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Semiopen sets on intuitionistic fuzzy topological spaces in Sostak's sense

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Abstract

We introduce the concepts of fuzzy (r,s)-semiopen sets and fuzzy (r,s)-semicontinuous mappings on intuitionistic fuzzy topological spaces in Sostak's sense and then we investigate some of their characteristic properties.

Key words : intuitionistic fuzzy topology in Sostak's sense, fuzzy (r,s)-semiopen set, fuzzy (r,s)-semicontinuity

1. Introduction

The concept of fuzzy sets was introduced by Zadeh [11]. Chang [2] defined fuzzy topological spaces. These spaces and its generalizations are later studied by several authors, one of which, developed by Sostak [10], used the idea of degree of openness. This type of generalization of fuzzy topological spaces was later rephrased by Chattopadhyay, Hazra, and Samanta [3], and by Ramadan [9].

As a generalization of fuzzy sets, the concept of intuitionistic fuzzy sets was introduced by Atanassov [1]. Recently, Coker and his colleagues [4,6,7] introduced intuitionistic fuzzy topological spaces using intuitionistic fuzzy sets.

Using the idea of degree of openness and degree of nonopenness, Coker and Demirci [5] defined intuitionistic fuzzy topological spaces in Sostak's sense as a generalization of smooth fuzzy topological spaces and intuitionistic fuzzy topological spaces.

In this paper, we introduce the concepts of fuzzy (r,s)-semiopen sets and fuzzy (r,s)-semicontinuous mappings on intuitionistic fuzzy topological spaces in Sostak's sense and then we investigate some of their characteristic properties.

2. Preliminaries

Let I be the unit interval $[0,1]$ of the real line. A member μ of I^X is called a fuzzy set of X. By $\tilde{0}$ and $\tilde{1}$ we denote constant maps on X with value 0 and 1, respectively. For any $\mu \in I^X$, μ^c denotes the complement $\tilde{1}-\mu$. All other notations are standard notations of fuzzy set theory.

Let X be a nonempty set. An *intuitionistic fuzzy set* A is an ordered pair

$$A=(\mu_A,\gamma_A)$$

where the functions $\mu_A: X\to I$ and $\gamma_A: X\to I$ denote the degree of membership and the degree of nonmembership, respectively, and $\mu_A+\gamma_A \le \tilde{1}$.

Obviously every fuzzy set μ on X is an intuitionistic fuzzy set of the form $(\mu, \tilde{1}-\mu)$.

Definition 2.1 [1] Let $A=(\mu_A,\gamma_A)$ and $B=(\mu_B,\gamma_B)$ be intuitionistic fuzzy sets on X. Then

(1) $A\subseteq B$ iff $\mu_A\le\mu_B$ and $\gamma_A\ge\gamma_B$.
(2) $A=B$ iff $A\subseteq B$ and $B\subseteq A$.
(3) $A^c=(\gamma_A,\mu_A)$.
(4) $A\cap B=(\mu_A\wedge\mu_B,\gamma_A\vee\gamma_B)$.
(5) $A\cup B=(\mu_A\vee\mu_B,\gamma_A\wedge\gamma_B)$.
(6) $0_\sim=(\tilde{0},\tilde{1})$ and $1_\sim=(\tilde{1},\tilde{0})$.

Let f be a map from a set X to a set Y. Let $A=(\mu_A,\gamma_A)$ be an intuitionistic fuzzy set of X and $B=(\mu_B,\gamma_B)$ an intuitionistic fuzzy set of Y. Then:

(1) The image of A under f, denoted by $f(A)$ is an intuitionistic fuzzy set in Y defined by

$$f(A)=(f(\mu_A),\tilde{1}-f(\tilde{1}-\gamma_A)).$$

(2) The inverse image of B under f, denoted by $f^{-1}(B)$ is an intuitionistic fuzzy set in X defined by

$$f^{-1}(B)=(f^{-1}(\mu_B),f^{-1}(\gamma_B)).$$

A *smooth fuzzy topology* on X is a map $T: I^X\to I$ which satisfies the following properties:

(1) $T(\tilde{0})=T(\tilde{1})=1$.

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(2) $T(\mu_1 \wedge \mu_2) \geq T(\mu_1) \wedge T(\mu_2)$.
(3) $T(\vee \mu_i) \geq \wedge T(\mu_i)$.
The pair (X, T) is called a *smooth fuzzy topological space.*

An *intuitionistic fuzzy topology* on X is a family T of intuitionistic fuzzy sets in X which satisfies the following properties:
(1) $0_\sim, 1_\sim \in T$.
(2) If $A_1, A_2 \in T$, then $A_1 \cap A_2 \in T$.
(3) If $A_i \in T$ for all i, then $\cup A_i \in T$.
The pair (X, T) is called an *intuitionistic fuzzy topological space.*

Let $I(X)$ be a family of all intuitionistic fuzzy sets of X and let $I \otimes I$ be the set of the pair (r, s) such that $r, s \in I$ and $r + s \leq 1$.

Definition 2.2 [5] Let X be a nonempty set. An *intuitionistic fuzzy topology in Sostak's sense* (SoIFT for short) $\tau = (\tau_1, \tau_2)$ on X is a map $\tau : I(X) \to I \otimes I$ which satisfies the following properties:
(1) $\tau_1(0_\sim) = \tau_1(1_\sim) = 1$ and $\tau_2(0_\sim) = \tau_2(1_\sim) = 0$.
(2) $\tau_1(A \cap B) \geq \tau_1(A) \wedge \tau_1(B)$ and $\tau_2(A \cap B) \leq \tau_2(A) \vee \tau_2(B)$.
(3) $\tau_1(\cup A_i) \geq \wedge \tau_1(A_i)$ and $\tau_2(\cup A_i) \leq \vee \tau_2(A_i)$.
The $(X, \tau) = (X, \tau_1, \tau_2)$ is said to be an *intuitionistic fuzzy topological space in Sostak's sense* (SoIFTS for short). Also, we call $\tau_1(A)$ a *gradation of openness* of A and $\tau_2(A)$ a *gradation of nonopenness* of A.

Definition 2.3 [8] Let A be an intuitionistic fuzzy set in a SoIFTS (X, τ_1, τ_2) and $(r, s) \in I \otimes I$. Then A is said to be
(1) *fuzzy (r, s)-open* if $\tau_1(A) \geq r$ and $\tau_2(A) \leq s$,
(2) *fuzzy (r, s)-closed* if $\tau_1(A^c) \geq r$ and $\tau_2(A^c) \leq s$.

Definition 2.4 [8] Let (X, τ_1, τ_2) be a SoIFTS. For each $(r, s) \in I \otimes I$ and for each $A \in I(X)$, the *fuzzy (r, s)-interior* is defined by

$$\mathrm{int}(A, r, s) = \cup \{B \in I(X) \mid A \supseteq B, \ B \text{ is fuzzy } (r, s)\text{-open}\}$$

and the *fuzzy (r, s)-closure* is defined by

$$\mathrm{cl}(A, r, s) = \cap \{B \in I(X) \mid A \subseteq B, \ B \text{ is fuzzy } (r, s)\text{-closed}\}.$$

The operators $\mathrm{int} : I(X) \times I \otimes I \to I(X)$ and $\mathrm{cl} : I(X) \times I \otimes I \to I(X)$ are called the *fuzzy interior operator* and *fuzzy closure operator* in (X, τ_1, τ_2), respectively.

Lemma 2.5 [8] For an intuitionistic fuzzy set A in a SoIFTS (X, τ_1, τ_2) and $(r, s) \in I \otimes I$,
(1) $\mathrm{int}(A, r, s)^c = \mathrm{cl}(A^c, r, s)$.
(2) $\mathrm{cl}(A, r, s)^c = \mathrm{int}(A^c, r, s)$.

Let (X, τ) be an intuitionistic fuzzy topological space in Sostak's sense. Then it is easy to see that for each $(r, s) \in I \otimes I$, the family $\tau_{(r,s)}$ defined by

$$\tau_{(r,s)} = \{A \in I(X) \mid \tau_1(A) \geq r \text{ and } \tau_2(A) \leq s\}$$

is an intuitionistic fuzzy topology on X.

Let (X, T) be an intuitionistic fuzzy topological space and $(r, s) \in I \otimes I$. Then the map $T^{(r,s)} : I(X) \to I \otimes I$ defined by

$$T^{(r,s)}(A) = \begin{cases} (1, 0) & \text{if } A = 0_\sim, 1_\sim, \\ (r, s) & \text{if } A \in T - \{0_\sim, 1_\sim\}, \\ (0, 1) & \text{otherwise} \end{cases}$$

becomes an intuitionistic fuzzy topology in Sostak's sense on X.

3. Fuzzy (r, s)-semiopen sets

Definition 3.1 Let A be an intuitionistic fuzzy set in a SoIFTS (X, τ_1, τ_2) and $(r, s) \in I \otimes I$. Then A is said to be
(1) *fuzzy (r, s)-semiopen* if there is a fuzzy (r, s)-open set B in X such that $B \subseteq A \subseteq \mathrm{cl}(B, r, s)$,
(2) *fuzzy (r, s)-semiclosed* if there is a fuzzy (r, s)-closed set B in X such that $\mathrm{int}(B, r, s) \subseteq A \subseteq B$.

It is obvious that every fuzzy (r, s)-open ((r, s)-closed) set is a fuzzy (r, s) semiopen ((r, s)-semiclosed) set but the converse need not be true which is shown by the following example.

Example 3.2 Let $X = \{x, y\}$ and let A_1 and A_2 be intuitionistic fuzzy set of X defined as

$$A_1(x) = (0.6, 0.3), \quad A_1(y) = (0.3, 0.5);$$

and

$$A_2(x) = (0.8, 0.1), \quad A_2(y) = (0.5, 0.4).$$

Define $\tau : I(X) \to I \otimes I$ by

$$\tau(A) = (\tau_1(A), \tau_2(A)) = \begin{cases} (1, 0) & \text{if } A = 0_\sim, 1_\sim, \\ (\frac{1}{2}, \frac{1}{3}) & \text{if } A = A_1, \\ (0, 1) & \text{otherwise.} \end{cases}$$

Then clearly (τ_1, τ_2) is a SoIFT on X. The intuitionistic fuzzy set A_2 is $(\frac{1}{2}, \frac{1}{3})$-semiopen which is not $(\frac{1}{2}, \frac{1}{3})$-open. Also, A_2^c is $(\frac{1}{2}, \frac{1}{3})$-semiclosed which is not

$(\frac{1}{2}, \frac{1}{3})$-closed.

Theorem 3.3 Let A be an intuitionistic fuzzy set in a SoIFTS (X, τ_1, τ_2) and $(r,s) \in I \otimes I$. Then the following statements are equivalent:

(1) A is a fuzzy (r,s)-semiopen set.
(2) A^c is a fuzzy (r,s)-semiclosed set.
(3) $\mathrm{cl}(\mathrm{int}(A,r,s),r,s) \supseteq A$.
(4) $\mathrm{int}(\mathrm{cl}(A^c,r,s),r,s) \subseteq A^c$.

Proof. (1) → (3) Let A be a fuzzy (r,s)-semiopen set of X. Then there is a fuzzy (r,s)-open set B in X such that $B \subseteq A \subseteq \mathrm{cl}(B,r,s)$. So

$$B = \mathrm{int}(B,r,s) \subseteq \mathrm{int}(A,r,s).$$

Hence

$$A \subseteq \mathrm{cl}(B,r,s) \subseteq \mathrm{cl}(\mathrm{int}(A,r,s)).$$

(3) → (1) Let $\mathrm{cl}(\mathrm{int}(A,r,s),r,s) \supseteq A$ and take $B = \mathrm{int}(A,r,s)$. Then B is a fuzzy (r,s)-open set and

$$B = \mathrm{int}(A,r,s) \subseteq A \subseteq \mathrm{cl}(\mathrm{int}(A,r,s),r,s) = \mathrm{cl}(B,r,s).$$

Thus A is a fuzzy (r,s)-semiopen set.

(2) ↔ (4) It is similar to the proof of (1) ↔ (3).

(3) ↔ (4) It follows from Lemma 2.5.

Theorem 3.4 Let (X, τ_1, τ_2) be a SoIFTS and $(r,s) \in I \otimes I$.

(1) If A is fuzzy (r,s)-semiopen and $\mathrm{int}(A,r,s) \subseteq B \subseteq \mathrm{cl}(A,r,s)$, then B is fuzzy (r,s)-semiopen.

(2) If A is fuzzy (r,s)-semiclosed and $\mathrm{int}(A,r,s) \subseteq B \subseteq \mathrm{cl}(A,r,s)$, then B is fuzzy (r,s)-semiclosed.

Proof. (1) Let A be a fuzzy (r,s)-semiopen set and $\mathrm{int}(A,r,s) \subseteq B \subseteq \mathrm{cl}(A,r,s)$. Then there is a fuzzy (r,s)-open set C such that $C \subseteq A \subseteq \mathrm{cl}(C,r,s)$. It follows that

$$C = \mathrm{int}(C,r,s) \subseteq \mathrm{int}(A,r,s) \subseteq A \subseteq \mathrm{cl}(A,r,s) \subseteq \mathrm{cl}(\mathrm{cl}(C,r,s),r,s) = \mathrm{cl}(C,r,s)$$

and hence

$$C \subseteq \mathrm{int}(A,r,s) \subseteq B \subseteq \mathrm{cl}(A,r,s) \subseteq \mathrm{cl}(C,r,s).$$

Thus B is a fuzzy (r,s)-semiopen set.

(2) Similar to (1).

Theorem 3.5 Let (X, τ_1, τ_2) be a SoIFTS and $(r,s) \in I \otimes I$.

(1) If $\{A_i\}$ is a family of fuzzy (r,s)-semiopen sets of X, then $\cup A_i$ is fuzzy (r,s)-semiopen.

(2) If $\{A_i\}$ is a family of fuzzy (r,s)-semiclosed sets of X, then $\cap A_i$ is fuzzy (r,s)-semiclosed.

Proof. (1) Let $\{A_i\}$ be a collection of fuzzy (r,s)-semiopen sets. Then for each i, there is a fuzzy (r,s)-open set B_i such that $B_i \subseteq A_i \subseteq \mathrm{cl}(B_i,r,s)$. Since $\tau_1(\cup B_i) \geq \wedge \tau_1(B_i) \geq r$ and $\tau_2(\cup B_i) \leq \vee \tau_2(B_i) \leq s$, $\cup B_i$ is a fuzzy (r,s)-open set. Moreover,

$$\cup B_i \subseteq \cup A_i \subseteq \cup \mathrm{cl}(B_i,r,s) \subseteq \mathrm{cl}(\cup B_i,r,s).$$

Hence $\cup A_i$ is a fuzzy (r,s)-semiopen set.

(2) It follows from (1) using Theorem 3.3.

Definition 3.6 Let (X, τ_1, τ_2) be a SoIFTS. For each $(r,s) \in I \otimes I$ and for each $A \in I(X)$, the *fuzzy (r,s)-semiinterior* is defined by

$$\mathrm{sint}(A,r,s) = \cup \{B \in I(X) \mid A \supseteq B, B \text{ is fuzzy } (r,s)\text{-semiopen}\}$$

and the *fuzzy (r,s)-semiclosure* is defined by

$$\mathrm{scl}(A,r,s) = \cap \{B \in I(X) \mid A \subseteq B, B \text{ is fuzzy } (r,s)\text{-semiclosed}\}.$$

Obviously $\mathrm{scl}(A,r,s)$ is the smallest fuzzy (r,s)-semiclosed set which contains A and $\mathrm{sint}(A,r,s)$ is the greatest fuzzy (r,s)-semiopen set which is contained in A. Also, $\mathrm{scl}(A,r,s) = A$ for any fuzzy (r,s)-semiclosed set A and $\mathrm{sint}(A,r,s) = A$ for any fuzzy (r,s)-semiopen set A. Moreover, we have

$$\mathrm{int}(A,r,s) \subseteq \mathrm{sint}(A,r,s) \subseteq A \subseteq \mathrm{scl}(A,r,s) \subseteq \mathrm{cl}(A,r,s).$$

Also, we have the following results:

(1) $\mathrm{scl}(0_\sim,r,s) = 0_\sim$, $\mathrm{scl}(1_\sim,r,s) = 1_\sim$.
(2) $\mathrm{scl}(A,r,s) \supseteq A$.
(3) $\mathrm{scl}(A \cup B,r,s) \supseteq \mathrm{scl}(A,r,s) \cup \mathrm{scl}(B,r,s)$.
(4) $\mathrm{scl}(\mathrm{scl}(A,r,s),r,s) = \mathrm{scl}(A,r,s)$.
(5) $\mathrm{sint}(0_\sim,r,s) = 0_\sim$, $\mathrm{sint}(1_\sim,r,s) = 1_\sim$.
(6) $\mathrm{sint}(A,r,s) \subseteq A$.
(7) $\mathrm{sint}(A \cap B,r,s) \subseteq \mathrm{sint}(A,r,s) \cap \mathrm{sint}(B,r,s)$.
(8) $\mathrm{sint}(\mathrm{sint}(A,r,s),r,s) = \mathrm{sint}(A,r,s)$.

Theorem 3.7 For an intuitionistic fuzzy set A of a SoIFTS (X, τ_1, τ_2) and $(r,s) \in I \otimes I$, we have:

(1) $\mathrm{sint}(A,r,s)^c = \mathrm{scl}(A^c,r,s)$.
(2) $\mathrm{scl}(A,r,s)^c = \mathrm{sint}(A^c,r,s)$.

Proof. (1) Since $\mathrm{sint}(A,r,s) \subseteq A$ and $\mathrm{sint}(A,r,s)$ is fuzzy (r,s)-semiopen in X, $A^c \subseteq \mathrm{sint}(A,r,s)^c$ and $\mathrm{sint}(A,r,s)^c$ is fuzzy (r,s)-semiclosed in X. Thus

$$\mathrm{scl}(A^c,r,s) \subseteq \mathrm{scl}(\mathrm{sint}(A,r,s)^c,r,s) = \mathrm{sint}(A,r,s)^c.$$

Conversely, since $A^c \subseteq \mathrm{scl}(A^c,r,s)$ and $\mathrm{scl}(A^c,r,s)$ is fuzzy (r,s)-semiclosed in X, $\mathrm{scl}(A^c,r,s)^c \subseteq A$ and

$\mathrm{scl}(A^c, r, s)^c$ is fuzzy (r, s)-semiopen in X. Thus

$$\mathrm{scl}(A^c, r, s)^c = \mathrm{sint}(\mathrm{scl}(A^c, r, s)^c, r, s) \subseteq \mathrm{sint}(A, r, s)$$

and hence $\mathrm{sint}(A, r, s)^c \subseteq \mathrm{scl}(A^c, r, s)$.

(2) Similar to (1).

Definition 3.8 Let $f: (X, \tau_1, \tau_2) \to (Y, w_1, w_2)$ be a mapping from a SoIFTS X to another SoIFTS Y and $(r, s) \in I \otimes I$. Then f is said to be

(1) *fuzzy (r, s)-semicontinuous* if $f^{-1}(B)$ is a fuzzy (r, s)-semiopen set of X for each fuzzy (r, s)-open set B of Y,

(2) *fuzzy (r, s)-semiopen* if $f(A)$ is a fuzzy (r, s)-semiopen set of Y for each fuzzy (r, s)-open set A of X,

(3) *fuzzy (r, s)-semiclosed* if $f(A)$ is a fuzzy (r, s)-semiclosed set of Y for each fuzzy (r, s)-closed set A of X.

It need not be true that f and g are fuzzy (r, s)-semicontinuous ((r, s)-semiopen and (r, s)-semiclosed, respectively) mappings then so is $g \circ f$. But we have the following theorem.

Theorem 3.9 Let (X, τ_1, τ_2), (Y, w_1, w_2) and (Z, σ_1, σ_2) be SoIFTSs and let $f: X \to Y$ and $g: Y \to Z$ be mappings and $(r, s) \in I \otimes I$. Then the following statements are true:

(1) If f is a fuzzy (r, s)-semicontinuous mapping and g is a fuzzy (r, s)-continuous mapping, then $g \circ f$ is a fuzzy (r, s)-semicontinuous mapping.

(2) If f is a fuzzy (r, s)-open mapping and g is a fuzzy (r, s) semiopen mapping, then $g \circ f$ is a fuzzy (r, s)-semiopen mapping.

(3) If f is a fuzzy (r, s)-closed mapping and g is a fuzzy (r, s)-semiclosed mapping, then $g \circ f$ is a fuzzy (r, s)-semiclosed mapping.

Proof. Straightforward.

Theorem 3.10 Let $f: (X, \tau_1, \tau_2) \to (Y, w_1, w_2)$ be a mapping and $(r, s) \in I \otimes I$. Then the following statements are equivalent:

(1) f is a fuzzy (r, s)-semicontinuous mapping.

(2) $f^{-1}(B)$ is a fuzzy (r, s)-semiclosed set of X for each fuzzy (r, s)-closed set B of Y.

(3) $\mathrm{int}(\mathrm{cl}(f^{-1}(B), r, s), r, s) \subseteq f^{-1}(\mathrm{cl}(B, r, s))$ for each intuitionistic fuzzy set B of Y.

(4) $f(\mathrm{int}(\mathrm{cl}(A, r, s), r, s)) \subseteq \mathrm{cl}(f(A), r, s)$ for each intuitionistic fuzzy set A of X.

Proof.(1) $\leftrightarrow$ (2) It is obvious.

(2) $\to$ (3) Let B be any intuitionistic fuzzy set of Y. Then $\mathrm{cl}(B, r, s)$ is a fuzzy (r, s)-closed set of Y. By (2), $f^{-1}(\mathrm{cl}(B, r, s))$ is a fuzzy (r, s)-semiclosed set of X. By Theorem 3.3,

$$f^{-1}(\mathrm{cl}(B, r, s)) \supseteq \mathrm{int}(\mathrm{cl}(f^{-1}(\mathrm{cl}(B, r, s)), r, s), r, s) \supseteq \mathrm{int}(\mathrm{cl}(f^{-1}(B), r, s), r, s).$$

(3) $\to$ (4) Let A be any intuitionistic fuzzy set of X. Then $f(A)$ is an intuitionistic fuzzy set of Y. By (3),

$$f^{-1}(\mathrm{cl}(f(A), r, s)) \supseteq \mathrm{int}(\mathrm{cl}(f^{-1}f(A), r, s), r, s) \supseteq \mathrm{int}(\mathrm{cl}(A, r, s), r, s).$$

Hence

$$\mathrm{cl}(f(A), r, s) \supseteq ff^{-1}(\mathrm{cl}(f(A), r, s)) \supseteq f(\mathrm{int}(\mathrm{cl}(A, r, s), r, s)).$$

(4) $\to$ (2) Let B be any fuzzy (r, s)-closed set of Y. Then $f^{-1}(B)$ is an intuitionistic fuzzy set of X. By (4),

$$f(\mathrm{int}(\mathrm{cl}(f^{-1}(B), r, s), r, s)) \subseteq \mathrm{cl}(ff^{-1}(B), r, s) \subseteq \mathrm{cl}(B, r, s) = B$$

and hence

$$\mathrm{int}(\mathrm{cl}(f^{-1}(B), r, s), r, s) \subseteq f^{-1}f(\mathrm{int}(\mathrm{cl}(f^{-1}(B), r, s), r, s)) \subseteq f^{-1}(B).$$

Thus $f^{-1}(B)$ is a fuzzy (r, s)-semiclosed set of X.

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