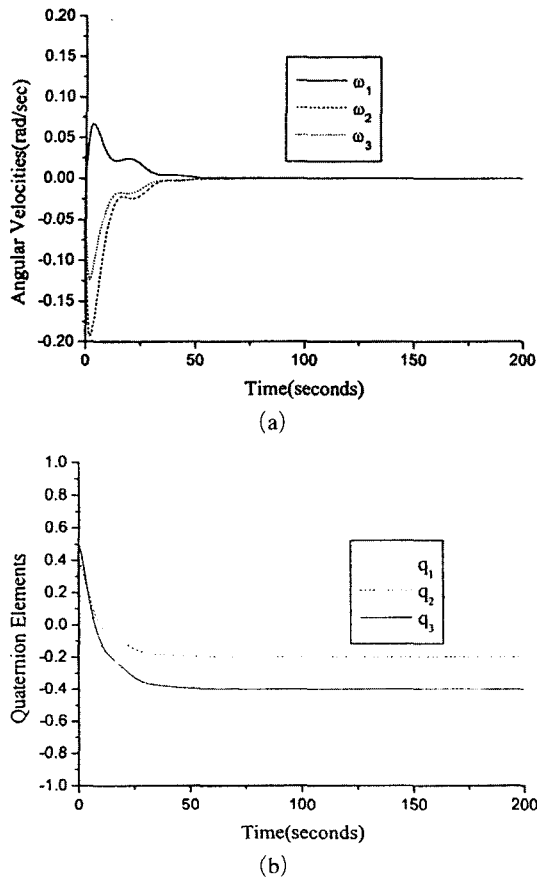


Fig. 7 Simulation results with the sliding mode control

presented in Fig. 7. Identical initial condition and control parameters are adopted as those in Fig. 2. It is not easy to see the difference between the results in Fig. 1 and Fig. 7. The sliding mode control needs to be analyzed from other aspects such as disturbance rejection and robustness with respect to modeling error, which is not pursued in this study.

5. Conclusions

It was shown that the feedback linearization technique could be applied to the large angle attitude maneuver of a rigid spacecraft model. Three quaternion attitude parameters are used to generate a set of reference motion. The simulation results demonstrate the useful merit of the proposed control law. Reference angular veloci-

ties are implicitly determined from the reference attitude. Stable output tracking performance was successfully achieved. The simple reference trajectory synthesis methodology proposed in this study could be used to construct corresponding output control command easily. Robustness issues for the proposed controller still leave room for further investigation.

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