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THE EFFECT OF DUST PARTICLES ON ION ACOUSTIC SOLITARY WAVES IN A DUSTY PLASMA

Cheong Rim Choi†, Dae Young Lee, and Yonggi Kim
Department of Astronomy and Space Science, College of Natural Sciences and Institute for Basic Sciences
Chungbuk National University, Cheongju 361-763, Korea
E-mail: crchoi@chungbuk.ac.kr

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ABSTRACT

In this paper we have examined the effect of dust charge density on nonlinear ion acoustic solitary wave which propagates obliquely with respect to the external magnetic field in a dusty plasma. For the dusty charge density below a critical value, the Sagdeev potential $\Psi(n)$ has a singular point in the region $n < 1$, where n is the ion number density divided by its equilibrium number density. If there exists a dust charge density over the critical value, the Sagdeev potential becomes a finite function in the region $n < 1$, which means that there may exist the rarefactive ion acoustic solitary wave. By expanding the Sagdeev potential in the small amplitude limit up to δn^4 near $n = 1$, we find the solution of ion acoustic solitary wave. Therefore we suggest that the dust charge density plays an important role in generating the rarefactive solitary wave.

Keywords: dust space plasma, ion acoustic solitary wave, soliton

1. INTRODUCTION

The physics of the dust particles or impurities in plasmas has been recently studied in many areas of space environments, such as planetary rings, comets, the interstellar medium, and the earth's ionosphere and magnetosphere (Goertz 1989, Mamun *et al.* 1996, Rao *et al.* 1990, Yinhua & Yu 1994, Mamun & Alam 1998, Spatschek *et al.* 1979, Mamun 1997). Through the processes such as ionization, field-emission, plasma currents, ultraviolet radiation, etc. the dust or impurity particles can be highly charged. Their presence in the plasma can significantly change the plasma parameters and modify the collective behavior of the plasma. Each of dusty particles has different shape, size, charge, and mass, and thus it is complicated to treat them individually. Therefore, many models of dusty plasma adopt an assumption that the dust-charged grains have constant charge and are stationary in contrast to ion's dynamical behavior (Shukla 2001, Mamun *et al.* 1996, Yu *et al.* 1980). In the present work, we consider solitary waves in a three-component plasma composed of electrons, ions and dusty particles. We assume that the size of dusty particles is much smaller than the Debye length and the average distance between the plasma particles.

† corresponding author

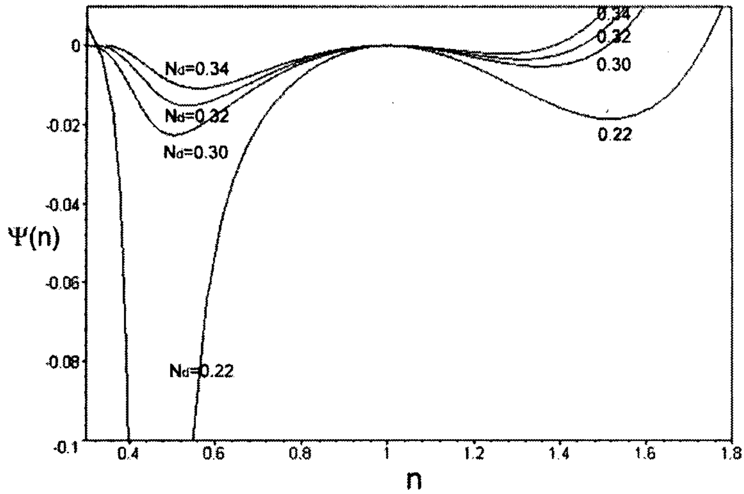

Figure 1. The Sagdeev potential when $M = 0.63$ and $l_{\|} = 0.47$ for $N_d = 0.22, 0.30, 0.32$, and 0.34.

Yinhua & Yu (1994) found the small amplitude compressive ion acoustic solitary wave in a dusty plasma by expanding Sagdeev potential up to δn^3 order near $n = 1$. However, we have noticed that a dusty plasma with its density over a critical value gives rise to a Sagdeev potential for which a higher order expansion than δn^3 is required. Figure 1 illustrates such a case in comparison to the Yinhua & Yu's case, about which we will discuss more below in section 3. In our work, we expand the Sagdeev potential up to δn^4 near $n = 1$ to see its effect on the ion acoustic solitary waves. From the nonlinear differential equation obtained in this approximation, we find that there can exist the rarefactive ion acoustic solitary wave as well as the compressive one.

In section 2, we briefly introduce the basic equations used and derive the Sagdeev potential. In section 3, we analyze the Sagdeev potential obtained in section 2 and explain the condition for the localized solution to exist. We then find the solutions of the ion acoustic solitary waves from the small amplitude expansion in section 4. The conclusion is given in section 5.

2. BASIC EQUATIONS AND THE SAGDEEV POTENTIAL

In this paper, we assume that the ion mass alone provides the inertia and the inertialess electrons follow the Boltzmann relation, and that the heavy impurity particles are stationary in a magnetized plasma.

The basic equations governing the ion dynamics in this plasma system are

$$\frac{\partial n_i}{\partial t} + \nabla \cdot (n_i \mathbf{v}_i) = 0. \tag{1}$$

and

$$\frac{\partial \mathbf{v_i}}{\partial t} + (\mathbf{v}_i \cdot \nabla)\mathbf{v}_i = -\frac{e\nabla\phi}{m_i} + \frac{eB_0}{m_i c}\mathbf{v}_i \times \hat{e}_z. \tag{2}$$

where subscript i stands for ions, n_i, $\mathbf{v}_i$, m_i, and ϕ are the number density, velocity, mass, and the electrostatic potential, respectively. The number density of the electrons is given by

$$n_e = n_{e0} \exp(e\phi/T_e). \tag{3}$$

We assume that the length scale L of the soliton should be greater than the Debye length λ_D and the gyro-radius r_g. Thus we use the charge neutrality condition instead of the Poisson equation. The charge neutrality condition is

$$n = N_d + N_e. \tag{4}$$

where $n = n_i/n_{i0}$, the normalized dust particles charge density $N_d = \frac{Z_d n_d}{n_{i0}}$, Z_d is the number of the charge residing on the dust particle, $N_e = n_e/n_{i0}$. So it is satisfied that $N_d + n_{e0}/n_{i0} = 1$ in equilibrium.

The basic equation of the system can be rewritten in the normalized form as follows

$$\frac{\partial n}{\partial t} + \frac{\partial (n v_x)}{\partial x} + \frac{\partial (n v_z)}{\partial z} = 0, \tag{5}$$

$$\frac{\partial v_x}{\partial t} + \left(v_x \frac{\partial}{\partial x} + v_z \frac{\partial}{\partial z} \right) v_x = -\frac{\partial \Phi}{\partial x} + v_y, \tag{6}$$

$$\frac{\partial v_y}{\partial t} + \left(v_x \frac{\partial}{\partial x} + v_z \frac{\partial}{\partial z} \right) v_y = -v_x, \tag{7}$$

and

$$\frac{\partial v_z}{\partial t} + \left(v_x \frac{\partial}{\partial x} + v_z \frac{\partial}{\partial z} \right) v_z = -\frac{\partial \Phi}{\partial z}. \tag{8}$$

Here, the normalized variables are $\Omega t \to t$, $(\frac{C_s}{\Omega})\nabla \to \nabla$, $\frac{\mathbf{v}_i}{C_s} \to \mathbf{v}$, $\frac{n_i}{n_{i0}} \to n$, and $\frac{e\phi}{T_e} \to \Phi$, where $C_s = (\frac{T_e}{m_i})^{1/2}$ is the ion acoustic velocity, $\Omega = \frac{eB_0}{m_i c}$ is the ion gyro-frequency, and $r_g = \frac{C_s}{\Omega}$ is the ion gyro-radius. Note that all variations are assumed to be in the x-z plane.

To obtain dimensionless linear dispersion relation for low frequency ($\omega \ll \Omega$) ion acoustic waves, we linearize the normalized basic equations by assuming all the perturbed quantities vary as $\exp[i(k_\perp x + k_\parallel z - \omega t)]$. Then, the dispersion can be written as

$$\omega = k_\perp (\epsilon + k_\parallel{}^2)^{-1/2}. \tag{9}$$

where $k_\perp$ and $k_\parallel$ are the wave vectors in the perpendicular and parallel direction to an external magnetic field $\mathbf{B_0}$, respectively, and $\epsilon = n_{e0}/n_{i0}$. This dispersion and the nonlinear steepening of the finite amplitude ion acoustic waves can balance each other to form solitary waves.

To find the solution of Eqs. (5)-(8), we define the moving coordinate as,

$$\xi = l_\perp x + l_\parallel z - Mt. \tag{10}$$

where $l_\perp$ and $l_\parallel$ are direction cosines, and M is the Mach number of localized wave. It is satisfied $l_\perp^2 + l_\parallel^2 = 1$. From the continuity equation (5), and using Eq. (10), we obtain the relation,

$$v_x \frac{\partial}{\partial x} + v_z \frac{\partial}{\partial z} = M \left(1 - \frac{1}{n} \right) \frac{d}{d\xi}. \tag{11}$$

Using the quasi-neutrality condition, we can write the normalized electric potential as

$$\Phi = \ln \left(\frac{n - N_d}{\epsilon} \right). \tag{12}$$

Then,

$$\frac{d\Phi}{d\xi} = \frac{1}{n - N_d}\frac{dn}{d\xi}. \tag{13}$$

Using the boundary condition at $\xi \to \infty$, $n \to 1$, $\phi = \mathrm{v} = 0$ and Eqs. (11)-(13), we can calculate v_y as

$$v_y = \frac{1}{l_\perp}\frac{d}{d\xi}\left(\frac{M^2}{2n^2} + \Phi\right). \tag{14}$$

We substitute Eq. (14) into the transformed Eq. (6) in a moving frame by using Eqs. (10) and (11), and integrate them. Then, v_x can be written as

$$v_x = -\frac{{l_\parallel}^2}{M l_\perp}\left\{(n-1) + N_d \ln\left(\frac{n - N_d}{1 - N_d}\right)\right\} + \frac{M}{l_\perp}\left(\frac{n-1}{n}\right). \tag{15}$$

Using Eqs. (10) and (11), we substitute v_x and Eq. (15) into the transformed Eq. (7) in a moving frame, then we obtain

$$\frac{d^2}{d\xi^2}\left(\Phi + \frac{M^2}{2n^2}\right) = -\frac{{l_\parallel}^2}{M^2}\left\{n(n-1) + nN_d \ln\left(\frac{n - N_d}{1 - N_d}\right)\right\} + (n-1). \tag{16}$$

Equation (16) can be expressed as

$$\frac{1}{2}\left(\frac{dn}{d\xi}\right)^2 + \Psi(n) = 0. \tag{17}$$

which is in the form of the the energy integral for a classical particle in a potential well (Sagdeev 1966), and where $\Psi(n)$ is the Sagdeev potential,

$$\Psi(n) = \frac{\frac{l_\parallel^2}{M^2}\Psi_1(n) + \Psi_2(n)}{\left(-\frac{M^2}{n^3} + \frac{1}{n - N_d}\right)^2} \tag{18}$$

and

$$\begin{aligned}
\Psi_1(n) = \;& N_d\left\{(n-1) + \tfrac{N_d}{2}\ln\left(\tfrac{n-N_d}{1-N_d}\right)\right\}\ln\left(\tfrac{n-N_d}{1-N_d}\right) + \tfrac{(n-1)(n+2N_d+1)}{2} \\
& -(n-1)(1+N_d) - M^2\left(\ln n - \tfrac{n-1}{n}\right) \\
\Psi_2(n) = \;& \left\{1 - N_d - {l_\parallel}^2\left(\tfrac{n-N_d}{n}\right)\right\}\ln\tfrac{n-N_d}{1-N_d} + {l_\parallel}^2 \ln n - (n-1)
\end{aligned}$$

3. CONDITION FOR THE EXISTENCE OF ION ACOUSTIC SOLITARY WAVES

We now examine the Sagdeev potential $\Psi(n)$ to determine the conditions for the ion acoustic solitary wave to exist and the behavior of possible localized solutions. The conditions for the existence of localized solutions is given by (Yinhua & Yu 1994, Mamun & Alam 1998, Popel & Yu 1995)

$$\begin{aligned}
\Psi(n)|_{n=1} &= \left.\frac{d\Psi(n)}{dn}\right|_{n=1} = 0 \\
\Psi(N) &= 0 \\
\Psi(n) &< 0, \quad N < n < 1 \quad \text{or} \quad 1 < n < N
\end{aligned} \tag{19}$$

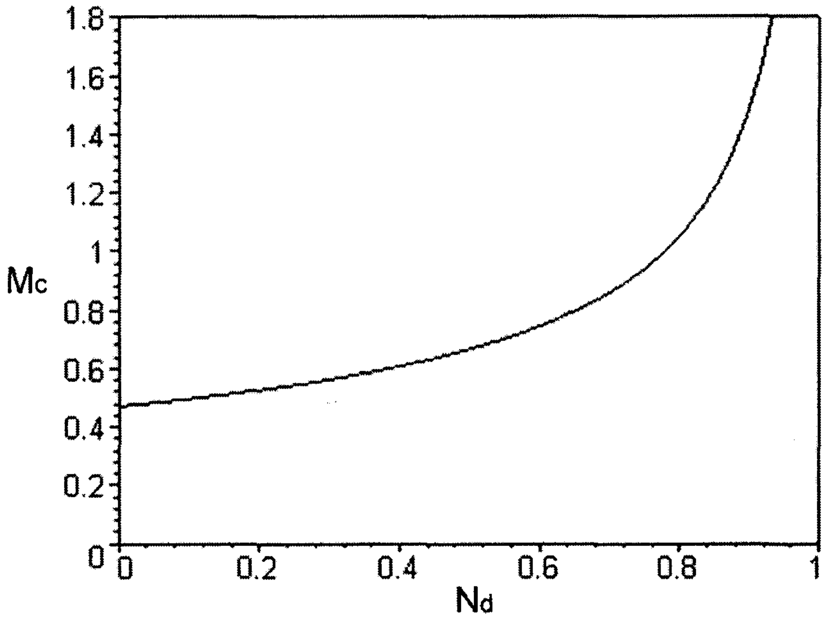

Figure 2. The critical Mach number M_c vs. N_d for $l_{\|} = 0.47$.

where N is the minimum or the maximum ion density within the localized structure, respectively.

In Figure 1, we plotted the Sagdeev potential for selected value of N_d. There are four solutions including one double solution. The rarefactive (compressive) solitary wave may exist in the regions with $N < n < 1$ $(1 < n < N)$. Most importantly, it is seen from Figure 1 that there must exist a critical value of N_d above which the rarefactive solitary wave exists in the region $N < n < 1$. For example, for $N_d = 0.22$, the potential becomes singular at within the region, as this value of N_d is smaller than the critical value. Thus, we suggest that the dust charge density N_d plays an important role in determining the rarefactive solitary wave solution. Also we see that the amplitude of ion acoustic solitary waves decreases as N_d increases as in Figure 1.

If the condition $\frac{d^2\Psi(n)}{dn^2}\Big|_{n=1} < 0$ is satisfied, there can exist ion acoustic solitary waves. In this case, the Mach number of the solitary wave lies in the range of

$$l_{\|}^2 \left(\frac{1}{1-N_d}\right) < M^2 < \frac{1}{1-N_d}. \tag{20}$$

For a dusty free plasma (Yu *et al.* 1980, Shukla & Yu 1978), the solitary wave propagates with a subsonic speed. From Eq. (20), we can calculate the maximum critical Mach number M_c to be

$$M_c = l_{\|}\sqrt{\frac{1}{1-N_d}}. \tag{21}$$

In the limit of $M \to M_c$, the amplitude of solitary wave becomes zero. As N_d increases, we know that the solitary wave can propagate with supersonic speed for $l_{\|} = 0.47$ which is noted in Figure 2.

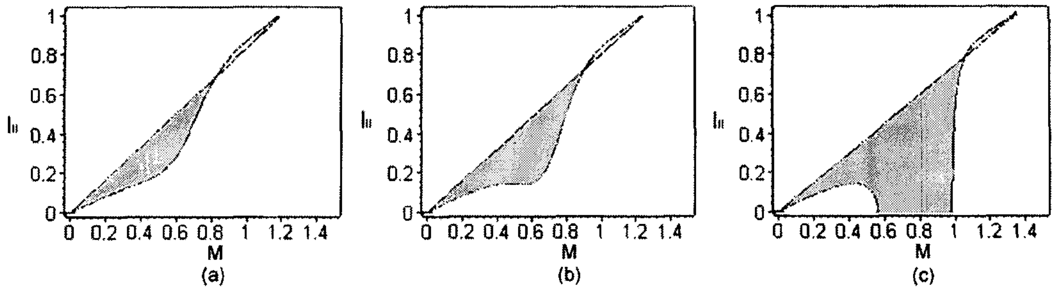

Figure 3. The change of the region can be existed both the compressive and rarefactive ion acoustic solitary wave when (a) $N_d = 0.30$, (b) $N_d = 0.36$, and (c) $N_d = 0.45$.

4. SMALL AMPLITUDE ION ACOUSTIC SOLITARY WAVE

In order to find both compressive and rarefactive solitary waves, the Sagdeev potential $\Psi(n)$ needs to be expanded up to fourth order in δn near $n = 1$,

$$\Psi(n) \approx A\delta n^2 + B\delta n^3 + C\delta n^4. \tag{22}$$

where $\delta n = n - 1$ and the coefficients are $A = \frac{1}{2}\frac{d^2\Psi(n)}{dn^2}\big|_{n=1}$, $B = \frac{1}{6}\frac{d^3\Psi(n)}{dn^3}\big|_{n=1}$, and $C = \frac{1}{24}\frac{d^4\Psi(n)}{dn^4}\big|_{n=1}$. The relationship among the coefficients A, B, and C, plays an important role in determining the localized solution to be the solitary waves of the bump type in Eq. (22). Expanding the Sagdeev potential up to lower order than δn^4 order will lead to only compressive solution, but no rarefactive one, as done previously by Yinhua & Yu (1994).

Comparing Eq. (22) and Figure 1, we find that the condition for a bump type solution is $A < 0$ and $C > 0$. The size of this region depends on N_d values, as shown in Figure 3. It is known that the region for the existence of the bump type (rarefactive and compressive) solitary waves increases as N_d increases. It means that the solitary waves can be more easily generated as N_d increases. For a dust free plasma, our result reduces to that of Yu *et al.* (1980).

Now we proceed to find the bump type solutions of ion acoustic solitary wave. Substituting the expression for Ψ of Eq. (22) into Eq. (17), we obtain

$$\frac{1}{2}\left(\frac{d\delta n}{d\xi}\right)^2 + A\delta n^2 + B\delta n^3 + C\delta n^4 = 0. \tag{23}$$

where $\delta n = n - 1$. Let $\delta n = \frac{1}{y}$, we obtain

$$\left(\frac{dy}{d\xi}\right)^2 + 2Ay^2 + 2By + 2C = 0. \tag{24}$$

As the condition for the bump type localized solutions to exist is $A < 0$ and $C > 0$, and so $B^2 - 4AC > 0$, the solutions of Eq. (24) are obtained as

$$\delta n = \frac{1}{-\frac{B}{2A} \pm |a|\cosh(\zeta)}. \tag{25}$$

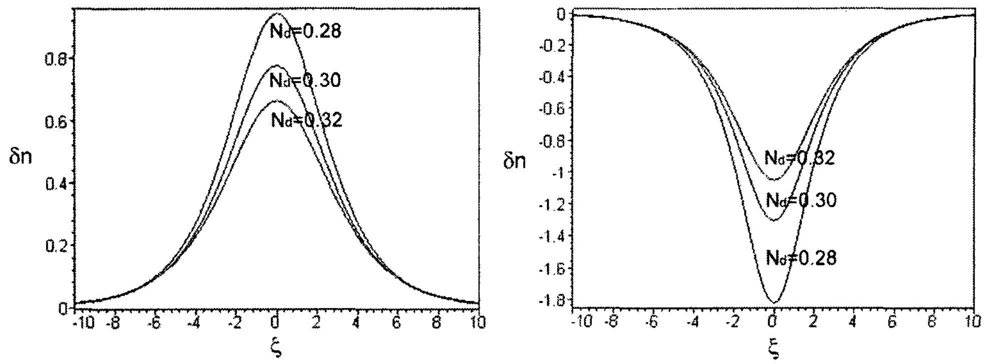

Figure 4. The bump type ion acoustic solitary waves for $M = 0.65$ and $l_{||} = 0.47$; (a) the compressive, (b) the rarefactive ion acoustic solitary waves for $N_d = 0.28$, 0.30, and 0.32, respectively.

where $a^2 = \frac{B^2-4AC}{4A^2}$ and $\zeta = \sqrt{|-2A|}\xi$. Each of the positive and negative sign means the solution of compressive and rarefactive solitary wave in Figure 4 (a) and (b), respectively. When $M = 0.65$ and $l_{||} = 0.47$, there are the compressive and the rarefactive ion acoustic solitary waves for $N_d =$ 0.28, 0.30, and 0.32 in Figure 4, respectively. It shows that the amplitude of the compressive solitary wave decreases as N_d increases. For the coefficient $C \rightarrow 0$, our result reduces to the solution of Yinhua & Yu (1994).

5. CONCLUSION

In this study, we have demonstrated the effect of dust particles on ion acoustic solitary wave in a dusty plasma. There can exist the rarefactive ion acoustic solitary wave in addition to the compressive ion acoustic solitary wave in a dusty plasma when dusty particle's charge density N_d is over a critical value. We calculated the Sagdeev potential exactly and demonstrated the existence of ion acoustic solitary wave. We calculated the rarefactive as well as the compressive solitary wave by expanding the Sagdeev potential in a small amplitude limit up to δn^4 order near $n = 1$. We also found that, as the dust charge density N_d increases, the parameter region where solitary waves exist increases, but the amplitude of solitary waves decreases.

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