J. Astron. Space Sci. 21(3), 209–220 (2004)

OPTIMAL FORMATION TRAJECTORY-PLANNING USING PARAMETER OPTIMIZATION TECHNIQUE

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(Received April 14, 2004; Accepted August 13, 2004)

ABSTRACT

Some methods have been presented to get optimal formation trajectories in the step of configuration or reconfiguration, which subject to constraints of collision avoidance and final configuration. In this study, a method for optimal formation trajectory-planning is introduced in view of fuel/time minimization using parameter optimization technique which has not been applied to optimal trajectory-planning for satellite formation flying. New constraints of nonlinear equality are derived for final configuration and constraints of nonlinear inequality are used for collision avoidance. The final configuration constraints are that three or more satellites should be placed in an equilateral polygon of the circular horizontal plane orbit. Several examples are given to get optimal trajectories based on the parameter optimization problem which subjects to constraints of collision avoidance and final configuration. They show that the introduced method for trajectory-planning is well suited to trajectory design problems of formation flying missions.

Keywords: trajectory-planning, formation flying, collision avoidance, parameter optimization

1. INTRODUCTION

Satellite formation flying (SFF) is the placing of micro-satellites into nearby orbits to form a cluster for the same mission. In recent years, it has become a topic of significant interest in the aerospace engineering. Formation flying system has several benefits compared to the large single spacecraft system with equivalent missions: low cost for launch and mass production, larger aperture size, greater launch flexibility, higher system reliability and easier expandibility (Lim et al. 2003). According to the characteristic of control purpose and design, SFF problem can be categorized into three phases: determination of initial conditions, satellite formation keeping and satellite formation configuration or reconfiguration. However, the determination of initial conditions can belong to satellite formation keeping problem because both problems are concerned with minimization of fuel consumption to maintain the formation against to the external forces such as the J_2 gravitational perturbation and air-drag.

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A critical issue in the satellite formation configuration or reconfiguration is to determine optimal maneuvers for fuel minimization when the formation needs to be changed. Beard et al. (2000) derived an open-loop control algorithm of reorienting a formation in free space by minimizing a cost function which is composed with fuel consumption and fuel equalization. Yang et al. (2002) modelled and analysed the satellite formation reconfiguration problem of fuel minimization as a multi-agent optimization problem. Tillerson et al. (2002) used convex optimization techniques to derive fuel/time optimal control algorithm of formation keeping and formation reconfiguration.

The spatial separation between satellites can be from a few meters to several kilometers for some SFF missions. It is very critical issue to avoid collisions between spacecrafts as they move in space under the mission of the configuration or reconfiguration. Many techniques have been developed for solving the problem of trajectory optimization with collision avoidance in collaborative systems. Especially the collision avoidance problem has been extensively investigated in the field of robot motion planning. A method based on potential functions is known to be a very effective and powerful technique for handling collision avoidance constraints (Gavin et al. 1995). This technique adds the proximity penalty to the cost function to account for collision avoidance constraints. Other techniques have been developed for path-planning with collision avoidance constraints such as randomized algorithms (Barraquand et al. 1997), splines (Singh & Hadaegh 2001) and a mixed-integer linear programming (Richards et al. 2002). Richards et al. (2002) introduced a method of finding fuel-optimal trajectories considering collision avoidances and plume impingements based on a mixed-integer linear programming (MILP) for the satellite formation reconfiguration.

For the configuration strategy, there are some literatures dealing with the final configuration. Some approaches compute the cost for many sets that are predefined to assign the final states and then select a set which gives the lowest cost. The problem of trajectory planning and configuration selection is decoupled in these approaches (Tillerson et al. 2002, Wang & Hadaegh 1998). MILP approach includes configuration selection in trajectory optimization problem. So selection and assignment are performed within MILP to achieve the subset of final states which give the lowest cost and are known as a global configuration (Richards et al. 2002).

This paper is concerned with optimal trajectory-planning aspects of fuel/time minimization. The optimal trajectory-planning problem is solved using a parameter optimization technique based on a sequential quadratic programming (SQP). Collision avoidance and final configuration constraints are considered in the optimal trajectory-planning. The final configuration constraints are that 3 or 4 satellites should make an equilateral polygon in the circular horizontal plane orbit (Sabol et al. 2001) as TechSat-21 mission. These constraints are applied to get the optimal trajectory-planning of satellite formation flying in a different way from the aforementioned configuration strategies.

2. PROBLEM FORMULATION

2.1 Relative Dynamics

A rotating local-vertical-local-horizontal (LVLH) frame is used to describe the relative motion with respect to the reference satellite. The x-axis points in the radial direction,'and the z-axis is perpendicular to the orbital plane and points in the direction of the angular momentum vector. Finally, the y-axis points in the along-track direction. In general, Clohessy-Wiltshire equations (Clohessy & Wiltshire 1960) based on LVLH frame are used to describe the relative motion and control strategies between satellites, which are known as Hill's equations. Hill's equations are a set of linearized equations that describe the relative motion between satellites, which were used to describe a relative motion of rendezvous mechanics in the past and a satellite formation flying these days. Hill's equations are derived under the assumption that the reference orbit is circular, the Earth is spherically symmetric, and the target satellite is very close to the reference orbit. From these assumptions, external perturbations such as the J_2 gravitation and nonlinear terms appeared in the relative motion can be ignored in Hill's equations. So Hill's equations can not accurately describe the relative motion under gravitational perturbation. However this study uses the Hill's equations as dynamic model to derive the constraints of final configuration. For the convenience of parameter optimization problem, we introduce a new time variable($\tau = \omega t$) using time(t) and mean motion(ω). Since $dx/dt = \omega(dx/d\tau)$ and $d^2x/dt^2 = \omega^2(d^2x/d\tau^2)$, the Hill's equations based on the new time variable can be rewritten as (Lim et al. 2003)

$$\begin{aligned} \ddot{x} - 2\dot{y} - 3x &= F_x/\omega^2 = u_x \\ \ddot{y} + 2\dot{x} &= F_y/\omega^2 = u_y \\ \ddot{z} + z &= F_z/\omega^2 = u_z \end{aligned} \tag{1}$$

where,

$$\begin{aligned} \dot{x} &= dx/d\tau, \quad \dot{y} = dy/d\tau, \quad \dot{z} = dz/d\tau \\ \ddot{x} &= d^2x/d\tau^2, \quad \ddot{y} = d^2y/d\tau^2, \quad \ddot{z} = d^2z/d\tau^2 \end{aligned}$$

$\mathbf{x} = [x\ y\ z\ \dot{x}\ \dot{y}\ \dot{z}]^T$ is the state vector, and $\mathbf{F} = [F_x\ F_y\ F_z]^T$ and $\mathbf{u} = [u_x\ u_y\ u_z]^T$ are the control input vectors. The terms of the first equation are total, Coriolis and centripetal acceleration from left to right. Note that the out-of-plane motion is decoupled from the in-plane motion in the Hill's equations.

2.2 Basic Parameter Optimization Problem

Trajectory optimization problems basically concern with how to find the control history and optimal trajectories for fuel/time minimization. It can be formulated to a parameter optimization problem. There are four general classes of methods for converting a trajectory optimization problem to a parameter optimization problem according to the unknowns in each class: 1) control parameters, 2) control parameters and some state parameters, 3) control parameters and state parameters, and 4) state parameters (Hull 1997). The parameter optimization problem in this paper is to find the free final time and the control history that minimize the cost function. So this problem can be categorized into the first class, because the free final time is one of the elements of the design parameters. To formulate a parameter optimization problem, nodes for time are defined according to the equal time interval as

$$\tau_0 < \tau_1 < \cdots < \tau_i < \tau_{i+1} < \cdots < \tau_N = \tau_f. \quad \forall i \in [0, 1, 2, \cdots N] \tag{2}$$

where τ_f is final time. And the time interval, $[\tau_i, \tau_{i+1}]$, can be divided into many sub intervals with the number of M to estimate control inputs at lots of nodes

$$\tau_{00} < \tau_{01} < \cdots < \tau_{10} < \tau_{11} < \cdots < \tau_{ij} < \cdots < \tau_{NM}, \quad \forall j \in [0, 1, 2, \cdots M] \tag{3}$$

Thus we can have the number (N × M) of nodes and τ_{i0} is equal to τ_i. If these are V number of satellites, we can define control inputs for the pth satellite and the sth component of a state variable at each node as

$$u_{psij} = u_{ps}(\tau_{ij}), \quad \forall p \in [1, 2 \cdots, V], \quad \forall s \in [1, 2, 3] \tag{4}$$

where p is the pth satellite and s means the component in the LVLH frame. If the subscript j for any parameter is omitted, the parameter means the value at time τ_i (for example, $u_{psi} = u_{psi0} =$

$u_{ps}(\tau_{i0})$). The control input vectors and final time should be within specified limits

$$\begin{aligned} -u_{max} &\leq u_{psij} \leq u_{max} \\ \tau_{min} &\leq \tau_f \leq \tau_{max} \end{aligned} \tag{5}$$

The problem is to find suitable control inputs and final time as to minimize the fuel and time for the formation configuration or reconfiguration. Thus cost function can be given by

$$J = \beta \left(\frac{u_{tmax}}{\tau_{tmax}} \right) \tau_f + (1-\beta) \sum_{p=1}^{V} \sum_{s=1}^{3} \sum_{i=0}^{N-1} \sum_{j=0}^{M} \left\{ u_{psij}^2 (\tau_{(i+1)(j+1)} - \tau_{ij}) \right\} \tag{6}$$

where β is a weighting factor and $0 \leq \beta \leq 1$. τ_{tmax} and u_{tmax} are introduced to keep a balance of the time term and fuel term in the cost function. They are define as

$$\begin{aligned} \tau_{tmax} &= \tau_{max} \\ u_{tmax} &= \sum_{p=1}^{V} \sum_{s=1}^{3} \sum_{i=0}^{N-1} \sum_{j=0}^{M} \left\{ u_{max}^2 \tau_{max} \right\} \end{aligned} \tag{7}$$

In this paper, control input vectors will be estimated at $\tau = \tau_i$ in the parameter optimization problem and it is assumed that the control inputs by constant thrusters are applied continuously in the time interval $[\tau_{(i+1)(j+1)} - \tau_{ij}.]$. Therefore the number of all unknown parameters for control input is (N + 1) × V × 3. Thus we can define the vector of unknown parameters including control inputs and final time to convert the trajectory optimization problem to the parameter optimization problem as

$$\begin{aligned} \mathbf{v} &= [u_{110}, u_{111}, \cdots, u_{11N}, u_{120}, u_{121}, \cdots, u_{12N}, u_{130}, u_{131}, \cdots, u_{13N}, \\ &\quad u_{210}, u_{211}, \cdots, u_{21N}, u_{220}, u_{221}, \cdots, u_{22N}, u_{230}, u_{231}, \cdots, u_{23N}, \\ &\quad u_{V10}, u_{V11}, \cdots, u_{V1N}, u_{V20}, u_{V21}, \cdots, u_{V2N}, u_{V30}, u_{V31}, \cdots, u_{V3N}, \tau_f]^T \\ &= [v_1, v_2, v_3, \cdots, v_{(N+1)\times V\times 3}, v_{(N+1)\times V\times 3+1}]^T \end{aligned} \tag{8}$$

Note that $v_1 = u_{110}$, $v_2 = u_{111}$ and so on. If collision avoidance and final configuration constraints are neglected, unknown parameters in Eq. (8) are chosen to minimize the cost function, Eq. (6). Control inputs, $u_{psij}(j \neq 0)$ in Eq. (6), are not estimated from this parameter optimization problem but they can be calculated by linear interpolation from $(u_{pqi}, u_{ps(i+1)})$.

2.3 Nonlinear Programming

There are two methods for solving trajectory optimization problem, direct and indirect methods. Indirect methods find a trajectory satisfying the necessary conditions such as the Euler-Lagrange equations or the Pontrygin's minimum principle, which can be solved by numerical methods such as shooting methods, multiple shooting methods and quasi-linearization methods. Direct methods recursively update the control and trajectory to reduce the value of the cost function while satisfying the boundary conditions and the terminal constraints. In direct methods, control and state variables are represented by piecewise polynomials and the discretized control and state variables at each time are considered as parameters for optimization. Thus direct methods can be converted into parameter optimization problems which are solved using nonlinear programming techniques. In this paper, SQP method is used to solve the parameter optimization problem, which is considered as one of the most promising techniques for solving optimization problems with nonlinear constraints (Onwubiko 2000).

The general constrained optimization problem is to minimize a nonlinear function ($f(\mathbf{v})$) subject to nonlinear constraints ($c_h(\mathbf{v})$), which can be descirbed as

$$\min\{f(\mathbf{v})\ :\ c_h(\mathbf{v}) \leq 0,\ h \in I,\ c_h(\mathbf{v}) = 0,\ h \in E\} \tag{9}$$

where I and E are index sets for inequality and equality constraints respectively. Using quadratic approximation for Lagrangian and linear approximation for constraint functions, Lagrangian for this problem can be rewritten as (Onwubiko 2000)

$$L(\bar{\mathbf{v}}, \lambda_k) = f(\mathbf{v}_k) + \nabla f(\mathbf{v}_k)^T \mathbf{d} + \frac{1}{2}\mathbf{d}^T \nabla^2_{vv}\, L(\mathbf{v}_k, \lambda_k)\mathbf{d} \tag{10}$$

where subscript k means the kth iteration, and λ and $\mathbf{d}$ are Lagrange multiplier and feasible descent direction, respectively. Unkonwn parameters are updated using $\bar{\mathbf{v}} = \mathbf{v}_k + \mathbf{d}_k$. The feasible descent direction can be obtained from the following quadratic programming

$$\begin{aligned} \text{minimize} \quad & \nabla f(\mathbf{v}_k)^T \mathbf{d} + \frac{1}{2}\mathbf{d}^T \nabla^2_{vv}\, L(\mathbf{v}_k, \lambda_k)\mathbf{d} \\ \text{subject} \quad & \nabla c_h^A(\mathbf{v}_k)^T \mathbf{d} = -c_h^A(\mathbf{v}_k) \end{aligned} \tag{11}$$

where c_h^A denotes the constraints in the active set. The process of updating unknown parameters is continued until $\| \mathbf{d} \| \leq \epsilon$, a specified tolerance.

3. CONSTRAINTS FOR FORMATION TRAJECTORY

3.1 Collision Avoidance

Collision avoidance is very critical in the configuration or reconfiguration of SFF system which consists of many satellites. Constraints of collision avoidances between different satellites are given in this section, which can be derived easily from a geometry based on the LVLH frame. A satellite has to be placed at least outside of 3-dimensional sphere whose center is the position of other satellite at each time step. Let the position vectors at the ith node to be $[x_{pi}\ y_{pi}\ z_{pi}]^T$ and $[x_{qi}\ y_{qi}\ z_{qi}]^T$ for the pth and the qth satellite. Then the constraint of collision avoidance between two satellites is given by

$$(x_{pi} - x_{qi})^2 + (y_{pi} - y_{qi})^2 + (z_{pi} - z_{qi})^2 \geq R_a^2 \tag{12}$$

where R_a is the radius of 3-dimensional sphere for collision avoidance.

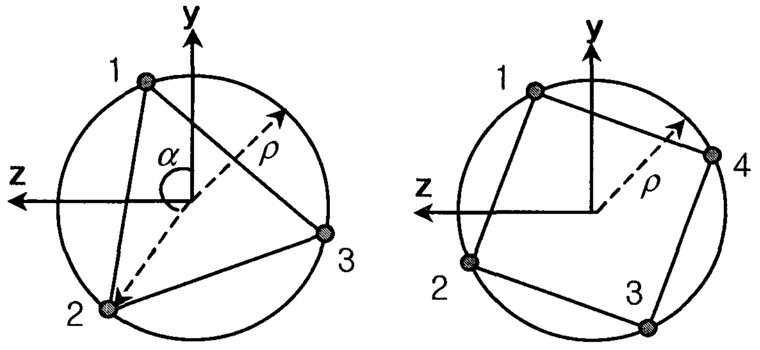
This constraint equation can be extended to the general case with many satellites at τ_{ij} using the same notation as before.

$$(x_{pij} - x_{qij})^2 + (y_{pij} - y_{qij})^2 + (z_{pij} - z_{qij})^2 \geq R_a^2,\ \forall p, \forall q \in [1, 2, \cdots, V];\ p \neq q \tag{13}$$

This equation is add to the parameter optimization problem as constraint of collision avoidance.

3.2 Final Configuration

There are some literatures dealing with the final configuration but most of them put the final states of satellites as fixed points. Richards et al. (2002) defined many subsets of final states and performed the assessment for all subsets within the trajectory optimization. And then they selected



(a) Equilateral triangle (b) Equilateral square

Figure 1. Configuration of satellites in the circular horizontal plane orbit (α: angle measured from y axis, ρ: radius of a circle).

only one subset which has the lowest overall fuel cost and is known as a global configuration. It is impossible to obtain the final configuration constraints for the general case. So in this section, the final configuration constraints are derived as the general form for the circular horizontal plane orbit in which three or four satellites have to be placed in an equilateral polygon of the y-z plane as Figure 1 (Alfriend et al. 2000, Sabol et al. 2001). It is very difficult to derive the final configuration constraints for satellites over 4 using equality constraints. Thus constraints of the final configuration are not given for more than 4 satellites in this paper.

All the satellites have to be placed in a circle of the y-z plane to generate the circular horizontal plane orbit. Eq. (14) can be derived from the simple geometry that the distance of the pth satellite from the origin is ρ.

$$y_p(\tau_f)^2 + z_p(\tau_f)^2 = \rho^2, \ \ \forall p \in \{1,2,3,4\} \tag{14}$$

where $y_p(\tau_f)$ is the y-component of the pth satellite at the final time and $z_p(\tau_f)$ is the z-component. Additional constraints are necessary for satellites to build up a equilateral polygon in the y-z plane. These constraints are given in Eqs. (15) which can be derived from the principle of center of mass.

$$\sum_{p=1}^{N} y_p(\tau_f) = 0, \ \text{and} \ \sum_{p=1}^{N} z_p(\tau_f) = 0, \ \ \forall p \in \{1,2,3,4\} \tag{15}$$

However, one constraint have to be added to build up a equilateral square in case of four satellite formation because the number of constraints in Eq. (14) and Eqs. (15) is lower than unknown parameters such as y and z components for 4 satellites. For example, 4 satellites which are placed in some inscribed rectangle of a circle can satisfy constraints of Eqs. (15). An additional constraint can be derived from the fact that the areas of all possible triangles which can be made from 3 out of 4 satellites located in a equilateral square is same.

$$\sqrt{s(s-a)(s-b)(s-c)} = \rho^2 \tag{16}$$

where,

$$s = \frac{1}{2}(a+b+c)$$

$$a = \sqrt{[y_1(\tau_f) - y_2(\tau_f)]^2 + [z_1(\tau_f) - z_2(\tau_f)]^2}$$
$$b = \sqrt{[y_1(\tau_f) - y_3(\tau_f)]^2 + [z_1(\tau_f) - z_3(\tau_f)]^2}$$
$$c = \sqrt{[y_2(\tau_f) - y_3(\tau_f)]^2 + [z_2(\tau_f) - z_3(\tau_f)]^2}$$

Eq. (16) means the area of a triangle constituted from any 3 satellites, which can be obtained from the Heron's formula that the area of a triangle can be calculated using lengths of three sides of a triangle. If 4 satellites are placed in some inscribed rectangle of a circle and the area of any triangle composed by 3 satellites among them is ρ^2, the 4 satellites make a square without exception. Thus this constraint is enough for four satellites placed in some inscribed rectangle of a circle to make a square.

If all the satellites are placed in an equilateral polygon of the y-z plane, additional constraints are necessary for x components and velocities of satellites to maintain the equilateral polygon as time goes by. These constraints can be derived from periodic solutions of Hill's equations. Periodic solutions of Hill's equations can be obtained by the requirement that the periods of reference satellite and follow satellites must be equal. Periodic solutions of relative motion (Alfriend et al. 2000) for a circular horizontal plane orbit are given below

$$\begin{aligned} x &= \tfrac{\rho}{2}\sin(\tau+\alpha) & \dot{x} &= \frac{\rho}{2}\cos(\tau+\alpha) \\ y &= \rho\cos(\tau+\alpha) & \dot{y} &= -\rho\sin(\tau+\alpha) \\ z &= \rho\sin(\tau+\alpha) & \dot{z} &= \rho\cos(\tau+\alpha) \end{aligned} \tag{17}$$

If y and z components of all satellites are calculated using Eqs. (14), (15), (16) and (17), constraints for velocities and x components of all satellites can be given by Eqs. (18) and Eq. (19) using Eqs. (17).

$$\begin{aligned} \dot{y}_p(\tau_f) + z_p(\tau_f) &= 0 \\ \dot{z}_p(\tau_f) - y_p(\tau_f) &= 0, \qquad \forall p \in \{1,2,3,4\} \end{aligned} \tag{18}$$

$$\begin{aligned} 2\dot{x}_p(\tau_f) - \dot{z}_p(\tau_f) &= 0 \\ 2x_p(\tau_f) + \dot{y}_p(\tau_f) &= 0, \qquad \forall p \in \{1,2,3,4\} \end{aligned} \tag{19}$$

where $x_p(\tau_f)$ is the x-component of pth satellite at the final time and $\dot{x}_p(\tau_f), \dot{y}_p(\tau_f)$ and $\dot{z}_p(\tau_f)$ are the velocities of pth satellite at the final time for x, y and z, respectively. These equations, Eqs. (14), (15), (16), (18) and (19), are add to the parameter optimization problem as constraints of final configuration for circular horizontal plane orbit.

4. NUMERICAL SIMULATION AND RESULTS

In this section, a optimal formation trajectory-planning is tested using parameter optimization technique with constraints of collision avoidance and final configuration. As mentioned in the previous section, all satellites have to make the equilateral polygon in the circular horizontal plane orbit at the final time and a satellite should be far from other satellites over a radius of a sphere for collision avoidances at all nodes. All data for numerical simulation is given in the Table 1 for the optimal formation trajectory-planning. If the number of nodes and satellites are given, the number of unknown parameters and constraints for collision avoidances and final configuration can be determined. We consider two cases of a configuration and a reconfiguration with 3 and 4 satellites respectively.

Table 1. Numerical data for the simulation.

		For 3 satellites	For 4 satellites
orbit	mean motion (ω)	1.095E-3 [rad/sec]	1.095E-3 [rad/sec]
	radius of a circle (ρ)	1000 [m]	1000 [m]
parameter	No. of nodes (N)	15	15
	No. of sub intervals (M)	5	5
	No. of unknown parameters	136	181
	maximum of control input (u_{max})	$0.003/(\omega * \omega)$	$0.003/(\omega * \omega)$
	minimum of time (τ_{min})	$2 * \pi/10$	$2 * \pi/10$
	maximum of time (τ_{max})	π	π
constraints	radius of sphere for collision avoidance (R_a)	50 [m]	50 [m]
	No. of constraints for collision avoidances (inequality)	210	420
	No. of constraints for final configuration (equality)	17	23

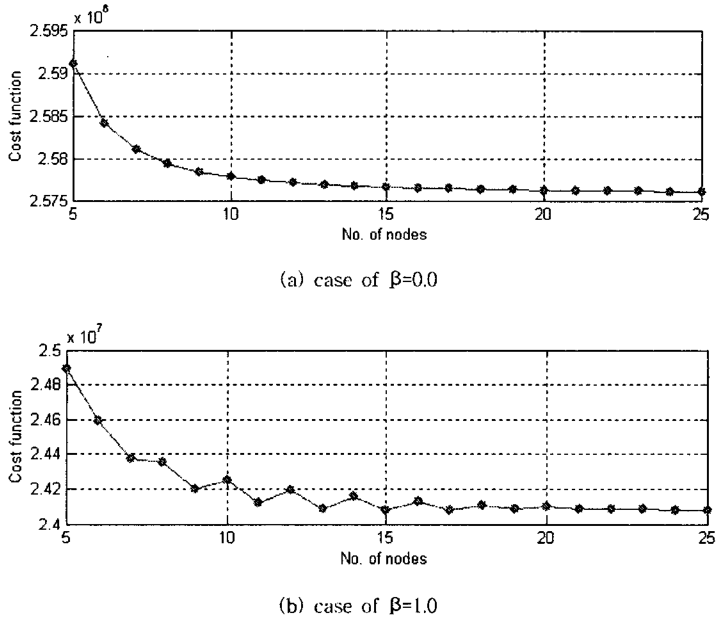
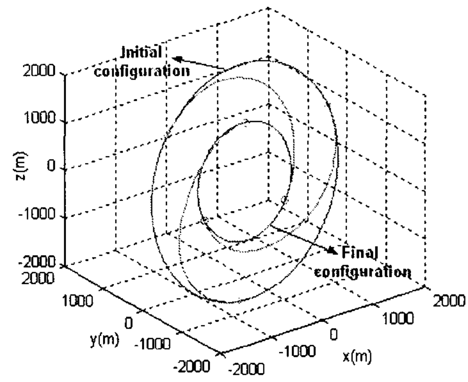
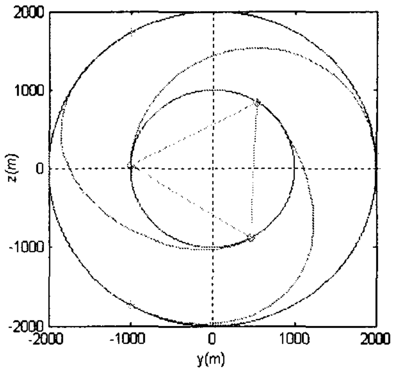


Figure 2. Relation of cost function and the number of nodes for 3 satellites.

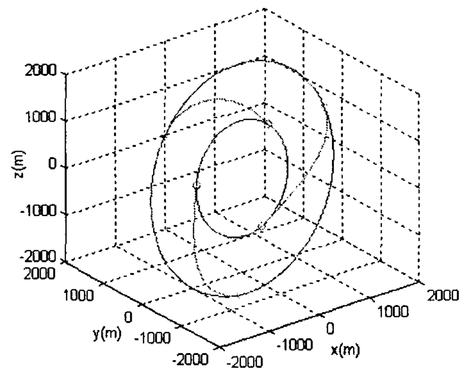
If the number of nodes increases, it takes longer time to get optimal trajectories because the number of unknown parameters and constraints for collision avoidances increases about twice even though the number of constraints of final configuration is same. For example, consider the cases of 10 and 20 nodes for 4 satellites. The number of unknown parameters and constraints for collision avoidances are 121 and 54 for 10 nodes and are 241 and 114 for 20 nodes. Figure 2 shows the relation of cost function and the number of nodes for configuration with 3 satellites. (a) is the case



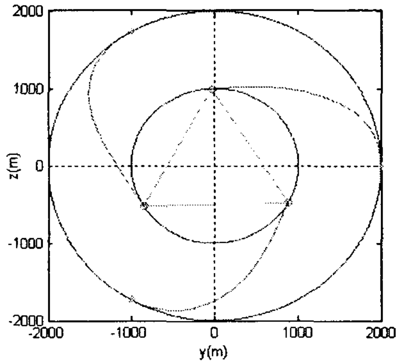
(a) Trajectories in 3-D space (β=0.0)



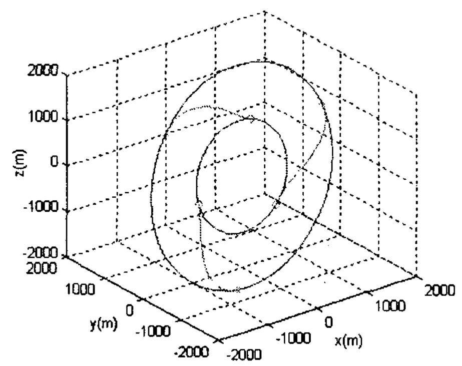
(b) Trajectories in y-z plane (β=0.0)



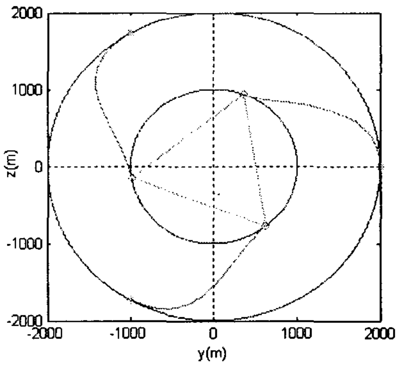
(c) Trajectories in 3-D space (β=0.5)



(d) Trajectories in y-z plane (β=0.5)

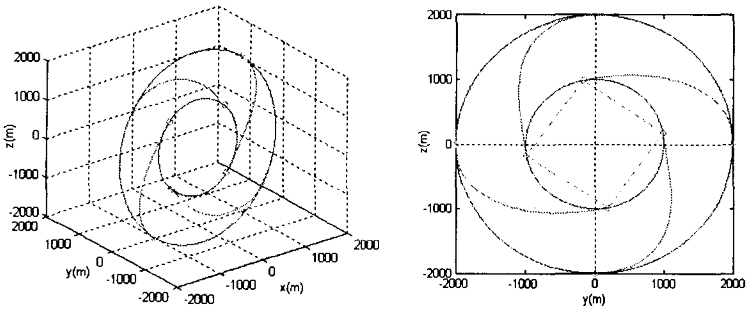


(e) Trajectories in 3-D space (β=1.0)



(f) Trajectories in y-z plane (β=1.0)

Figure 3. Trajectories of a reconfiguration with 3 satellites.



(a) Trajectories in 3-D space (β=0.5) (b) Trajectories in y-z plane (β=0.5)

Figure 4. Trajectories of a reconfiguration with 4 satellites.

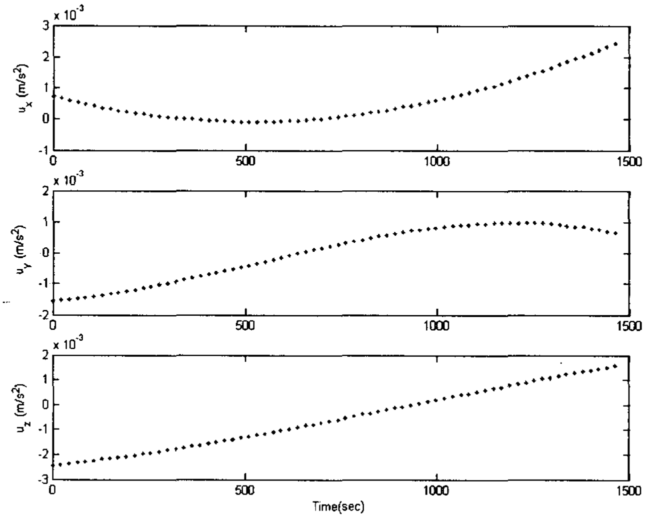
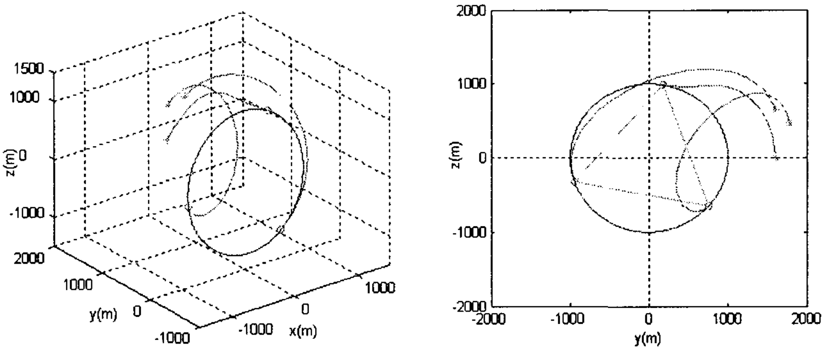


Figure 5. Control inputs of a reconfiguration with 3 satellites ($\beta = 0.5$).

considering the minimization of fuel and (b) is for the minimization of final time in Figure 2. The number of nodes, 15 was used for a optimal formation trajectory-planning in this paper because cost function is nearly equal over 15 nodes from this results.

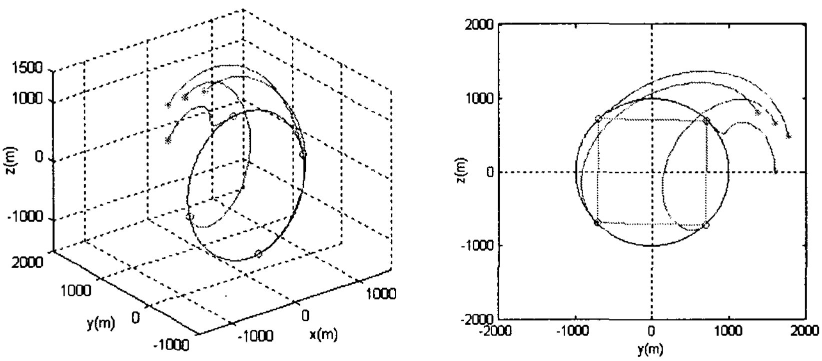
Case A. Reconfiguration of three and four satellite formation

In this example, it is assumed that 3 and 4 satellites are evenly spaced in a circle of 2,000 m radius in y-z plane at the initial time. At the final time, positions of all satellites have to form an equilateral polygon on a circle of 1,000 m radius in the y-z plane. Simulation is performed for three cases of the minimization of fuel ($\beta = 0.0$), the minimization of time ($\beta = 1.0$) and the minimization of fuel and time ($\beta = 0.5$) using reconfiguration problem with 3 satellites. And the minimization of fuel and time ($\beta = 0.5$) is considered for reconfiguration problem with 4 satellites. Figure 3 shows the trajectories of reconfiguration in three dimensional space and y-z plane for three satellite formation according to three minimization strategies mentioned above. Positions of all the satellites



(a) Trajectories in 3-D space (β=0.5) (b) Trajectories in y-z plane (β=0.5)

Figure 6. Trajectories of a configuration with 3 satellites.



(a) Trajectories in 3-D space (β=0.5) (b) Trajectories in y-z plane (β=0.5)

Figure 7. Trajectories of a configuration with 4 satellites.

form a equilateral triangle in the y-z plane at the final time from Figure 3. As a weighting factor (β) increases, the length of trajectories of all the satellites becomes shorter because the increasement of a weighting factor means from Eq. (5) that the cost function puts more weight on the final time than a fuel consumption. Figure 4 shows the trajectories of reconfiguration for four satellite formation according to minimization strategy of final time and fuel consumption ($\beta = 0.5$). From Figure 4, all satellites form a square in the y-z plane at the final time.

Control inputs of a satellite is shown in Figure 5 for reconfiguration problem of Figure 3c. The number of unknown control inputs is 15 for x component and a node is divided into 5 sub intervals in which control inputs are calculated by linear interpolation not by estimation.

Case B. Configuration of three and four satellite formation

In this case, it is assumed that 3 and 4 satellites can have any position at the initial time. However positions of all the satellites should form an equilateral polygon on a circle of 1,000 m radius in the y-z plane at the final time. Trajectories of three dimensional space and y-z plane are shown in Figure 6 for configuration problem with 3 satellites and Figure 7 for configuration problem with 4 satellites.

5. CONCLUSIONS

A new method is presented to get optimal trajectories in the configuration and reconfiguration mission of satellite formation flying based on parameter optimization technique which is solved using SQP method. New constraints of collision avoidance and final configuration are derived and applied to the trajectory optimization problem which minimizes fuel consumption and time required for the formation. From the results of numerical simulation, optimal trajectories of configuration and reconfiguration could be obtained for a formation flying system with 3 and 4 satellites. If the number of nodes increases, cost function can decrease but time required for estimation of unknown parameters will be longer because the number of unknown parameters and constraints for collision avoidance increases. Thus the suitable number of nodes can be selected according to the cost function. It is important to determine the suitable radius of a sphere for collision avoidance because the large radius of a sphere cannot generate the optimal trajectories for the given limit of control inputs. This method is useful not for real-time trajectory generation but for optimal trajectory-planning aspects of mission management.

ACKNOWLEDGEMENTS: The present work was supported by National Research Lab. (NRL) Program (2002) by the Ministry of Science and Technology, Korea. Authors fully appreciate the financial support.

REFERENCES

Alfriend, K. T., Schaub, S., & Gim, D-W. 2000, 23rd Annual AAS Guidance and Control Conference, AAS 00-012

Barraquand, J., Kavraki, L., Latombe, J. C., Motwani, R., Li, T. Y., & Raghavan, P. 1997, International Journal of Robotics Research, 16, 759

Beard, R. W., McLain, T. W., & Hadaegh, F. Y. 2000, Journal of Guidance, Control, and Dyanmics, 23, 339

Clohessy, W. H., & Wiltshire, R. S. 1960, Journal of the Aerospace Sciences, 27, 653

Gavin, H., Johnson, Y., & McInnes, C. R. 1995, Advances in Astronautical Sciences, AAS 95-104, 2093

Hull, D. G. 1997, Journal of Guidance, Control, and Dyanmics, 20, 57

Lim, H. C., Bang, H. C., Park, K. D., & Park, P. H. 2003, JA&SS, 20, 365

Onwubiko, C. 2000, Introduction to Engineering Design Optimization (New Jersey: Prentice-Hall Inc.), pp.175-177.

Richards, A., Schouwenaars, T., How, J. P., & Ferson, E. 2002, Journal of Guidance, Control, and Dyanmics, 25, 755

Sabol, C., Burns, R., & McLaughlin, C. A. 2001, Journal of Spacecraft and Rockets, 38, 270

Singh, G., & Hadaegh, F. Y. 2001, AIAA Guidance, Navigation and Control Conference and Exhibit, AIAA 2001-4088

Tillerson, M., Inalhan, G., & How, J. 2002, International Journal of Robust and Nonlinear Control, 12, 207

Yang, G., Yang, Q., Kapila, V., Palmer, D., & Vaidyanathan, R. 2002, International Journal of Robust and Nonlinear Control, 12, 243

Wang, P. K. C., & Hadaegh, F. Y. 1998, AIAA Guidance, Navigation and Control Conference and Exhibit. AIAA 1998-4226