Evaluation of Manufacturing Cost Considering Reliability of Manufacturing Facilities

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설비 신뢰성을 고려한 제조경비 평가

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Abstract

In this study, new way of evaluating manufacturing cost is organized and applied. In real manufacturing circumstances, tolerances of parts and assemblies are closely related to the cost. Several researches have been tried to identify the relations and set models. Moreover tolerances have influences on the maintenance of the manufacturing facilities. However past researches have not considered the processing cost for the failed products. Therefore maintenance costs are represented as stochastic expressions, which include reliability of assembly and facilities. The stochastic nature of the maintenance cost is modeled and solved using Markov chain approach. Results show that this approach gives reliable estimations with remarkable computing time reduction.

Key Words : Yield, Reliability, Tolerances, Production Cost, Markov Chain, Recursive Formulation

1. 서 론

In manufacturing processes, the dimensions have distributions around the desired values. In the assembly process, the parts are selected from the stack randomly. The sum dimensions appear as the sum of the dimensions of the parts under consideration put into a given assembly. The characteristics describing the statistical behavior of the sum dimension can be calculated based on the statistical parameters of the individual dimensions: In real manufacturing circumstances, tolerances of parts and assemblies are closely related to the cost. Several researches have been tried to identify the relations and set models.

Several cost function models for tolerances is the mathematical representation of the manufacturing costs

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in terms of the tolerances. In most cases monotonically decreasing functions are selected as cost functions because of the monotonic behavior of manufacturing costs for tolerances[1-3]. Using the models, researches to optimize the overall assembly cost were also tried[4]. Because the stochastic nature of the tolerances and yield, stochastic mathematical programmings[5,6] and simulation methods[7,8] were tired to solve the problem. However the researches have not considered the processing cost for the failed products. Moreover tolerances have influences on the maintenance of the manufacturing facilities. Therefore the cost models for producing certain amount of products have to include the considerations on facility maintenance.

Several researches were performed to represent the production cost as the probabilistic maintenance model. The most frequently used probabilistic model of production costing is due to Baleriaux, Jamoulle and Guertechin[9]. In the Baleriaux model, the hourly loads for the forecast time horizon are considered deterministic[10]. Methods for computing the expected production costs are well developed[11]. However, the variance of the cost has only recently been studied[12,13]. Mazumdar and Yin developed formulas for the variance of production cost[14]. Bloom[15] has developed the recursive equation for the rate of production cost.

In this paper, conventional cost function model is modified by adding the facility maintenance cost. Markov chain is introduced to distinguish the failure state to repairing state. The recursive equation for the production cost is developed further to be applied to modified cost function model. A numerical example is selected and solved. The resulting cost is compared with the simulation result.

2. Manufacturing cost models

2.1 General cost function

The domain of dimensions is divided into a safe region and a failure region by inequalities. Those inequalities are the design functions (i.e. constraints on the sum dimensions). The intersection of the safe region and the acceptable tolerance region is referred to as the reliable region. The reliable region depends on the standard deviation σj of each dimension since the tolerance region varies with σj. An important concept called yield is computed as the probability of x being in reliable region. Let xiu and xil represent the upper and lower limits of an individual dimension xi in an assembly. Then the yield is represented as

$$Y = \int_{X \in R_R} \phi(X)dX \tag{1}$$

where $\phi(x_1,\cdots,x_n)$ is the multivariate normal probability density function

$\mathbf{R_R}$ represents the reliable region.

As discussed earlier it is not easy to compute the yield Y from the set of tolerances (standard deviations) that constitute the multivariate normal probability density function in equation (1) when the dimensionality becomes high

The cost function model for tolerances is the mathematical representation of the manufacturing costs in terms of the tolerances. In most cases monotonically decreasing functions are selected as cost functions because of the monotonic behavior of manufacturing costs for tolerances[1]. In practical modeling, the costs are estimated for several manufacturing processes. Then the coefficients of the cost function are calculated using curve fitting techniques. There are several cost function models for tolerance variables. Two common models are the reciprocal squared model[2] and the exponential model[3]. In the reciprocal squared model, the cost function is represented as

$$C(t) = \frac{a}{t^2} + f \tag{2}$$

In the exponential model:

$$C(t) = a\exp(-\frac{t}{b}) + f \tag{3}$$

where a and b are constants for variable manufacturing cost, and f is a constant for fixed manufacturing cost. In multi-dimensional model, total manufacturing cost can be obtained by summing the individual costs for each dimension:

$$C(t) = \sum_{j=1}^{n} C(t_j) \quad (4)$$

or in the standard deviation domain:

$$C(\sigma) = \sum_{j=1}^{n} C(\sigma_j) \quad (5)$$

In a real design situation, the aim of solving the optimal tolerance allotment problem is to determine the cost minimizing tolerances of each dimension while guaranteeing the desired yield (spec yield) of the assembly. In order to guarantee the spec yield, a yield calculating method, such as Monte Carlo simulation, should be implemented as an inner calculation procedure. However, Monte Carlo simulation requires many sampling points to result in a desired accuracy. In real manufacturing processes, the failures in assembly and manufacturing facilities cause additional cost increases. Therefore real manufacturing cost, *Creal*, is expressed as

$$Creal = C(\sigma) + r < -\left(\int_{x \in R_R} \phi(x)dx - Y_{spec}\right) >^2 \quad (6)$$

$$where < a >= \begin{cases} a, & x \geq 0 \\ 0, & a < 0 \end{cases}$$

r is cost coefficient and positive

or

$$Creal = C(\sigma) + \Lambda \quad (7)$$

where repairing cost

$$\Lambda = r < -\left(\int_{x \in R_R} \phi(x)dx - Y_{spec}\right) >^2$$

Simulation schemes are suggested to overcome the difficulties of domain and dimension. Monte Carlo methods[7,8] are the best known methods.

2.2 Recursive formulation of manufacturing cost

It is assumed that the costs are being calculated for a manufacturing system consisting of N generating units. For two manufacturing state x_0, y_0, we define the transition probability by $p_0(y_0 \mid x_0)$. The limiting probability for the state x_0 is denoted by $\pi_0(x_0)$. We define $p_i^{(1)}(y_i \mid x_i)$ to be the one-step transition probability for moving from state x_i to y_i for unit i (x_i, y_i = 0,1; state 0 corresponds to a capacity value of 0, and state 1 corresponds to a capacity value of c_i.).

For $i \neq j$, $X_i(t)$ and $X_j(s)$ are independent for all values of t and s. The process $X_0(t)$ is independent of each $X_i(t)$. Let λ_i be the failure rate and μ_i be the repair rate for unit i. Then, the unavailability or F.O.R. of this unit is

$$p_i = \frac{\lambda_i}{\lambda_i + \mu_i} \quad (8)$$

From the formula by Ross[16] on the transition probabilities, we approximate the transition probabilities at the end of each process to be

$$P_i^{(1)} \approx P_i(1) = \begin{bmatrix} p_i + q_i e^{-(\lambda_i + \mu_i)} & q_i - q_i e^{-(\lambda_i + \mu_i)} \\ p_i - p_i e^{-(\lambda_i + \mu_i)} & q_i + p_i e^{-(\lambda_i + \mu_i)} \end{bmatrix} \quad (9)$$

When λ_i^{-1} and μ_i^{-1} are both large, the one-step transition probability matrix $p_i^{(1)}$ for the unit can also be approximated as follows:

$$P_i^{(1)} = \begin{bmatrix} 1-\mu_i & \mu_i \\ \lambda_i & 1-\lambda_i \end{bmatrix} \quad (10)$$

Both approximations yield the limiting probability for the available capacity of unit i to be

$$\pi_i(0) = p_i$$
$$\pi_i(1) = q_i$$

Having defined the Markov chains for the manufacturing units and the load, we can combine these two chains to form the Markov chain for the manufacturing system. Let $\tilde{x} = [x_0, x_1, \cdots, xN]$ be the state vector where x_0 represents the state of the load and x_i, i = 1, 2, ..., N, denotes the available capacity of each unit i such that $x_i = c_i$ if unit i is up and $x_i = 0$ if unit i is down. The transition probabilities and limiting probabilities for the system are given by

$$p_{sys}(y_0, y_1, ..., y_N | x_0, x_1, ..., x_N) = p_0(y_0|x_0).\prod_{i=1}^{N} p_i^{(1)}(y_i|x_i) \quad (11)$$

$$\pi_{sys}(x_0, x_1, ..., x_N) = \pi_0(x_0).\prod_{i=1}^{N} \pi_i(x_i) \quad (12)$$

Defining the transition probability matrix and the matrix of limiting state probabilities by P_{sys} *and* Π_{sys}, the fundamental matrix for the Markov chain[17] is given by

$$F = \sum_{k=0}^{\infty} (P_{sys} - \Pi_{sys})^k = [I - (P_{sys} - \Pi_{sys})]^{-1} \quad (13)$$

The number of rows and columns in P_{sys} *and* Π_{sys} equals $L \cdot 2^N$. Thus the state space increases exponentially with N, the number of manufacturing units. We define $\Lambda(\tilde{x})$ to be the manufacturing cost incurred by the system per hour when the system is in state $[x_0, x_1, ..., x_N]$. To apply the asymptotic formulas for the evaluation of production cost mean and variance, let $\tilde{x}(t) = [x_0(t), x_1(t), ..., x_N(t)]$ denote the state vector of the generation system at process t.

Define

$\tilde{x}, \tilde{y}$: states of the Markov chain

$\Lambda(\tilde{x})$: cost rate for state $\tilde{x}$ i.e., the manufacturing costof the system per unit time when the system is in state $\tilde{x}$ $\pi(\tilde{x})$: limiting probability for state $\tilde{x}$

$f(\tilde{y} | \tilde{x})$: element of the fundamental matrix where the state of the row is $\tilde{x}$ and the state of the column is $\tilde{y}$.

While applying the above formulas to estimating the production costs of a system consisting of a large number of generating units, one faces the problem of how to evaluate Λ, f and π in an effective manner, taking into account the large state space of the Markov chain. From (11) and (12), the transition probabilities and the limiting probabilities have nice product forms where the individual terms represent probabilities for the load and the manufacturing units. This property provides a means for easily developing recursive procedures for computing these quantities by considering one unit at a time. Unfortunately, to obtain the fundamental matrix, the matrix inversion cannot be done without first constructing the entire transition probability and the limiting probability matrices for the entire generation system. The approximate recursive procedure given below precludes the need for matrix inversion, and makes the computation of the asymptotic variance of the production cost feasible for large systems.

Suppose that we have already calculated the marginal cost $\lambda(x_0, x_{i+1}, ..., x_N)$ of the units i+1,i+2,...,N, expressed as the corresponding availability states of these units, and we are now ready to incorporate unit i. Suppose that the available generating capacity of unit i is x_i . If the load of x_0 is less than x_i ($x_0 > 0$), then unit i is the system's marginal unit and its running cost k_i is the system's marginal cost. Then the marginal cost in the interval $0 < x_0 < x_i$ is k_i which can be represented by

$$k_i\{R^0(x_0) - R^0(x_0 - x_i)\},$$

where

$$R^0(x) = \begin{cases} 1; & x \geq 0 \\ 0; & x < 0 \end{cases}$$

If the load x_0 exceeds x_i, then one of the units i+1, i+2,...,N is marginal; since the addition of x_i reduces the effective load by this amount for the remaining units, the marginal cost is $\lambda(x_0 - x_i, x_{i+1}, ..., x_N)$. Thus, noting that $\lambda(x_0, x_i, ..., x_N) = 0$ when $x_0 < 0$, we have

$$\lambda(x_0, x_i, ..., x_N)$$
$$= \lambda(x_0 - x_i, x_{i+1}, ..., x_N) + k_i\{R^o(x_0) - R^0(x_0 - x_i)\}$$
$$\lambda(x_0) = k_{N+1} R^0(x_0)$$

Now, let $\overline{\Lambda}_i(x_0)$ be the expected value of $\Lambda(x_0, x_i,$

..., x_N), with respect to $\pi(x_i,x_{i+1},...,x_N)$, the limiting distribution of $x_i,...,x_N$. Then, using the property that the limiting probability has a product form,

$$\begin{aligned}\overline{\Lambda}_i(x_0) &= \sum_{x_i,...,x_N} \Lambda(x_0,x_i,...,x_N)\pi(x_i,...,x_N) \\ &= \sum_{x_i} \sum_{x_{i+1},...,x_N} \Lambda(x_0-x_i,x_{i+1},...,x_N)\pi(x_{i+1},...x_N)\pi_i(x_i) \\ &+ \sum_{x_i} \sum_{x_{i+1},...,x_N} k_i\left\{R^1(x_0)-R^1(x_0-x_i)\right\}\pi(x_{i+1},...,x_N)\pi_i(x_i) \\ &= \sum_{x_i} \left\{\overline{\Lambda}_{i+1}(x_0-x_i)-k_iR^1(x_0-x_i)\right\}\pi_i(x_i)+k_iR^1(x_0)\end{aligned} \quad (14)$$

and $\overline{\Lambda}_{N+1}(x_0)=k_{N+1}R^1(x_0)$.

Thus, the expected manufacturing cost of the system is:

$$\begin{aligned}&\sum_{x_0,x_1,...,x_N} \Lambda(x_0,x_1,...,x_N)\pi(x_0,x_1,...,x_N) \\ &= \sum_{x_0} \overline{\Lambda}_1(x_0)\pi_0(x_0)\end{aligned} \quad (15)$$

3. Numerical results

For the numerical example, a problem from Lee and Woo[4] was selected. The shape of the assembly is shown in Figure 1. The linear design functions for this example are:

$$F_1(x)=-x_4-x_5+5.005$$
$$F_2(x)=x_2-x_1-x_8+x_7-0.0003$$
$$F_3(x)=x_7-x_6-x_3+x_2+0.001$$
$$F_4(x)=x_4-x_3-x_6-0.0003$$

$F_1(\mathbf{x})$ is the constraint on the size of the base part. The other design functions represent the clearance condition for assembly. The nominal dimensions are given as $\mathbf{x}^T$ = (1.0 2.0 3.0 4.0 1.0 0.998 2.0 2.998). The reciprocal squared cost function model, equation (5), was modified and used to define total manufacturing cost as

$$C_i(\sigma_i)=\frac{a_i\times 10^{-3}}{(6\sigma_i)^{b_i}} \quad (16)$$

The coefficients in equation (16) were set by Lee and Woo as: $a_1 = a_2 = 1.0$, $a_3 = a_4 = 1.5$, $a_5 = 0.8$, $a_6 = 0.9$, $a_7 = 0.8$, and $a_8 = 0.6$; and $b_1 = 2.0$, $b_2 = 1.8$, $b_3 = 1.7$, $b_4 = 2.0$, $b_5 = 3.0$, $b_6 = 2.0$, and $b_7 = b_8 = 1.9$. The spec yield is 95%.Tolerances for each dimensions were set as: $t_1 = 0.00333$, $t_2 = 0.00133$, $t_3 = 0.00143$, $t_4 = 0.00305$, $t_5 = 0.01429$, $t_6 = 0.00171$, $t_7 = 0.00133$, $t_8 = 0.00143$.

Tables 1 provides an example of a manufacturing system that has been patterned after the Reliability Test System[18]. Table 1 shows the manufacturing units(j) in their loading order with their capacity(c_j), cost(k_j) and reliability parameters(λ_j^{-1} and μ_j^{-1}).

Let x_0 and y_0 denote two load states That is, we assume

$$p_0(y_0|x_0)=\begin{cases}1 & if\, y_0=x_0+1 \\ 0 & otherwise\end{cases}, \quad x_0=1,...,23;\ y_0=2,3,...,24$$

and

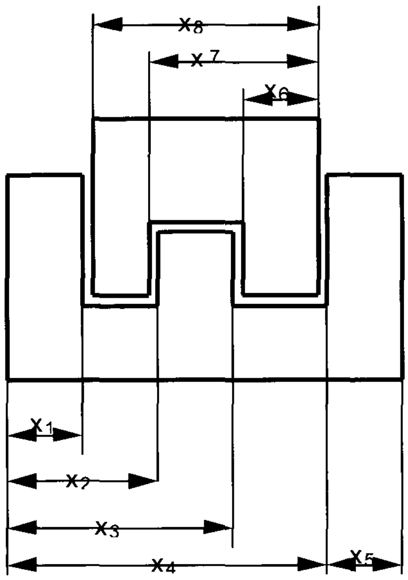

Fig. 1 Assembly feature

Table 1 Manufacturing facilities

Unit (j)	Xi	c_j	p_j	$1/\lambda_j$	$1/\mu_j$	Cost, k_j
1-2	X1	1000	.1000	1440	160	4.50
3	X2	700	.0977	1200	130	5.50
4-5	X3	600	.0909	1100	110	5.75
6-8	X4	500	.0873	1150	110	6.00
9-14	X5	300	.0654	1000	70	10.00
15-19	X6	200	.0535	850	48	14.50
20-26	X7	100	.0741	600	48	22.50
27-32	X8	100	.0331	350	12	44.00

Table 2 Comparison of the Recursive equation with Simulation

Comparison		Total cost, *Creal*
Expected Value	Simulation	2628.4
	Recursive	2632.2
CPU (seconds)	Simulation	336.92
	Recursive	3.52

$$p_0(1|24) = 1$$
$$p_0(y_0|24) = 0, y_0 = 2,3,...,24.$$

The limiting probabilities of each manufacturing state are

$$\pi_0(x_0) = 1/24, \qquad x_0 = 1,2,...,24.$$

For the manufacturing situation described above, *Creal* in equation (7) can be calculated by adding the costs described in equation (16) and (15). The results are shown in Table 2 which also gives the corresponding point estimates obtained from Monte Carlo simulation with 8000 runs. The respective CPU-times are also shown. Simulations and calculations are done by Pentium 4 PC. It is seen that the recursive procedure provides very accurate evaluations where they are almost indistinguishable from the Monte Carlo results. The computer time needed by the approximation compares well with the Monte Carlo simulation. Under the assumption of periodic load, the recursive formulation is much faster compared to Monte Carlo with 8000 runs.

4. Conclusion

New approach to evaluate the manufacturing cost is tried. The failure of the manufacturing facilities is related to the tolerances of the products and assemblies. To overcome the computational complexities in evaluating the cost, Markov chain process is introduced and modeled for the problem. Results show that this approach gives reliable estimations with remarkable computing time reduction.

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