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Some properties of fuzzy net-convergences

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Abstract

In this paper, we introduce the fuzzy limit degrees in L-fuzzy topologies using complete MV-algebras. We investigate the properties of net-convergences (the degrees of fuzzy convergent, fuzzy cluster, fuzzy adherent points and fuzzy limits).

Key Words : Complete MV-algebra, Neighborhood systems, The degrees of fuzzy adherent (cluster, convergent, limit) points

Ⅰ. Introduction

Ward and Dilworth [14] introduced residuated lattices as the foundation of the algebraic structures of fuzzy logics. There have been developed in many directions [2-4,7,9,11-12,14-16]. Recently, Hohle [3,4] extended the fuzzy set $f:X\to L$ where L is a complete MV-algebra in stead of an unit interval I or a lattice L. It is a remarkable work to apply fuzzy topologies to fuzzy logics. On the other hand, Pu and Liu [10] introduced the convergence theory in fuzzy topologies with quasi coincident neighborhood systems. Ying [17] introduced the neighborhood systems as a new method from different points of view in [10]. Kim and Ko [5-6] introduced neighborhood systems in L-fuzzy topologies in a view of [17] using complete MV-algebras.

In this paper, we introduce the fuzzy limit degrees in L-fuzzy topologies using complete MV-algebras. We investigate the relationships among the degrees of fuzzy convergent, fuzzy cluster, fuzzy adherent points and fuzzy limit net.

2. Preliminaries

Definition 2.1 [4.12] A lattice $(L,\le,\wedge,\vee,\odot,\to,0,1)$ is called an MV-algebra if it satisfies the following conditions: for each $x,y,z\in L$,

(M1) $(L,\odot,1)$ is a commutative monoid,

(M2) if $x\le y$, then $x\odot z\le y\odot z$ ($\odot$ is an isotone operation),

(M3) (Galois correspondence): $(x\odot y)\le z$ iff $x\le y\to z$.

(M4) $x\wedge y=x\odot(x\to y)$,

(M5) $x\vee y=[(x\to y)\to y]\wedge[(y\to x)\to x]$,

(M6) $(x\to y)\vee(y\to x)=1$.

(M7) $x=x^{**}$ where $x^*=(x\to 0)$.

An MV-algebra L is called *complete* if $\bigwedge_{i\in\Gamma}x_i\in L$ and $\bigvee_{i\in\Gamma}x_i\in L$ for any $x_i\in L$.

Throughout this paper, let L be a complete MV-algebra and $L_0=L-\{0\}$. The class of all fuzzy sets on a set X will be denoted by L^X and the fuzzy sets by the Greek symbols λ,μ,ν, etc. All algebraic operations on L can be extended pointwise to the set L^X as follows:

$\mu\to\rho$ iff $\mu(x)\to\rho(x)$, for all $x\in X$,

$(\mu\odot\rho)(x)=\mu(x)\odot\rho(x)$, for all $x\in X$.

The set of all fuzzy points in X is denoted by $Pt(X)$. For $x_t\in Pt(X)$, $x_t\in\lambda$ iff $t\le\lambda(x)$. A lattice L is called order dense if for each $x,y\in L$ such that $x<y$, there exists $z\in L$ such that $x<z<y$.

All the other notations and the other definitions are standard in fuzzy set theory.

Definition 2.2 [1,4] A subset T of L^X is called an L-fuzzy *topology* on X if it satisfies the following conditions:

(O1) $\bar{0},\bar{1}\in T$, where $\bar{0}(x)=0$ and $\bar{1}(x)=1$ $\forall x\in X$.

(O2) If $\mu_1,\mu_2\in T$, $\mu_1\wedge\mu_2\in T$.

(O3) If $\mu_i\in T$ for each $i\in\Gamma$, $\bigvee_{i\in\Gamma}\mu_i\in T$.

The pair (X,T) is called an L-fuzzy *topological space.*

Definition 2.3 [6] Let $\lambda\in L^X$ and $x_p\in Pt(X)$.

Then the degree to which x_p belongs to λ is

$$[x_p\to\lambda]=p\to\lambda(x).$$

Definition 2.4 [6] Let (X,T) be an L-fuzzy topological space, $\mu\in L^X$ and $e\in Pt(X)$.Then the degree

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to which λ is *a neighborhood* of e is defined by

$$N_e(\lambda)=\bigvee\{[e\rightarrow\mu] \mid \mu\le\lambda,\ \mu\in T\}.$$

A mapping $N_e: L^X\rightarrow L$ is called the fuzzy *neighborhood system* of e.

Theorem 2.5 [6] Let (X,T) be an L-fuzzy topological space and N_e the fuzzy neighborhood system of e. For $\lambda,\mu\in L^X$, it satisfies the following properties:

(1) $N_e(\overline{0})=[e\rightarrow\overline{0}]$ and $N_e(\overline{1})=1$.

(2) $N_e(\lambda)\le[e\rightarrow\lambda]$.

(3) $N_e(\lambda)\le N_e(\mu)$, if $\lambda\le\mu$.

(4) $N_e(\lambda)\wedge N_e(\mu)\le N_e(\lambda\wedge\mu)$.

(5) $N_e(\lambda)\le\bigvee\{N_e(\mu)\mid\mu\le\lambda,[d\rightarrow\mu]\le N_d(\mu,r)\,\forall d\in Pt(X)\}$.

(6) $N_{x_p}(\lambda)=p\rightarrow N_{x_1}(\lambda)$, for each $x_p\in Pt(X)$.

Definition 2.6 [5,6] Let (X,T) be an L-fuzzy topological space, $\lambda\in L^X$ and $e\in Pt(X)$. Then the degree to which e is an adherent point of λ is defined by

$$Ad_e(\lambda)=N_e(\lambda^*)^*.$$

Definition 2.7 [8,10] Let D be a directed set. A function $S:D\rightarrow Pt(X)$ is called *a fuzzy net*. Let $\lambda\in I^X$.

(1) S is *a fuzzy net in* λ if $S(n)\in\lambda$ for every $n\in D$.

(2) S is *often in* λ if for each $n\in D$, there exists $n_0\in D$ such that $n_0\ge n$ and $S(n_0)\in\lambda$.

(3) S is *finally in* λ if there exists $n_0\in D$ such that for each $n\in D$ with $n\ge n_0$, we have $S(n)\in\lambda$.

Definition 2.8 [8,10] Let $S:D\rightarrow Pt(X)$ and $T:E\rightarrow Pt(X)$ be two fuzzy nets. A *fuzzy net* T is called a *subnet* of S if there exists a function $N:E\rightarrow D$, called by a *cofinal selection* on S, such that:

(1) $T=S\circ N$.

(2) For every $n_0\in D$, there exists $m_0\in E$ such that $N(m)\ge n_0$, for $m\ge m_0$.

Definition 2.9 [5] Let (X,T) be an L fuzzy topological space, $\lambda\in I^X$ and $e\in Pt(X)$.

(1) The *fuzzy convergent degree* of S to e is defined by $Con_e(S)=\bigwedge\{N_e(\lambda)^*\mid S$ is often in $\lambda^*\}$,

(2) The *fuzzy cluster degree* of S to e is defined by

$$Cl_e(S)=\bigwedge\{N_e(\lambda)^*\mid S \text{ is finally in } \lambda^*\}.$$

Theorem 2.10 [5] Let (X,T) be an L-fuzzy topological space.Let $S:D\rightarrow Pt(X)$ be a fuzzy net and $W:E\rightarrow Pt(X)$ a subnet of S. Then we have the following properties.

(1) $Con_e(S)\le Cl_e(S)$.

(2) $Cl_e(W)\le Cl_e(S)$.

(3) $Con_e(S)\le Con_e(W)$.

(4) $Con_{x_t}(S)=[x_t\odot Con_{x_1}(S)]$.

(5) $Cl_{x_t}(S)=[x_t\odot Cl_{x_1}(S)]$.

Theorem 2.11 [5] Let (X,T) be an L-fuzzy topological space and $S,U:D\rightarrow Pt(X)$ fuzzy nets such that for each $n\in D$.

$$S(n)\vee U(n),\ S(n)\wedge U(n)\in Pt(X).$$

Define fuzzy nets $S\vee U, S\wedge U:D\rightarrow Pt(X)$ by, for each $n\in D$,

$$(S\vee U)(n)=S(n)\vee U(n),$$
$$(S\wedge U)(n)=S(n)\wedge U(n).$$

Then the following properties hold:

(1) If $S(n)\le U(n)$ for all $n\in D$, then $Cl_e(S)\le Cl_e(U)$, $Con_e(S)\le Con_e(U)$.

(2) $Cl_e(S\wedge U)\le Cl_e(S)\wedge Cl_e(U)$.

(3) $Con_e(S\vee U)\ge Con_e(S)\vee Con_e(U,r)$.

(4) $Con_e(S\wedge U)\le Con_e(S)\wedge Con_e(U)$.

(5) If L is an order dense totally ordered lattice, then

$$Cl_e(S\vee U)=Cl_e(S)\vee Cl_e(U).$$

Theorem 2.12 [5] Let (X,T) be an L-fuzzy topological space and L be a totally ordered lattice.For $\lambda\in I^X$ and $e\in Pt(X)$, we have:

$$Ad_e(\lambda)=\bigvee\{Cl_e(S)\mid S \text{ is a fuzzy net in } \lambda\}$$
$$=\bigvee\{Con_e(S)\mid S \text{ is a fuzzy net in } \lambda\}.$$

3. The some properties of fuzzy net-convergences

Theorem 3.1 Let (X,T) be an L-fuzzy topological space.Let $S:D\rightarrow Pt(X)$ be a fuzzy net. Then we have:

(1) $Cl_e(S)=\bigwedge_{n_0\in D}Ad_e(\bigvee_{n\ge n_0}S(n))$.

(2) $Ad_e(\bigwedge_{n\in D}S(n))\le Con_e(S)$.

(3) $\bigvee_{n_0\in D}Ad_e(\bigwedge_{n\ge n_0}S(n))\le Con_e(S)$.

Proof. (1) For each $n_0\in D$, S is finally in $\bigvee_{n\ge n_0}S(n)$. Hence $Cl_e(S)\le\bigwedge_{n_0\in D}Ad_e(\bigvee_{n\ge n_0}S(n))$.

Let S be finally in λ^*, that is , for each $n\ge n_0\in D$, $S(n)\in\lambda^*$. Hence $\bigvee_{n\ge n_0}S(n)\le\lambda^*$. It follows

$$Ad_e(\bigvee_{n\ge n_0}S(n))=(N_e((\bigvee_{n\ge n_0}S(n))^*))^*\le N_e(\lambda)^*.$$

From the definition Cl_e,

$$Cl_e(S) \geq \bigwedge_{n_0 \in D} Ad_e(\bigvee_{n \geq n_0} S(n)).$$

(2) Let S is often in λ^*, for each $n \in D$, there exists $n_0 \geq n$ with $S(n_0) \in \lambda^*$. Hence $\bigwedge_{n \in D} S(n) \leq \lambda^*$.

It follows

$$Ad_e(\bigwedge_{n \in D} S(n)) = (N_e((\bigwedge_{n \in D} S(n))^*)^* \leq (N_e(\lambda))^*.$$

From the definition $Con_e(S)$, Thus,

$$Ad_e(\bigwedge_{n \in D} S(n)) \leq Con_e(S).$$

(3) From (2), if S is often in λ^*, for each $n_0 \in D$, there exists $n_1 \geq n$ with $S(n_1) \in \lambda^*$. Hence $\bigwedge_{n \geq n_0} S(n) \leq \lambda^*$. It follows

$$\bigvee_{n_0 \in N} Ad_e(\bigwedge_{n \geq n_0} S(n)) = (N_e((\bigwedge_{n \geq n_0} S(n))^*)^* \leq (N_e(\lambda))^*.$$

Thus, $\bigvee_{n_0 \in D} Ad_e(\bigwedge_{n \geq n_0} S(n)) \leq Con_e(S)$.

Example 3.2 Let $L = ([0,1], \leq, \wedge, \vee, \odot, \rightarrow, 0, 1, {}^*)$ be a complete MV-algebra defined by (called a Lukasiewicz logic, ref.[6,11])

$$a \rightarrow b = \min\{1, 1-a+b\}$$
$$a \odot b = \max\{0, a+b-1\}.$$

Then $a^* = a \rightarrow 0 = 1-a$.

Let $X = \{x, y\}$ be a set and $\mu \in L^X$ as follows:

$$\mu(x) = 0.3, \quad \mu(y) = 0.4.$$

We define an L-fuzzy topology

$$T = \{\overline{0}, \overline{1}, \mu\}.$$

(1) In general, $Cl_e(S) \neq Ad_e(\bigvee_{n \in D} S(n))$.

Define a fuzzy net $S: N \rightarrow Pt(X)$ by

$$S(n) = x_{a_n}, \quad a_n = 0.6 + \frac{0.2}{n}$$

where N is a natural number.

Then $\bigvee_{n \in N} S(n) = x_{0.8}$.

Let $e = x_{0.5}$. From Definition 2.6, we have

$Ad_e(x_{0.8}) = N_e(x^*_{0.8})^* = ([x_{0.5} \rightarrow \overline{0}])^* = 0.5.$

Since S is often in μ^* or $\overline{1}$, for $\mu, \overline{0} \in T$, we have

$$\begin{aligned} Cl_e(S) &= \bigwedge\{N_e(\lambda)^* \mid S \text{ is finally in } \lambda^*\} \\ &= (\bigvee\{N_e(\lambda) \mid S \text{ is finally in } \lambda^*\})^* \\ &= N_e(\mu)^* \\ &= 1 - N_e(\mu) \\ &= 1 - [x_{0.5} \rightarrow \mu] = 0.2. \end{aligned}$$

(2) In general, $Ad_e(\bigwedge_{n \in D} S(n)) \neq Con_e(S)$.

Define a fuzzy net $S: N \rightarrow Pt(X)$ by

$$S(n) = x_{a_n}, \quad a_n = 0.8 + (-1)^n \frac{0.2}{n}$$

where N is a natural number.

Then $\bigwedge_{n \in N} S(n) = x_{0.6}$.

Let $e = x_{0.5}$. From Definition 2.6, we have

$$Ad_e(x_{0.6}) = N_e(x^*_{0.6})^* = ([x_{0.5} \rightarrow \mu])^* = 0.2.$$

Since S is often in $\overline{1}$, for $\overline{0} \in T$, we have

$$\begin{aligned} Con_e(S) &= \bigwedge\{N_e(\lambda)^* \mid S \text{ is often in } \lambda^*\} \\ &= (\bigvee\{N_e(\lambda) \mid S \text{ is often in } \lambda^*\})^* \\ &= N_e(\overline{0})^* \\ &= 1 - N_e(\overline{0}) \\ &= 1 - [x_{0.5} \rightarrow \overline{0}] = 0.5. \end{aligned}$$

Theorem 3.3 Let (X, T) be an L-fuzzy topological space.Let $S: D \rightarrow Pt(X)$ be an increasing fuzzy net. Then we have:

$$Con_e(S) = Cl_e(S) = Ad_e(\bigvee_{n \in D} S(n)).$$

Proof. From Theorem 2.10(1) and Theorem 3.1(1),

$$Con_e(S) \leq Cl_e(S) \leq Ad_e(\bigvee_{n \in D} S(n)).$$

Let S be often in λ^*, for each $n_0 \in D$, there exists $n \in D$ such that $S(n) \in \lambda^*$. Since S is an increasing fuzzy net, $\bigvee_{n \in D} S(n) \leq \lambda^*$. It follows

$$Ad_e(\bigvee_{n \in D} S(n)) = (N_e((\bigvee_{n \in D} S(n))^*)^* \leq N_e(\lambda)^*.$$

From the definition Con_e,

$$Con_e(S) \geq Ad_e(\bigvee_{n \in D} S(n)).$$

4. The degrees of fuzzy limit net

Definition 4.1 Let (X, T) be an L-fuzzy topological space. Let $S: D \rightarrow Pt(X)$ be a fuzzy net and $e \in Pt(X)$. The *fuzzy limit degree* of S to e is defined by , denoted by $\underset{e}{\text{Lim}}(S) = t$, if $Cl_e(S) = Con_e(S) = t$.

Theorem 4.2 Let (X, T) be an L-fuzzy topological space. Let $S, U: D \rightarrow Pt(X)$ be fuzzy nets such that

$$S(n) \vee U(n) \in Pt(X), \text{ for each } n \in D.$$

If $Cl_e(S) = Con_e(S)$ and $Cl_e(U) = Con_e(U)$, then $\underset{e}{\text{Lim}}(S \vee U) = \underset{e}{\text{Lim}}(S) \vee \underset{e}{\text{Lim}}(U)$.

Proof. From Theorem 2.11, $S \vee U$ is a fuzzy net. We easily proved it from the followings:

$$Cl_e(S\vee U)=Cl_e(S)\vee Cl_e(U)$$

(by Theorem 2.11(2))

(since $Cl_e(S)=Con_e(S)$ and $Cl_e(U)=Con_e(U)$,)

$$=Con_e(S)\vee Con_e(U)$$

$$\leq Con_e(S\vee U)$$

(by Theorem 2.11(4))

$$\leq Cl_e(S\vee U).$$

(by Theorem 2.8(2))

Example 4.3 We define an L-fuzzy topology T and L as Example 3.2. Let N be a natural number set. Define a fuzzy net $S: N\to Pt(X)$ by

$$S(n)=x_{a_n},\quad a_n=0.6+(-1)^n 0.2.$$

Let $e=x_{0.3}$. Since S is often in μ^*, for $\mu\in T$,

$$Con_e(S)=N_e(\mu)^*=1-[x_{0.3}\to\mu]=0.$$

Since S is finally in $\overline{1}$, for $\overline{0}\in T$,

$$Cl_e(S)=N_e(\overline{0})^*=1-[x_{0.3}\to\overline{0}]=0.3.$$

Thus, since $Con_e(S)\neq Cl_e(S)$, $\underset{e}{\mathrm{Lim}}(S)$ does not exists.

Theorem 4.4 Let L be an order dense totally ordered lattice and (X, T) be an L- fuzzy topological space. Let $S: D\to Pt(X)$ be a fuzzy net and

$$H=\{W \mid W \text{ is a subnet of } S\}.$$

Then the following statements hold:

(1) $Con_e(S)=\bigwedge_{W\in H}Cl_e(W)$.

(2) $Cl_e(S)=\bigvee_{W\in H}Con_e(T)$.

(3) If $\underset{e}{\mathrm{Lim}}(S)=t$, then $\underset{e}{\mathrm{Lim}}(W)=t$ for each $W\in H$.

Proof. (1) For each $W\in H$, by Theorem 2.10(1-3), we have

$$Con_e(S)\leq Con_e(W)\leq Cl_e(W)\leq Cl_e(S) \qquad \text{(I)}$$

Hence

$$Con_e(S)\leq \bigwedge_{W\in H}Cl_e(W).$$

Suppose

$$Con_e(S)\ngeq \bigwedge_{W\in H}Cl_e(W).$$

Since L is an order dense totally ordered lattice, there exist $x_p\in Pt(X)$ and $t\in L_0$ such that

$$Con_{x_p}(S)<t<\bigwedge_{W\in H}Cl_{x_p}(W).$$

Since $Con_{x_p}(S)<t$, there exists $\mu\in L^X$ with S is often in μ^* such that

$$Con_{x_p}(S)\leq N_{x_p}(\mu)^*<\bigwedge_{W\in H}Cl_{x_p}(W). \qquad \text{(II)}$$

Since S is often in μ^*, for each $n\in D$ there exists $N(n)\in D$ with for $N(n)\geq n$ and $S(N(n))\in\mu^*$.
Hence there exists a cofinal selection $N: E\to D$ such that $W=S\circ N$. Thus W is a subnet of S and W is finally in μ^*. So, $Cl_{x_p}(W)\leq N_{x_p}(\mu^*)$. It is a contradiction for (II).
Hence $Con_e(S)\geq \bigwedge_{W\in H}Cl_e(W)$.

(2) From (I) of (1), we have

$$\bigvee_{W\in H}Con_e(W)\leq Cl_e(S).$$

Conversely, let $Cl_e(S)=t>0$.
Then $N_e(\lambda)\leq 1-t$, for S is finally in λ^*. Let

$$F=\{\mu \mid N_e(\mu)>1-t\}.$$

Define a relation on $E=D\times F$ by

$$(m,\mu_1)\leq(n,\mu_2) \text{ iff } m\leq n,\ \mu_1\geq\mu_2.$$

Then $(E,\leq)$ is a directed set.
If $\mu\in F$, then S is not finally in μ^*, that is,
for each $n\in D$, there exists $n_0\in D$ with $n_0\geq n$ such that $S(n_0)\notin\mu^*$. So, for each $(n,\mu)\in E$, there exists $N(n,\mu)=n_0\in D$ with $N(n,\mu)\geq n$ such that $S(N(n,\mu))\notin\lambda^*$. Thus, we can define $N: E\to D$.
For each $n_0\in D$ and $\mu_0\in F$, there exists $N(n_0,\mu_0)\in D$ with $N(n_0,\mu_0)\geq n_0$ such that $S(N(n_0,\mu_0))\notin\mu_0^*$. Hence for every $(n,\mu)\geq(n_0,\mu_0)$, since $n\geq n_0$, we have $N(n,\mu)\geq n\geq n_0$ and $S(N(n,\mu))\notin\mu^*$. (III)
Therefore N is a cofinal selection on S. So, $T=S\circ N$ is a fuzzy subnet of S. If T is often in λ^*, by (III), $\lambda\notin F$. Thus

$$\bigvee_{T\in H}Con_e(T)=\bigwedge\{N_e(\lambda)^* \mid T \text{ is often in } \lambda^*\}\geq t.$$

Since t is arbitrary, we complete the proof.

(3) From (I) of (1), we easily prove it.

Theorem 4.5 Let L be an order dense totally ordered lattice and (X, T) be an L fuzzy topological space. Let $S: D\to Pt(X)$ be a fuzzy net. If $Con_e(W)=t$ for every subnet W of S, then $\underset{e}{\mathrm{Lim}}(S)=t$.

Proof. Let $H=\{W \mid W \text{ is a subnet of } S\}$. For each $W\in H$, since W has a subnet K with $\underset{e}{\mathrm{Lim}}(K)=t$, by Theorem 2.10(3), we have

$$Con_e(W)\leq Con_e(K)=Cl_e(K)=t.$$

Hence, by Theorem 4.4(2),

$$Cl_e(S)=\bigvee_{W\in H}Con_e(W)\leq t. \qquad \text{(IV)}$$

Conversely, by Theorem 2.10(2),

$$t = Con_e(K) = Cl_e(K) \le Cl_e(W).$$

Hence, by Theorem 4.4(1),

$$t \le \bigwedge_{W \in H} Cl_e(T) = Con_e(S). \qquad \text{(V)}$$

By (IV) and (V), $Cl_e(S) \le Con_e(S)$.

Since $Con_e(S) \le Cl_e(S)$ from Theorem 2.10(1), $Cl_e(S) = Con_e(S)$, that is, $\underset{e}{\mathrm{Lim}}(S) = t$.

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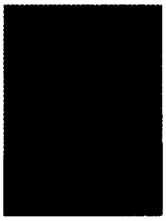

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