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구간값 퍼지집합, Intuitionistic 퍼지집합, Bipolar-valued 퍼지집합의 비교

Comparison of Interval-valued fuzzy sets, Intuitionistic fuzzy sets, and bipolar-valued fuzzy sets

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Abstract

There are several kinds of fuzzy set extensions in the fuzzy set theory. Among them, this paper is concerned with interval-valued fuzzy sets, intuitionistic fuzzy sets, and bipolar-valued fuzzy sets. In interval-valued fuzzy sets, membership degrees are represented by an interval value that reflects the uncertainty in assigning membership degrees. In intuitionistic fuzzy sets, membership degrees are described with a pair of a membership degree and a nonmembership degree. In bipolar-valued fuzzy sets, membership degrees are specified by the satisfaction degrees to a constraint and its counter-constraint. This paper investigates the similarities and differences among these fuzzy set representations.

Key words : interval-valued fuzzy sets, intuitionistic fuzzy sets, bipolar-valued fuzzy sets

1. Introduction

Fuzzy sets are a kind of useful mathematical structure to represent a collection of objects whose boundary is vague. In fuzzy sets, membership degrees indicate the degree of belongingness of elements to the collection or the degree of satisfaction of elements to the property corresponding to the collection.

There have been proposed several kinds of extensions for fuzzy sets.[1] Type 2 fuzzy sets represent membership degrees with fuzzy sets. L-fuzzy sets are a kind of fuzzy set extension to enlarge the range of membership degree [0,1] into a lattice structure. Interval-valued fuzzy sets represent the membership degree with interval values to reflect the uncertainty in assigning membership degrees.[6] Intuitionistic fuzzy sets have membership degrees that are a pair of membership degree and nonmembership degree.[2] Bipolar-valued fuzzy sets have membership degrees that represent the degree of satisfaction to the property corresponding to a fuzzy set and its counter-property.[4] In this study, we are concerned with these three fuzzy set extensions: interval-valued fuzzy sets, intuitionistic fuzzy sets, and bipolar-valued fuzzy sets. These fuzzy sets have some similarities and some differences in their representation and semantics.

This paper is organized as follows: Section 2, 3, and 4 briefly describe the interval-valued fuzzy sets, the intuitionistic fuzzy sets, and the bipolar-valued fuzzy sets, respectively. Section 5 compares the interval-valued fuzzy sets with intuitionistic fuzzy sets, and Section 6 compares intuitionistic fuzzy sets with bipolar-valued fuzzy sets. Section 7 gives some examples to use these fuzzy set representations. Finally Section 8 draws conclusions.

2. Interval-valued Fuzzy Sets

Interval-valued fuzzy sets are an extension of fuzzy sets, where membership degrees of elements can be intervals of real numbers in [0,1]. An interval-valued fuzzy set A is formally defined by membership functions of the form

$$A = \{(x,\ \mu_A(x)) \mid x \in X\}$$
$$\mu_A(x) : X \rightarrow P([0,1]),$$

where $\mu_A(x)$ is a closed interval in [0,1] for each $x \in X$.[6]

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Suppose that A and B are interval-valued fuzzy sets whose membership degrees of elements x are represented like this:

$$\mu_A(x) = [\mu_A^l(x), \mu_A^r(x)]$$
$$\mu_B(x) = [\mu_B^l(x), \mu_B^r(x)]$$

The basic set operations for interval-valued fuzzy sets are defined as follows:

$$A \cup B = \{(x, \mu_{A\cup B}(x)) \mid x \in X\}$$
$$\mu_{A\cup B}(x) = [\mu_{A\cup B}^l(x), \mu_{A\cup B}^r(x)]$$
$$\mu_{A\cup B}^l(x) = \max\{\mu_A^l(x), \mu_B^l(x)\}$$
$$\mu_{A\cup B}^r(x) = \max\{\mu_A^r(x), \mu_B^r(x)\}$$
$$A \cap B = \{(x, \mu_{A\cap B}(x)) \mid x \in X\}$$
$$\mu_{A\cap B}(x) = [\mu_{A\cap B}^l(x), \mu_{A\cap B}^r(x)]$$
$$\mu_{A\cap B}^l(x) = \min\{\mu_A^l(x), \mu_B^l(x)\}$$
$$\mu_{A\cap B}^r(x) = \min\{\mu_A^r(x), \mu_B^r(x)\}$$
$$\overline{A} = \{(x, \mu_{\overline{A}}(x)) \mid x \in X\}$$
$$\mu_{\overline{A}}(x) = [\mu_{\overline{A}}^l(x), \mu_{\overline{A}}^r(x)]$$
$$\mu_{\overline{A}}^l(x) = 1 - \mu_A^r(x)$$
$$\mu_{\overline{A}}^r(x) = 1 - \mu_A^l(x)$$

In interval-valued fuzzy sets, interval values are used as membership degrees in order to express some uncertainties in assigning membership degrees. The larger the interval is, the more uncertainty there is in assigning membership degrees.

3. Intuitionistic Fuzzy Sets

The intuitionistic fuzzy set theory is an extension of the fuzzy set theory by Atanassov[2]. Here we give some basic definitions for the intuitionistic fuzzy sets. Let a set X be the universe of discourse. An intuitionistic fuzzy set A in X is an object having the form

$$A = \{(x,\ \mu_A(x),\ \nu_A(x)) \mid x \in X\},$$

where the functions $\mu_A(x) : X \rightarrow [0,1]$ and $\nu_A(x) : X \rightarrow [0,1]$ define the degree of membership and the degree of non-membership respectively of the element $x \in X$ to the set A, which is a subset of X, and for every $x \in X$,

$$0 \le \mu_A(x) + \nu_A(x) \le 1.$$

The amount $\pi_A(x) = 1 - (\mu_A(x) + \nu_A(x))$ is called the hesitation part or intuitionistic index, which may cater to either membership degree or non-membership degree. It means that the intuitionistic fuzzy sets are a representation to express the uncertainty in assigning membership degrees to elements.

If A and B are two intuitionistic fuzzy sets on the set X, their basic set operations are defined as follows[2]:

$$A \cup B = \{(x, \mu_{A\cup B}(x), \nu_{A\cup B}(x)) \mid x \in X\}$$
$$\mu_{A\cup B}(x) = \max\{\mu_A(x), \mu_B(x)\}$$
$$\nu_{A\cup B}(x) = \min\{\nu_A(x), \nu_B(x)\}$$
$$A \cap B = \{(x, \mu_{A\cap B}(x), \nu_{A\cap B}(x)) \mid x \in X\}$$
$$\mu_{A\cap B}(x) = \min\{\mu_A(x), \mu_B(x)\}$$
$$\nu_{A\cap B}(x) = \max\{\nu_A(x), \nu_B(x)\}$$
$$\overline{A} = \{(x, \mu_{\overline{A}}(x), \nu_{\overline{A}}(x)) \mid x \in X\}$$
$$\mu_{\overline{A}}(x) = \nu_A(x)$$
$$\nu_{\overline{A}}(x) = \mu_A(x)$$

4. Bipolar-valued Fuzzy Sets

Bipolar-valued fuzzy sets are an extension of fuzzy sets whose membership degree range is enlarged from the interval [0, 1] to [−1, 1]. In a bipolar-valued fuzzy set, the membership degree 0 means that elements are irrelevant to the corresponding property, the membership degrees on (0,1] indicate that elements somewhat satisfy the property, and the membership degrees on [−1,0) indicate that elements somewhat satisfy the implicit counter-property. [4]

In bipolar-valued fuzzy sets, two kinds of representation are used: canonical representation and reduced representation. In the canonical representation, membership degrees are expressed with a pair of a positive membership value and a negative membership value. That is, the membership degrees are divided into two parts: positive part in [0, 1] and negative part in [−1, 0]. In the reduced representation, membership degrees are presented with a value in [−1, 1]. The following gives the definitions for those representation methods. Let X be the universe of discourse. The canonical representation of a bipolar-valued fuzzy set A on the domain X has the following shape:

$$A = \{(x, (\mu_A^P(x), \mu_A^N(x))) \mid x \in X\}$$
$$\mu_A^P(x) : X \rightarrow [0,1]$$
$$\mu_A^N(x) : X \rightarrow [-1,0]$$

The positive membership degree $\mu_A^P(x)$ denotes the satisfaction degree of an element x to the property corresponding to a bipolar-valued fuzzy set A, and the negative membership degree $\mu_A^N(x)$ denotes the satisfaction degree of x to some implicit counter-property of A. If $\mu_A^P(x) \neq 0$ and $\mu_A^N(x) = 0$, it is the situation that x is regarded as having only positive satisfaction for A. If $\mu_A^P(x) = 0$ and $\mu_A^N(x) \neq 0$, it is the situation that x does not satisfy the property of A but somewhat satisfies the counter-property of A. In the canonical representation, it is possible for elements x to be $\mu_A^P(x) \neq 0$ and $\mu_A^N(x) \neq 0$ when the membership function of the property overlaps that of its counter-property over some portion of the domain.

The reduced representation of a bipolar-valued fuzzy set A on the domain X has the following shape:

$$A = \{(x, \mu_A^R(x)) \mid x \in X\}$$
$$\mu_A^R : X \to [-1, 1]$$

The membership degree $\mu_A^R(x)$ for the reduced representation can be derived from its canonical representation as follows:

$$\mu_A^R(x) = \begin{cases} \mu_A^P(x) & \text{if } \mu_A^N(x) = 0 \\ \mu_A^N(x) & \text{if } \mu_A^P(x) = 0 \\ f(\mu_A^P(x), \mu_A^N(x)) & \text{otherwise} \end{cases}$$

Here $f(\mu_A^P(x), \mu_A^N(x))$ is an aggregation function to merge a pair of positive and negative membership values into a value. Such aggregation functions $f(\mu_A^P(x), \mu_A^N(x))$ can be defined in various ways. The choice of the aggregation function may depend on the application domains.[4]

Suppose that there are two bipolar-valued fuzzy sets A and B expressed in the canonical representation as follows:

$$A = \{(x, (\mu_A^P(x), \mu_A^N(x))) \mid x \in X\}$$
$$B = \{(x, (\mu_B^P(x), \mu_B^N(x))) \mid x \in X\}$$

The set operations for bipolar-valued fuzzy sets are defined as follows:

$$A \cup B = \{(x, \mu_{A\cup B}(x)) \mid x \in X\}$$
$$\mu_{A\cup B}(x) = (\mu_{A\cup B}^P(x), \mu_{A\cup B}^N(x))$$
$$\mu_{A\cup B}^P(x) = \max\{\mu_A^P(x), \mu_B^P(x)\}$$
$$\mu_{A\cup B}^N(x) = \min\{\mu_A^N(x), \mu_B^N(x)\}$$
$$A \cap B = \{(x, \mu_{A\cap B}(x)) \mid x \in X\}$$
$$\mu_{A\cap B}(x) = (\mu_{A\cap B}^P(x), \mu_{A\cap B}^N(x))$$
$$\mu_{A\cap B}^P(x) = \min\{\mu_A^P(x), \mu_B^P(x)\}$$
$$\mu_{A\cap B}^N(x) = \max\{\mu_A^N(x), \mu_B^N(x)\}$$
$$\overline{A} = \{(x, \mu_{\overline{A}}(x)) \mid x \in X\}$$
$$\mu_{\overline{A}}(x) = (\mu_{\overline{A}}^P(x), \mu_{\overline{A}}^N(x))$$
$$\mu_{\overline{A}}^P(x) = 1 - \mu_A^P(x)$$
$$\mu_{\overline{A}}^N(x) = -1 - \mu_A^N(x)$$

5. Comparison of Interval-valued Fuzzy Sets with Intuitionistic Fuzzy Sets

Intuitionistic fuzzy sets can be regarded as another expression for interval-valued fuzzy sets. According to this interpretation, we can convert an intuitionistic fuzzy set into an interval-valued fuzzy set as follows:

Intuitionistic fuzzy sets

$$A = \{(x, \mu_A(x), \nu_A(x)) \mid x \in X\},$$

Interval valued fuzzy sets

$$A = \{(x, [\mu_A^l(x), \mu_A^r(x)]) \mid x \in X\}$$

where, $\mu_A^l(x) = \mu_A(x)$, $\mu_A^r(x) = 1 - \nu_A(x)$

From the correspondence between boundary values of interval membership degrees in interval-valued fuzzy sets and the pairs of membership and nonmembership degrees in intuitionistic fuzzy sets, we can deduce that the basic set operations for interval-valued fuzzy sets and intuitionistic fuzzy sets have the same roles. To begin with, let us see the case of union operations.

$$A \cup B = \{(x, [\mu_{A\cup B}^l(x), \mu_{A\cup B}^r(x)]) \mid x \in X\}$$
$$\mu_{A\cup B}^l(x) = \max\{\mu_A^l(x), \mu_B^l(x)\}$$
$$\mu_{A\cup B}^r(x) = \max\{\mu_A^r(x), \mu_B^r(x)\}$$

The lower bound $\mu_{A\cup B}^l(x) = \max\{\mu_A^l(x), \mu_B^l(x)\}$ of interval-valued fuzzy set union can be transformed by the correspondence relationship $\mu_A^l(x) = \mu_A(x)$ like this:

$$\mu_{A\cup B}^l(x) = \max\{\mu_A^l(x), \mu_B^l(x)\}$$
$$= \max\{\mu_A(x), \mu_B(x)\} = \mu_{A\cup B}(x)$$

This is the same with the union $\mu_{A\cup B}(x)$ of the intuitionistic fuzzy sets. The upper bound $\mu_{A\cup B}^r(x) = \max\{\mu_A^r(x), \mu_B^r(x)\}$ can be transformed by the relationship $\mu_A^r(x) = 1 - \nu_A(x)$ as follows:

$$\mu_{A\cup B}^r(x) = \max\{\mu_A^r(x), \mu_B^r(x)\}$$
$$= \max\{1 - \nu_A(x), 1 - \nu_B(x)\}$$
$$= 1 - \min\{\nu_A(x), \nu_B(x)\}$$

When we rewrite the above equation using the relationship $\mu_A^r(x) = 1 - \nu_A(x)$, we can see that the upper bound of the union operation of interval-valued fuzzy sets corresponds to the nonmembership degree $\nu_{A\cup B}(x) = \min\{\nu_A(x), \nu_B(x)\}$. It means that both union operations of interval-valued fuzzy sets and intuitionistic fuzzy sets are the same. In a similar way, we can prove that the intersection operations for both kinds of fuzzy sets are the same. The following shows the equivalence in negation operations.

$$\overline{A} = \{(x, [\mu^l_{\overline{A}}(x), \mu^r_{\overline{A}}(x)]) \mid x \in X\}$$
$$\mu^l_{\overline{A}}(x) = 1 - \mu^r_A(x)$$
$$\mu^r_{\overline{A}}(x) = 1 - \mu^l_A(x)$$

$\mu^l_{\overline{A}}(x)$ and $\mu^r_{\overline{A}}(x)$ can be rewritten as follows:

$$\mu^l_{\overline{A}}(x) = 1 - \mu^r_A(x) = 1 - (1 - \nu_A(x)) = \nu_A(x)$$
$$\mu^r_{\overline{A}}(x) = 1 - \mu^l_A(x) = 1 - \mu_A(x)$$

We can see that $\mu^l_{\overline{A}}(x)$ and $\mu^r_{\overline{A}}(x)$ correspond to $\mu_{\overline{A}}(x)$ and $\nu_{\overline{A}}(x)$ respectively.

From those observations, we can see that interval-valued fuzzy sets and intuitionistic fuzzy set have the same expressive power and the same basic set operations.

6. Comparison of Intuitionistic Fuzzy Sets with Bipolar-valued Fuzzy Sets

When we compare a bipolar-valued fuzzy set $A = \{(x, (\mu^P_A(x), \mu^N_A(x))) \mid x \in X\}$ with an intuition-istic fuzzy set $A = \{(x, \mu_A(x), \nu_A(x)) \mid x \in X\}$ under the conditions $\mu^P_A(x) = \mu_A(x)$ and $\mu^N_A(x) = -\nu_A(x)$, bipolar-valued fuzzy sets and intuitionistic fuzzy sets look similar each other. However, they are different each other in the following senses: In bipolar-valued fuzzy sets, the positive membership degree $\mu^P_A(x)$ characterizes the extent that the element x satisfies the property A, and the negative membership degree $\mu^N_A(x)$ characterizes the extent that the element x satisfies an implicit *counter*-property of A. On the other hand, in intuitionistic fuzzy sets, the membership degree $\mu_A(x)$ denotes the degree that the element x satisfies the property A and the membership degree $\nu_A(x)$ indicates the degree that x satisfies the *not*-property of A. Since a *counter*-property is not usually equivalent to *not*-property, both bipolar-valued fuzzy sets and intuitionistic fuzzy sets are the different extensions of fuzzy sets.

Their difference can be manifested in the interpretation of an element x with membership degree (0, 0). In the perspective of bipolar-valued fuzzy set A, it is interpreted that the element x does not satisfy both the property A and its implicit counter-property. It means that it is indifferent (i.e., neutral) from the property and its implicit *counter*-property. In the perspective of intuitionistic fuzzy set A, it is interpreted that the element x does not satisfy the property and its *not*-property. When we regard an intuitionistic fuzzy set as an interval-valued fuzzy set, the element with the membership degree (0, 0) in intuitionistic fuzzy set has the membership degree [0, 1] in interval-valued fuzzy set. It means that we have no knowledge about the element. On the other hand, their set operations union, intersection, and negation are also different each other.

These things differentiate bipolar-valued fuzzy sets from intuitionistic fuzzy sets. The intuitionistic fuzzy set representation is useful when there are some uncertainties in assigning membership degrees. The bipolar-valued fuzzy set representation is useful when irrelevant elements and contrary elements are needed to be discriminated.

7. Examples

This section gives some examples to use the three fuzzy set representations for a fuzzy concept *frog's prey*. The next is an interval-valued fuzzy set for *frog's prey*:

frog's prey = {(*mosquito*, [1,1]), (*dragon fly*, [0.4,0.7]), (*turtle*, [0,0]), (*snake*, [0,0])}

The following shows an intuitionistic fuzzy set corresponding to the above interval-valued fuzzy set:

frog's prey = {(*mosquito*, 1, 0), (*dragon fly*, 0.4,0.3), (*turtle*, 0, 1), (*snake*, 0, 1)}

From those examples, we can see that interval-valued fuzzy sets and intuitionistic fuzzy sets have the same expressive power.

The next shows a bipolar-valued fuzzy set for *frog's prey*:

frog's prey = {(*mosquito*, (1,0)), (*dragon fly*, (0.4,0)), (*turtle*, (0,0)), (*snake*, (0,–1))}

For the element *snake*, the above interval-valued fuzzy set and the intuitionistic fuzzy set have 0 membership degree which just means that *snake* does not satisfy the property corresponding to *frog's prey* despite that *snake* is a predator of *frog*. On the other hand, the above bipolar-valued fuzzy set has –1 membership degree which indicates that *snake* satisfies some counter-property with respect to *frog's prey*. Meanwhile, interval-valued fuzzy sets and intuitionistic fuzzy sets can express uncertainties in assigning membership degrees to elements.

8. Conclusions

This paper compared three fuzzy set representations: interval-valued fuzzy sets, intuitionistic fuzzy sets, and bipolar-valued fuzzy sets. It showed that interval-valued fuzzy sets and intuitionistic fuzzy sets have the same expressive power and the same basic set operations. Interval-valued fuzzy sets and intuitionistic fuzzy sets can represent uncertainties in membership degree assignments, but they cannot represent the satisfaction degree to counter-property. On the other hand, bipolar-valued fuzzy sets can represent the satisfaction degree to counter-property, but they cannot express uncertainties in assigning membership degrees.

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