동적 카오틱 뉴런의 수렴 특성에 관한 연구

A Study on the Convergence Characteristics Analysis of Chaotic Dynamic Neuron

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요 약

생체 뉴론은 일반적으로 지속적 또는 과도적인 카오틱 특성을 가지고 있다. 생체 뉴론의 카오틱 반응에 대한 분석적인 해석은 아직까지 이루어지지 않고 있다. 동적 카오틱 반응에 대한 카오틱 뉴런의 과도 카오틱 특성은 지역 수렴 문제를 극복하는데 도움이 되지만 일반적으로 지속적인 카오틱 응답은 최적화 문제에 악영향을 미치게 되므로 초기 카오틱 특성은 사라져야 한다. 패턴 인식, 확인, 예측, 그리고 제어에 사용되는 대부분의 신경회로망 응용에 있어서 필요한 최적화 문제를 해결하기 위해서는 뉴론은 한 개의 안정적인 고정점을 가지고 있어야 한다. 본 논문에서는 동적 카오틱 뉴런의 동적 특성과 카오틱 응답을 발생시키는 조건을 분석하고, 카오틱 뉴런의 수렴조건을 제안하였다.

Abstract

Biological neurons generally have chaotic characteristics for permanent or transient period. The effects of chaotic response of biological neuron have not yet been verified by using analytical methods. Even though the transient chaos of neuron could be beneficial to overcoming the local minimum problem, the permanent chaotic response gives adverse effect on optimization problems in general. To solve optimization problems, which are needed in almost all neural network applications such as pattern recognition, identification or prediction, and control, the neuron should have one stable fixed point. In this paper, the dynamic characteristics of the chaotic dynamic neuron and the condition that produces the chaotic response are analyzed, and the convergence conditions are presented.

Key words : Dynamic Chaotic Neuron, Convergence Condition, Transient Chaos

I. INTRODUCTION

Biological neurons generally have chaotic characteristics for permanent or transient period[1]. The chaotic responses of biological neurons have been modeled quantitatively by many researchers. The primitive model was the Hodgkin-Huxley equation.

Caianiello[2,3] and Nagumo-Sato[4] modified this primitive model to make chaotic neural networks. Aihara et al. proposed a discrete time model with continuous output, and applied it to chaotic neural networks[5]. They showed some possibility that chaotic neural networks could be used to solve optimization problems such as traveling salesman problem(TSP). However, the effects of chaotic response have not yet been verified by using analytical methods. The chaotic characteristics of neuron generally cause an adverse effect on optimization problems, but the transient chaos of neuron model could be beneficial to overcoming the local minimum problem. Aihara proposed that the transient chaotic characteristics of neuron could be helpful for global optimization[6,7]. Even though some modifications were made, those previously proposed chaotic neuron models are still complicate to apply to neural networks, and need more dynamic characteristics in neuron itself and

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learning algorithm[8]. In previous paper, we presented a novel modified chaotic neural network for simplification of structure and enforcement of dynamic characteristics, and applied this network to system identification and adaptive control[9]. In this paper, the traditional chaotic neuron model is studied for the analysis of the chaotic characteristics of the chaotic neural network(CNN), and the convergence condition is presented.

II. CHAOTIC NEURON MODELS

1. Traditional Chaotic Neuron Model

The traditional model of a chaotic neuron was suggested by Caianiello[1]. Past excitation inputs give inhibitory influence to inside neurons for refractory period. This inhibitory influence of past firing decreases exponentially with time. Under this assumption, the behavior of a neuron was modeled by a nonlinear differential equation as shown in eqn. (1).

$$x(k+1)=u(A(k)-\alpha\sum_{r=0}^{t}K^{r}x(k-r)-\theta) \tag{1}$$

where $x(k+1)$ is the output of a neuron at discrete time $k+1$, and $x(k)$ takes either 0 or 1. $u(\cdot)$ is a unit step function, $A(k)$ is the strength of the activation input at discrete time k, and K^r is the damping factor of the refractoriness having values between 0 and 1. The constant α is a positive parameter, and θ is the threshold of a chaotic neuron. If the internal state of a neuron i at time $k+1$ is assumed to be given by the following equation[3],

$$y(k+1)=A(k)-\alpha\cdot\sum_{r=0}^{t}K^{r}x(k-r)-\theta \tag{2}$$

Then eqn. (1) can be rewritten as

$$x(k+1)=u[y(k+1)] \tag{3}$$

By using a bifurcation parameter $a(k)$, eqn. (2) can be rewritten as

$$y(k+1)=K^{r}y(k)-\alpha\cdot u[y(k)]+a(k) \tag{4}$$

where $a(k)$ is defined as

$$a(k)=A(k)-K^{r}A(k-1)-(1-K^{r})\cdot\theta \tag{5}$$

The conventional chaotic neuron model, suggested by Nagumo and Sato[4], has two different types of input simultaneously: one from same layer and another form outside. It also has a refractory term, which is self-feedback. The refractory term performs effective dynamic characteristics through repeated signal control of one of three terms, which affect output of the chaotic neuron. The neuron model is shown in fig. 1.

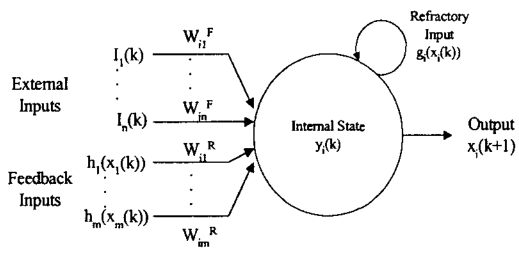


Fig. 1. Chaotic neuron unit

Generally, the dynamics of the ith chaotic neuron in networks at discrete time $k+1$ is expressed as

$$\begin{aligned} x_i(k+1)=f_N[&\sum_{j=1}^{n}w_{ij}^{F}\sum_{r=0}^{k}K_s^rI_j(k-r)\\ &+\sum_{j=1}^{m}w_{ij}^{R}\sum_{r=0}^{k}K_m^rh_j(x_i(k-r))\\ &-\alpha\cdot\sum_{r=0}^{k}K_r^rg_i(x_i(k-r))-\theta_i] \end{aligned} \tag{6}$$

where $f_N(\cdot)$ is a sigmoid function, w_{ij}^F and w_{ij}^R are coupling coefficients(weights) from the jth external neuron and the jth feedback neuron to the ith neuron, respectively. $I_j(k-r)$ is the strength of the jth externally applied input at time $k-r$, $h_j(x_i(k-r))$ is a transfer function of the axon connected to the jth chaotic neuron, and $g_i(x_i(k-r))$ is a refractory function of the ith chaotic neuron at time $k-r$, usually that is an identity function. The n and m are the numbers of external and feedback inputs applied to the chaotic neuron. The decay parameters, K_s^r, K_m^r, and K_r^r are the damping factors of the external, feedback, and refractoriness, respectively. In this paper, we assumed the constant value for decay parameters K. The θ_i is the threshold of the ith chaotic neuron.

Aihara represented the ith chaotic neuron equation in a reduced form by dividing it into feedback, external, and refractory term[5]. Each term is expressed as

$$\xi_i(k+1)=K\cdot\xi_i(k)+\sum_{j=1}^{n}w_{ij}^{F}I_j(k) \tag{7}$$

$$\eta_i(k+1) = K \cdot \eta_i(k) + \sum_{j=1}^{m} w_{ij}^R h_j(f(y_i(k))) \quad (8)$$

$$\zeta_i(k+1) = K \cdot \zeta_i(k) - \alpha g_i(f_i(y_i(k))) - \theta_i(1-K) \quad (9)$$

If the internal state of a chaotic neuron at time $k+1$ is expressed as

$$y_i(k+1) = \xi_i(k+1) + \eta_i(k+1) + \zeta_i(k+1) \quad (10).$$

then the eqn. (10) can be written as follows

$$\begin{aligned} y_i(k+1) &= K \cdot (\xi_i(k) + \eta_i(k) + \zeta_i(k)) \\ &+ \sum_{j=1}^{n} w_{ij}^F I_j(k) + \sum_{j=1}^{m} w_{ij}^R h_j(f_N(y_i(k))) \\ &- \alpha g_i(f_N(y_i(k))) - \theta_i(1-K) \end{aligned} \quad (11).$$

Since $y_i(t)$ is defined as $y_i(t) = \xi_i(t) + \eta_i(t) + \zeta_i(t)$, eqn. (11) can be expressed as

$$\begin{aligned} y_i(k+1) = K \cdot y_i(k) &+ \sum_{j=1}^{n} w_{ij}^F I_j(k) + \sum_{j=1}^{m} w_{ij}^R h_j(f_N(y_i(k))) \\ &- \alpha g_i(f_N(y_i(k))) - \theta_i(1-K) \end{aligned} \quad (12).$$

In order to apply the continuous Hopfield neural network structure to the recurrent inputs, Aihara et al. assumed the symmetric structure of recurrent weights, therefore $w_{ij}^R = w_{ji}^R,\ w_{ii}^R = 0$. This neural network used two kinds of learning rules in the same network. Since this structure reduces the efficiency of learning and dynamic characteristics of network, this model is not appropriate for modeling dynamic systems.

2. Chaotic Dynamic Neuron Model

Since the chaotic neuron model is including complicate nonlinear function, it requires simplification to reduce the computation time. This paper presents a chaotic dynamic neuron unit with same chaotic characteristics. Since the $\alpha g_i(f_N(y_i(k)))$ term in eqn. (12) is overlapped with the term $\sum_{j=1}^{m} w_{ij}^R h_j(f_N(y_i(k)))$ in the case of $i=j$, the $\alpha g_i(f_N(y_i(k)))$ is abbreviated in this modified model. For more simplification, the threshold term $\theta_i(1-K)$, is set to '0', and the nonlinear function $h_j(\cdot)$, is set '1'. Fig. 2 shows modified chaotic neuron unit.

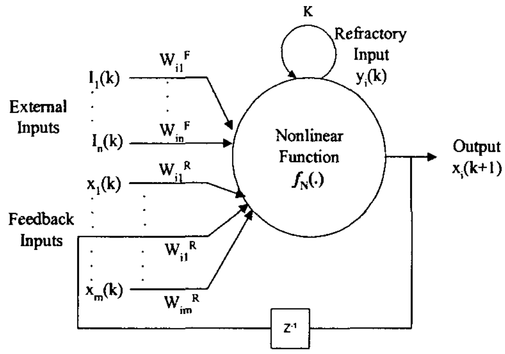


Fig. 2. Chaotic dynamic neuron unit

Then eqn. (6) is reduced to eqn. (14).

$$y_i(k+1) = K \cdot y_i(k) + \sum_{j=1}^{n} w_{ij}^F \cdot I_j(k) + \sum_{j=1}^{m} w_{ij}^R \cdot x_i(k) \quad (13)$$

$$x_i(k+1) = f_N[y_i(k+1)] \quad (14)$$

$$f_N[y_i(k+1)] = \frac{1}{1+e^{-y_i(k+1)/\varepsilon}} \quad (15)$$

where ε is the slope of a sigmoid function.

To increase dynamic characteristics, the nonsymmetric weights are applied to recurrent inputs such that $w_{ij}^R \neq w_{ji}^R,\ w_{ii}^R \neq 0$. The chaotic neuron sums three inputs: the refractoriness $K \cdot y_i(k)$, the activation $\sum_{j=1}^{n} w_{ij}^F \cdot I_j(k)$, and the recurrent input $\sum_{j=1}^{m} w_{ij}^R \cdot x_i(k)$ The summation result is passed to the nonlinear sigmoid function. This model is similar to the transiently chaotic neural network(TCNN) except for the recurrent input term[6], and has similar characteristics as TCNN.

3. Analyzing The Chaotic Characteristics of Chaotic Neuron

The single neuron model in fig. 3 is the traditional chaotic neuron model suggested by Nagumo-sato and Aihara. The simplified single chaotic neuron model is described in eqn. (16) and (17).

$$y(k+1) = K \cdot y(k) - x(k) + A(k) \quad (16)$$

$$x(k+1) = f_N[y_i(k+1)] = \frac{1}{1+e^{-y_i(k+1)/\varepsilon}} \quad (17)$$

where $y(k+1)$ is internal state of chaotic neuron at discrete time $k+1$, $x(k+1)$ is the output of chaotic neuron at discrete time $k+1$, K is the refractory parameter, A(k) is the activation of chaotic neuron at discrete time *k*, and ε is the slope of a sigmoid function. As shown in eqn. (16), the internal state of chaotic neuron is determined by summation of three inputs: refractory, recurrent and activational inputs.

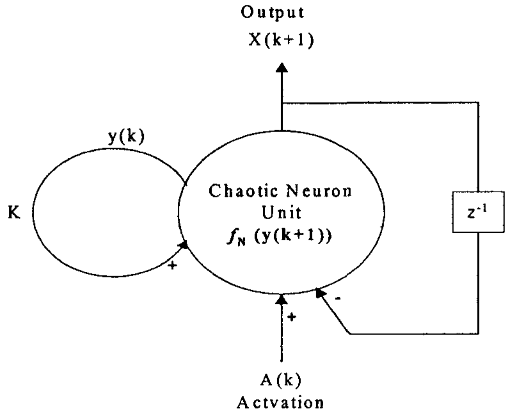
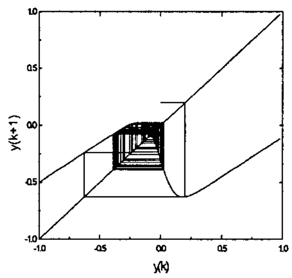
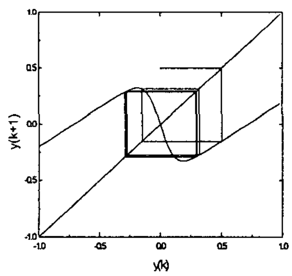


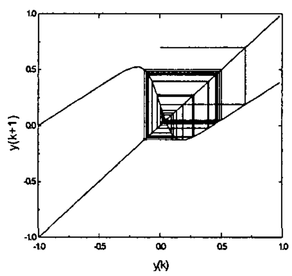
Fig. 3. Single chaotic neuron unit



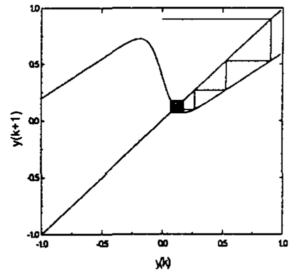
(a) A=0.2



(b) A=0.5



(c) A=0.7



(d) A=0.9

Fig. 4 Output characteristics of CNN for different activation values (In case of refractory K=0.7)

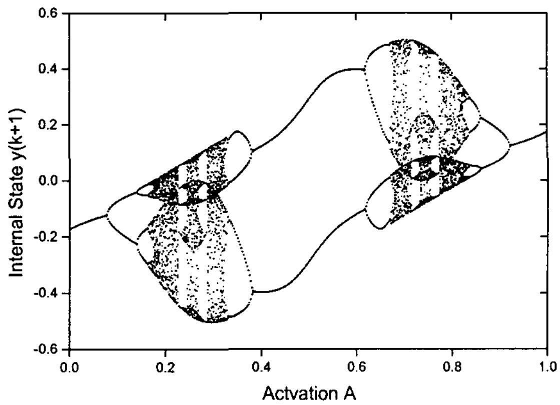


Fig. 5 Bifurcation diagram of logistic map for CNN

The output of neuron is determined by a nonlinear sigmoid function in eqn. (17). The output of neuron has different output characteristics depending on activation values as shown in fig. 4. The fig. 4 shows the poincare map for identification of chaotic characteristics. The slope is fixed at $\varepsilon = 0.06$, the refractory value is also fixed at *K=0.7,* and the activation value has four different values: *A=0.2, A=0.5, A=0.7,* and *A=0.9*. In cases of *A=0.2* and *A=0.7*, the graphs show chaotic outputs. In cases of *A=0.5* and *A=0.9*, the outputs have fixed point. Fig. 4.b shows a limit cycle, and fig. 4.d shows a typical orbit which spirals into the fixed point as $k \to \infty$. Depending on activation values, the output of chaotic neuron shows chaotic characteristics or fixed points.

Fig. 5 shows the bifurcation diagram for logistic map based on modified chaotic neurons. In this case, the slope is fixed at $\varepsilon = 0.06$, the refractory value is also fixed at *K=0.7,* and the activation value varies from 0 to 1. The output is split like period doubling bifurcation, and the map becomes chaotic intermittently.

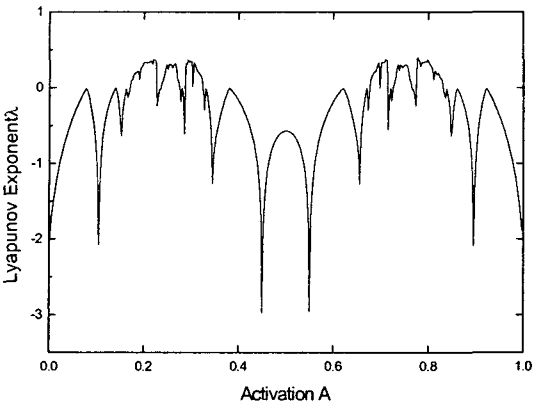


Fig. 6 Lyapunov exponent diagram λ

Even though logistic map in fig. 5 exhibits aperiodic orbits for certain activation values, Lyapunov exponent is more computationally useful for verifying chaotic characteristics. A positive Lyapunov exponent is a signature of chaos. Fig. 6 shows Lyapunov exponent diagram. The formula is defined as

$$\lambda = \lim_{n\to\infty}\{\frac{1}{n}\sum_{i=0}^{n-1}\ln|f'(x_i)|\} \tag{18}$$

where λ is Lyapunov exponent, x_i is the state of orbit in ith iterations, and $f(\cdot)$ is a smooth function. For limited p cycle, Lyapunov exponent can be defined as

$$\lambda \approx \frac{1}{p}\sum_{i=0}^{p-1}\ln|f'(x_i)| \tag{19}$$

In this modified chaotic neural network application, Lyapunov exponent could be defined as

$$\lambda \approx \frac{1}{p}\sum_{i=0}^{p-1}\ln|K-\frac{x_p^2\cdot e^{y_p/\varepsilon}}{\varepsilon}| \tag{20}$$

Fig. 7 shows three dimensional diagram of Lyapunov exponent with variations of slope and activation. Fig. 8 shows the contour map of Lyapunov exponent. These results show that chaotic output is produced intermittently depending on the slope and activation values.

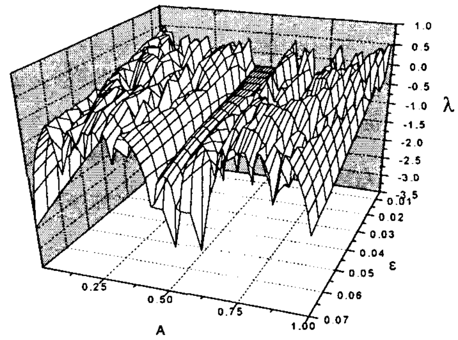


Fig. 7 Lyapunov exponents with variation of slope ε

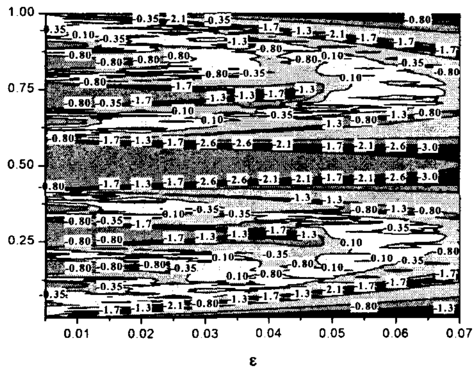


Fig. 8 Contour map of lyapunov exponent

III. CONVERGENCE AND STABILITY OF CHAOTIC DYNAMIC NEURON

1. Single Chaotic Dynamic Neuron Model

A single chaotic dynamic neuron model shown in fig. 9 has three inputs: weighted recurrent input $w^R\cdot f_N(y(k))$, weighted input $w^F\cdot I(k)$, and refractory input $K\cdot y(k)$. The output is determined by a sigmoid function of weighted sum. It can be expressed by eqn. (21) and (22).

$$y(k+1) = K\cdot y(k) + w^F\cdot I(k) + w^R\cdot f_N(y(k)) \tag{21}$$

$$x(k+1) = f_N[y(k+1)] \tag{22}$$

where $f_N(\cdot)$ is a sigmoid function $f_N[y(k+1)] = \frac{1}{1+e^{-y(k+1)/\varepsilon}}$, and K, w^F and w^R are refractory variable, weight for external input, and weight for recurrent input respectively. *I(k)*, *y(k)*, and *x(k)* are the external input, the internal state, and the output of single neuron respectively at time k.

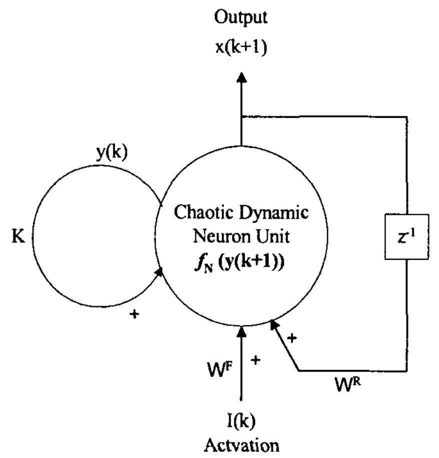


Fig. 9 Single chaotic dynamic neuron unit

2. Convergence and Stability of Chaotic Dynamic Neuron

To solve optimization problems, which are needed in almost all neural network applications such as pattern recognition, identification or prediction, and control, this neuron should have one stable fixed point. Chaotic characteristics or limit cycle in the output of neuron should disappear in permanent in order to solve optimization problems. Therefore, the chaotic characteristics of chaotic neuron should be transient.

Definition 1: Suppose $y^*(k)$ satisfies $y^*(k+1)=f(y^*(k))$, then $y^*(k)$ is an equilibrium point (fixed point); because if $y(k)=y^*(k)$ then $y(k+1)=f(y(k+1))=f(y^*(k))=y^*(k)$. Hence the orbit remains at $y^*(k)$ for all further iterations.

Definition 2: If the derivative of any one dimensional system $y(k+1)=f(y(k))$ has a negative value at the fixed point $y^*(k)$, then the fixed point is stable.

Theorem 1: Let w^R be the recurrent weight for the internal state of chaotic dynamic neural network(CDNN) $y^*(k)$ and S_{f^*} be defined as $S_{f^*}=\partial f_N(y(k))/\partial y(k)|_{y(k)=y^*(k)}$, where $f_N(y(k))$ is the output of neuron with a sigmoid function. The convergence is guaranteed for only one fixed point $y^*(k)$ if w^R has the following condition

$$-(1-K)\cdot S_{f^*}<w^R\le\frac{1-K}{S_{f^*}} \quad (23).$$

Proof: From eqn. (21) and definition 1, the internal state $y(k)$ at the fixed point can be represented as

$$y^*(k)=K\cdot y^*(k)+w^F\cdot I(k)+w^R\cdot f_N(y^*(k)) \quad (24).$$

Eqn. (24) can be rewritten as

$$\frac{(1-k)}{w^R}\cdot y^*(k)-\frac{w^F}{w^R}\cdot I(k)=f_N(y^*(k)) \quad (25).$$

Let $a=\frac{(1-k)}{w^R}$ and $b=\frac{w^F}{w^R}\cdot I(k)$, then

$$a\cdot y^*(k)-b=f_N(y^*(k)) \quad (26).$$

Let $F(y^*(k))=a\cdot y^*(k)-b$ and $G(y^*(k))=f_N(y^*(k))$, then eqn. (26) can be rewritten as

$$F(y^*(k))=G(y^*(k)) \quad (27).$$

If $a\ge 0$, the always stable condition at the fixed point with only one fixed point can be defined by using definition 2 as follows

$$\max|\frac{\partial G(y^*(k))}{\partial y^*(k)}|\le\frac{\partial F(y^*(k))}{\partial y^*(k)}\le\infty \quad (28).$$

Eqn. (28) can be rewritten as

$$S_{f,MAX}\le a\le\infty \quad (29).$$

From eqn. (29), the stability condition related to w^R can be made as

$$0\le w^R\le\frac{1-K}{S_{f,MAX}} \quad (30)$$

If $a\le 0$, the always stable condition at the fixed point with only one fixed point can be defined by using definition 2 as follows

$$-\infty\le a<-\frac{1}{S_{f,MAX}} \quad (31)$$

From eqn. (31), the stability condition related to w^R can be made as

$$-(1-K)\cdot S_{f,MAX}<w^R\le 0 \quad (32)$$

By using eqn. (30) and (31), the stability condition expressed in eqn. (23) is made.

In case $0<a\le S_{f,MAX}$, the fixed points can be classified as three types: 1)two fixed points case, 2) three fixed points case, and 3) one fixed point case.

1) Two fixed point case:

Two fixed points case can happen when the slopes of two functions, $F(y^*(k)), G(y^*(k))$, have the same value as follows

$$\frac{\partial G(y^*(k))}{\partial y^*(k)} \cong \frac{\partial F(y^*(k))}{\partial y^*(k)} \quad (33)$$

In this case eqn. (33) can be represented as

$$a = x^*(k) \cdot (1- x^*(k))/s \quad (34)$$

where $x^*(k)$ is the output of chaotic dynamic neuron at a fixed point, and s is the slope of a sigmoid function. From eqn. (34), $x^*(k)$ and $y^*(k)$ are resolved to

$$x^*(k) = \frac{1 \pm \sqrt{1-4as}}{2a} \quad (35)$$

$$y^*(k) = -s \cdot \ln[\frac{2a}{1 \pm \sqrt{1-4as}} - 1] \quad (36)$$

And eqn. (26) can be rewritten as

$$b = a \cdot y^*(k) - x^*(k)$$

$$= -as \ln[\frac{2a}{1 \pm \sqrt{1-4as}} - 1] - \frac{1 \pm \sqrt{1-4as}}{2a} \quad (37)$$

The fixed point a in eqn. (35) is half-stable, and another fixed point is stable but it locates at near 0 or 1.

2) Three fixed points case:

$$-as \ln[\frac{2a}{1 - \sqrt{1-4as}} - 1] - \frac{1 - \sqrt{1-4as}}{2a} < b$$

$$< -as \ln[\frac{2a}{1 + \sqrt{1-4as}} - 1] - \frac{1 + \sqrt{1-4as}}{2a} \quad (38)$$

Two fixed points near 0 and 1 are stable, and the other fixed point between 0 and 1 is unstable.

3) One fixed point case:

$$b > -as \ln[\frac{2a}{1 + \sqrt{1-4as}} - 1] - \frac{1 + \sqrt{1-4as}}{2a} \quad (39)$$

This case has one stable fixed point which is located at near 1

$$b < -as \ln[\frac{2a}{1 - \sqrt{1-4as}} - 1] - \frac{1 - \sqrt{1-4as}}{2a} \quad (40)$$

This case has one stable fixed point which is located at near 0

In case of $-\frac{1}{S_{f,MAX}} < a \le 0$, there is one fixed point which is unstable by definition 2.

w^R can be limited as

$$-\infty \le w^R < -(1-K) \cdot S_{f,MAX} \quad (41)$$

3. Chaos and limit cycle condition

If $a = -\frac{1}{S_f^*}$, the slopes of $F(y^*(k)) = a \cdot y^*(k) - b$ and $G(y^*(k)) = f_N(y^*(k))$ are perpendicular. The output of chaotic dynamic neuron shows stable limit cycle.

If $a \approx -\frac{1}{S_f^*}$, the slopes of $F(y^*(k)) = a \cdot y^*(k) - b$ and $G(y^*(k)) = f_N(y^*(k))$ are almost perpendicular. The output of chaotic dynamic neuron shows chaos. The result of chaotic response is shown in fig. 4.a and fig. 4.c.

IV. CONCLUSION

In this paper, we presented the convergence condition of the chaotic dynamic neuron. Even though the chaotic response may be helpful to overcoming local minimum problem, the initial chaos should disappear in time such as in the transiently chaotic neuron. Since the chaotic neural network has highly dynamic characteristics and the high slope in the sigmoid function, the network shows fast learning, but also shows some stability problem. To overcome this problem, the convergence and stability condition were presented in this paper. This condition could be applied to chaotic neural networks.

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