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CCD PHOTOMETRY USING MULTIPLE COMPARISON STARS

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ABSTRACT

The accuracy of CCD observations obtained at the Korean 1.8 m telescope has been studied. Seventeen comparison stars in the vicinity of the cataclysmic variable BG CMi have been measured. The "artificial" star has been used instead of the "control" star, what made possible to increase accuracy estimates by a factor of 1.3-2.1 times for "good" and "cloudy" nights, respectively. The algorithm of iterative determination of accuracy and weights of few comparison stars contributing to the artificial star, has been presented. The accuracy estimates for 13-mag stars are around $0.^{m}002$ mag for exposure times of 30 sec.

Keywords: data analysis, CCD photometry, BG CMi.

1. INTRODUCTION

The aim of this study is to check accuracy of the CCD observations obtained at the Korean 1.8 m telescope and to improve it using multiple variable stars. Despite the idea of using more photons from many stars seem to be obvious, usually another scheme is applied. One of the stars, usually close in color to the variable, is used as the comparison star. Another star is used as the check star to be sure that the comparison star is not variable and to use it, if the magnitude of the variable star undergoes abrupt changes owed to cosmic rays, hot pixels etc.

In this work, we study dependence of accuracy estimates as the function of stellar magnitude, which may be used for planing further observations. Such study for improving the observational accuracy with BOAO 1.8 m telescope has not been reported until now. Therefore this method of multiple comparison stars will be used for our further analysis of BOAO data.

2. OBSERVATION

The CCD images were obtained with a thinned SITe 2k CCD camera attached to the BOAO 1.8 m telescope. The instrumental V system has been used. To determine instrumental magnitudes of stars, the IRAF/DAOPHOT package has been used. An integration time was 30^{s}.

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Table 1. Journal of observations: begin and end of the run, the number of observations n.

t_{start}	t_{end}	n	Range of C4	
53035.0390	53035.1655	217	15.042	0.176
53036.1058	53036.2677	278	14.953	0.163

Table 2. Mean brightness and color of the variable and comparison stars.

Star	V_{in}	σ_{in}	$\Delta(V-R)_{in}$	$\Delta(R-I)_{in}$	V_{HH}	$(B-V)_{HH}$	ID_{HH}
BG	14.997	0.169	–0.092	–0.194	14.92*	–0.05*	
C1	14.142	0.011	0.172	0.150	14.117	1.033	11
C2	12.666	0.007	0.230	0.221	12.630	1.171	02
C3	12.958	0.006	–0.048	–0.065	12.969	0.571	04
C4	12.457	0.007	0.000	0.000	12.457	0.707	05
C5	14.439	0.021	0.118	0.065	14.426	0.851	09
C6	13.479	0.018	–0.086	–0.077	13.465	0.473	03

*data from Patterson & Thomas (1993), whereas the rest V_{HH} and $(B-V)_{HH}$ are from Henden & Honeycutt (1995). ID_{HH} shows an identification number from the latter paper.

For our study, we have chosen two nights of observations of the cataclysmic variable BG CMi= 3A 0729 + 103. Results of investigation of this intermediate polar will be presented elsewhere (Kim et al. 2004). We have chosen two nights of observations obtained on January 30 and 31, 2004. The weather conditions during the first night were good, with an expectation of normal atmospheric extinction. During the second night, there were light clouds, so the measured signal varied occasionally by $\sim 1.^m2$. These data allow to study separately extinction in transparent atmosphere and in clouds.

The journal of observations is presented in Table 1. As the characteristic of atmospheric transparency variations, we list the range of instrumental magnitudes of the "main" comparison star C4 (cf. Pych et al. 1996). According to Henden & Honeycutt (1995), it's magnitude V=$12.^m457$, so the difference between the standard and instrumental magnitudes $V_{st} \approx V_{in}$-$1.^m511$. However, the "instrumental" magnitudes are affected by atmospheric transparency, and so apparently vary with time, so this relation is only approximate to scale the magnitudes. Fig. 1 shows instrumental magnitudes of 17 comparison stars and the variable BG CMi at the same scale. Horizontal lines are delimiters of groups of 6 stars. The data have been obtained at JD 2453035 with good atmospheric conditions. Fig. 2. represents instrumental magnitudes of 6 comparison stars with smaller scatter. Differences between instrumental magnitudes of these 6 comparison stars are shown as well as the light curves of 6 comparison stars restored using the multi–comparison star method.

3. USING FEW COMPARISON STARS

To obtain better accuracy, we have used few comparison in the field. Their BV magnitudes have been estimated by Henden & Honeycutt (1995) as summarized in Table 2. The following procedure has been applied. Let $X_{k\alpha}$ be the $k^{\rm th}$ ($k = 1..N$) measurement of the $\alpha^{\rm th}$ star. Then the values $\langle X_\alpha \rangle$ averaged over all observations are computed and adopted as mean instrumental magnitudes of stars. Because of changing atmospheric transparency, the scatter of individual points is much larger, than the internal accuracy of observations σ_α for every star. The latter has been estimated using weighted mean of all m comparison stars $\bar{X}_k = \sum_{\alpha=1}^{m} w_\alpha X_{k\alpha}$, where the sum of the weights is normalized to unity. Assuming that the statistical errors have zero mean and r.m.s. deviation σ_α, one

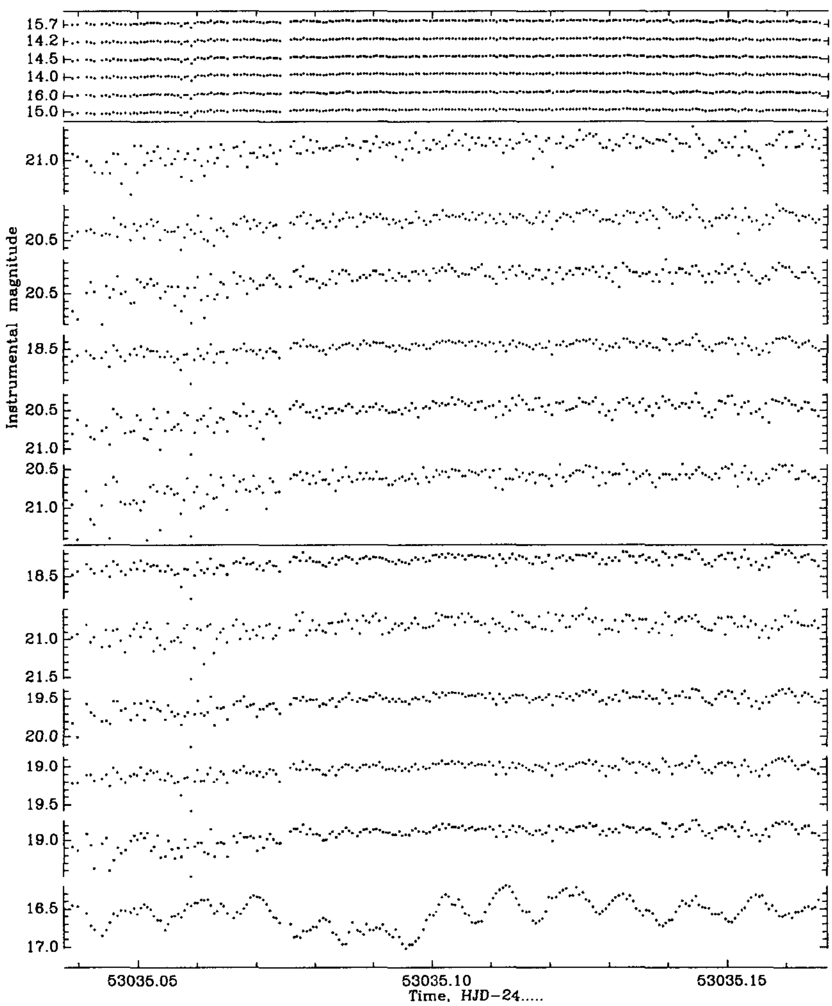

Figure 1. Instrumental magnitudes of 17 comparison stars and the variable BG CMi (bottom) at the same scale. Horizontal lines are delimiters of groups of 6 stars. The data have been obtained at JD 2453035 with good atmospheric conditions.

may estimate an accuracy of each comparison star

$$\sigma_\alpha^2 = \langle (X_{k\alpha} - \bar{X}_k)^2 \rangle, \quad \tilde{\sigma}_\alpha^2 = \sigma_\alpha^2/(1 - w_\alpha), \tag{1}$$

where the mathematical expectation of the squared difference is replaced by a sample mean. Here we have additionally suggested a statistical weight $w_\alpha = \sigma_0^2/\sigma_\alpha^2$ (cf. Korn & Korn 1961). The parameters σ_α^2 and $\tilde{\sigma}_\alpha^2$ are biased and unbiased estimates of the variance of residuals of brightness from the artificial star.

Obviously, we do not know the accuracies (weights) *a priori*, thus may compute them using some iterations: initial values of the weights W_α are all set to unity, then normalizing $w_k = W_\alpha \ / \ \sum_{\alpha=1}^m W_\alpha$, using these weights to estimate σ_α^2 using Eq.(1). Next step is to make correction

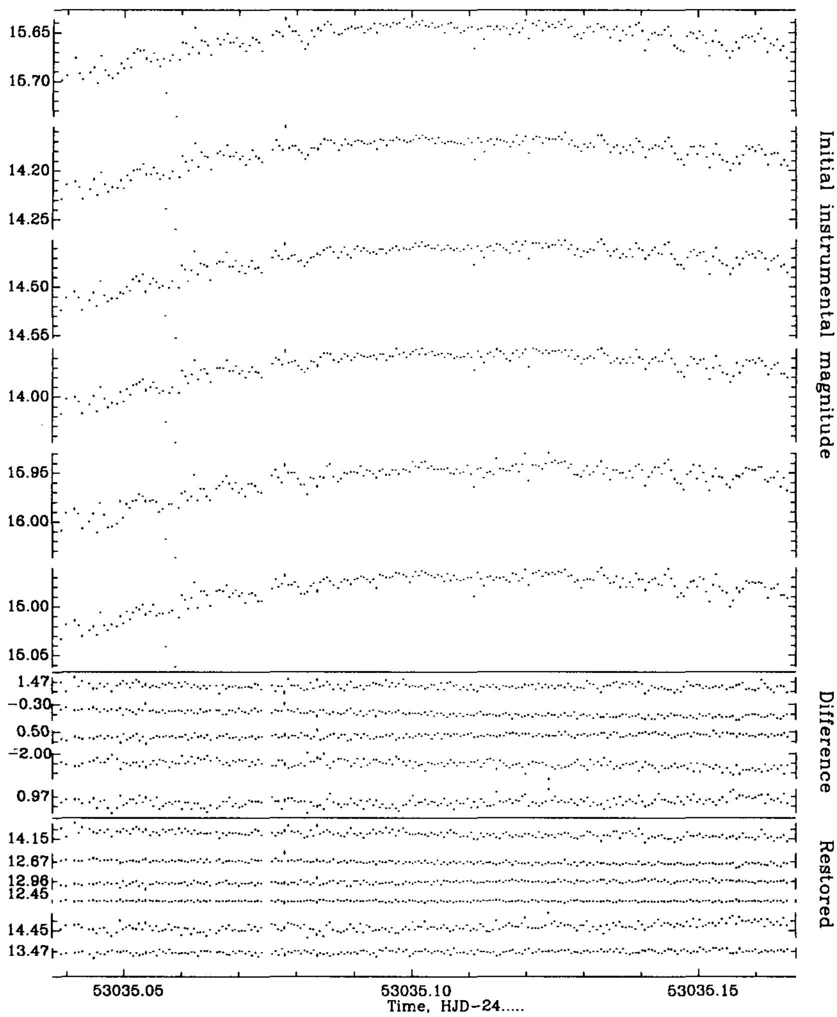

Figure 2. *Up:* Instrumental magnitudes of 6 comparison stars with smaller scatter. *Middle:* Differences between instrumental magnitudes of these 6 comparison stars. *Bottom:* The light curves of 6 comparison stars restored using the "multi–comparison star" method.

of the weights $W_\alpha = \sigma_\alpha^{-2}$, normalization etc until after some iterations the weight will converge to their self-consistent values. Then an unit weight error (error of the weighted mean) for each exposure had been computed using the formula

$$\sigma_{0k}^2 = \frac{1}{m-1} \sum_{\alpha=1}^{m} w_\alpha (X_{k\alpha} - \bar{X}_k)^2. \tag{2}$$

As an example, the accuracy estimates σ_α and the corresponding normalized weights w_α for comparison stars 1-6 are listed in the Table 3.

The weight of the "artificial" comparison star is $w = 1$ with a corresponding r.m.s. error estimate $\sigma = 0.^{m}00072$. One may note, that the weights are very different in value. The largest weight 0.544

Table 3. The accuracy estimates σ_α and the corresponding normalized weights w_α for comparison stars.

Star	1	2	3	4	5	6
σ_α	$0.^m0034$	$0.^m0020$	$0.^m0019$	$0.^m0010$	$0.^m0042$	$0.^m0021$
w_α	0.044	0.128	0.141	0.544	0.029	0.114

corresponds to the star $C4$, whereas the merged weight of stars $C1$ and $C5$ is 7 percent only. The lowest weight for the star $C5$ is an indicator of it's possible variability, which will be checked later. instrumental magnitudes of the "mean weighted" comparison star and the variable BG CMi are shown in Fig. 3.

4. CHARACTERISTICS OF COMPARISON STARS

In the Table 2, the following characteristics of the measured stars are listed: V_{in} - instrumental V magnitude shifted to the main comparison star $C4$ with brightness of V=$12.^m457$ (Henden & Honeycutt 1995), σ_{in} is the r.m.s. value of the deviations from the mean value for a given star. The instrumental color differences are computed as $\Delta(V-R)_{in} = (V-R)_{in,star} - (V-R)_{in,c4}$ and similarly $\Delta(R-I)_{in}$. For these estimates, the measurements in the VRI filter have been made at HJD 2453036.10405. For comparison, we also list the values V_{HH} and $(B-V)_{HH}$ published by Henden & Honeycutt (1995). Instrumental color indices $(V-R)_{in}$ and standard color index $(B-V)_{HH}$ are shown in Fig. 4.

Of course, the scatter is largest for the variable star and is much smaller for the comparison stars. The mean squared accuracy of the "mean" star for all observations is $\sigma_0 = 0.^m0036$, twice better, than of the comparison star $C4$ alone. The latter star is closest to the variable, being by 16.4" to the north, and is among closest to the variable in colors. It was also used as "Comp 1" by Pych et al. (1996).

The differences between our instrumental magnitudes V_{in} and the standard ones V_{HH} (Henden & Honeycutt 1995) are well explained by the color reduction formula

$$V_{in} = V_{HH} - 0.^m0025(16) + 0.161(11)\Delta(V-R)_{in} \quad (3)$$

The correlation coefficient between the instrumental colors $(V-I)_{in}$ and the standard colors $(B-V)_{HH}$ (Henden & Honeycutt 1995) is $r = 0.991 \pm 0.066$. The "color–color" diagram is shown in Fig.5. The regression line is

$$(V-R)_{in} = 0.^m064(8) + 0.469(31)((B-V)_{HH} - 0.^m801). \quad (4)$$

Such a good correlation argues for normal colors of all stars studied.

5. EXTINCTION COEFFICIENT

For the "good" night JD 2453035, the variations of the instrumental magnitudes V seem to be caused by the variations of the air mass. For each comparison star, we have computed the coefficients of the regression line

$$m(t) = m_{00} + k_{00} \cdot M(z), \quad (5)$$

where $M(z)$ is the air mass corresponding to the zenith distance z. This formula from the course of General Astronomy corresponds to the constant extinction coefficient k_{00}. This smoothed dependence significantly deviates from the real data making such a fit senseless even for the "good" night.

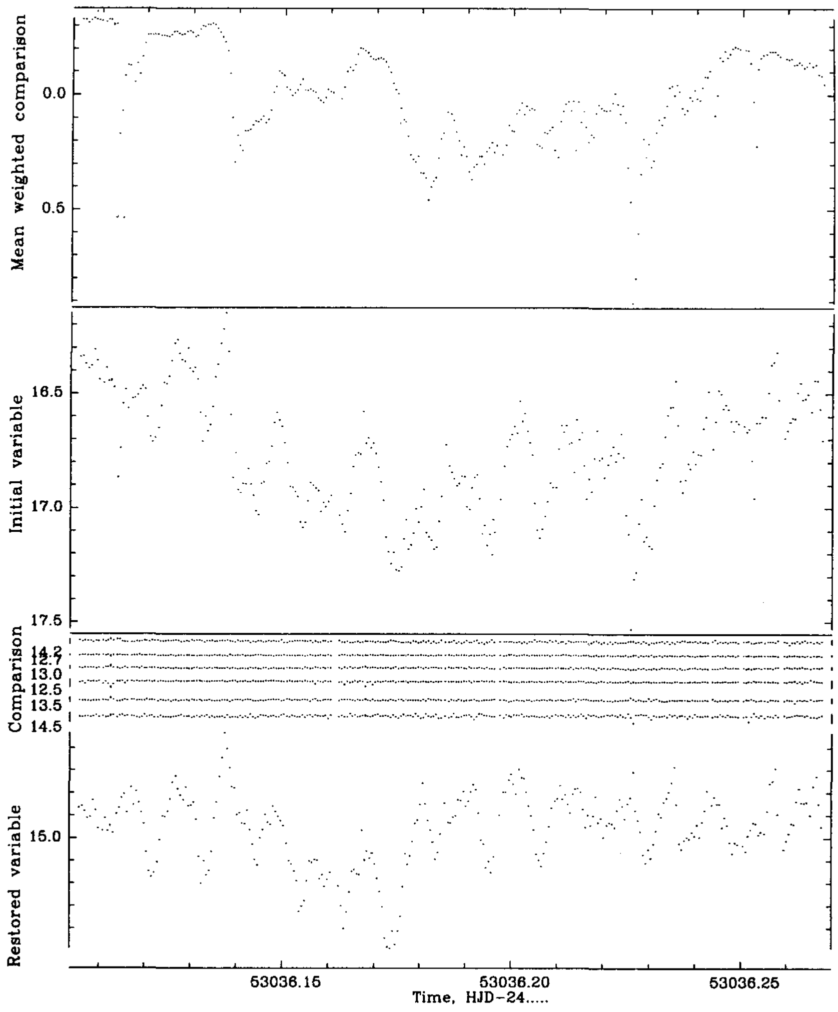

Figure 3. *Up:* Instrumental magnitudes of the "mean weighted" comparison star and the variable BG CMi (bottom) at the same scale. The data have been obtained at JD 2453036 with bad weather and stronly variable atmospheric transparency. *Middle:* The light curves of 6 comparison stars restored using the "multi–comparison star" method. *Bottom:* The final light curve of the variable BG CMi.

Assuming that the extinction coefficient changes smoothly with time, we have applied the simplest formula $k(t) = k_{10} - k_{11} \cdot (t - \bar{t})$, where $\bar{t}$ is the mean time for the moments of observations. The minus sign has been chosen, because the extinction seemed to decrease with time, so the atmosphere was becoming more transparent with decreasing temperature. Thus the final fit is

$$m(t) = m_{10} + k_{10} \cdot M(z) - k_{11} \cdot ((t - \bar{t}) \cdot M(z)). \quad (6)$$

Both fits are shown in Fig.5 for the adopted comparison star $C4$. The r.m.s deviations from the fit σ_{10} are by 1.4-1.9 times smaller than that σ_{00} for the assumption of the constant extinction. The values of the coefficients k_{11} exceed their error estimates by a factor from 21 to 30, showing their

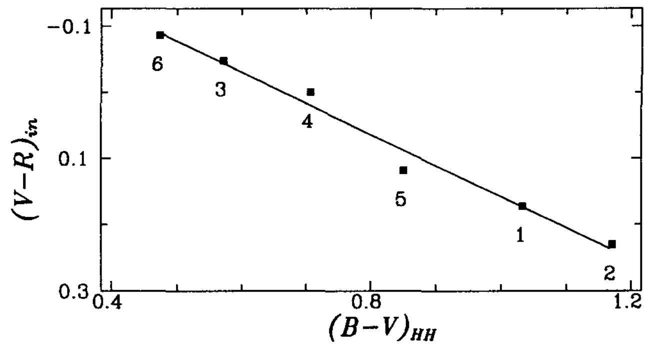

Figure 4. Instrumental color indices $(V-R)_{in}$ vs standard color index $(B-V)_{HH}$ from Henden & Honeycutt (1995). The line corresponds to the best linear least squares fit.

very high statistical significance. We also include results for the "artificial" star, which is marked by number 0.

Table 4 shows the estimated extinction coefficients and airmasses for the adopted comparison stars and artificial star. The analysis of this table shows that the extinction coefficients k_{10} for the mid-run $\bar{t}$ are by $\sim$ 10 per cent larger than the "mean" ones k_{00}. This systematic difference is owed to the changes of the atmospheric transparency described by the parameter k_{11}. These coefficients are equal for all stars within error estimates, arguing for absence of statistical dependence on the color index. The only star which deviates from other comparison stars, is $C5$. However, this may be explained by it's intrinsic variability and an apparent rise during the observations, as one may see from Fig.3. We assume that the "artificial star" is not the real star, but a deviation of the star from the sample mean value. So zero for the data WITH atmospheric extinction, and -0.467 and -0.520 in Table 4 as the "out-of-atmosphere" value.

It is important to note that even for the model with variable atmospheric extinction, the r.m.s. deviation from the fit $\sigma_{10} = 0\overset{m}{.}0062$ exceeds the internal accuracy of this "artificial" star $\sigma = 0\overset{m}{.}00072$ by a factor of 8.6. This means that the atmospheric flickering is very significant, and usage of "artificial" comparison star is preferrable over time smoothing of any individual comparison star.

The value of m_{10} is the most accurate approximation of the instrumental magnitude outside the atmosphere. It may be used for further determination of the trial extinction coefficient

$$k(t) = (m(t) - m_{00})/M(z). \tag{7}$$

Obviously, for each star it will show an apparent trend with $\mathrm{d}k(t)/\mathrm{d}t = -k_{11}$ onto which the flickering is superimposed.

6. DEPENDENCE OF ACCURACY ON BRIGHTNESS

Using the artificial comparison star, we have computed the "restored" brightness of 17 comparison stars in the field for both "good" (JD 2453035) and "cloudy" (JD 2453036) nights using the fixed weights described above. We have preferred to determine the weights using the "good night" instead of using all available data with larger scattering. For these nights, the mean values and r.m.s. deviations have been computed using Eq.(1). For the stars, which have not been used for computation of the artificial star "0", we have estimated the unbiased variance by subtracting the mean variance of

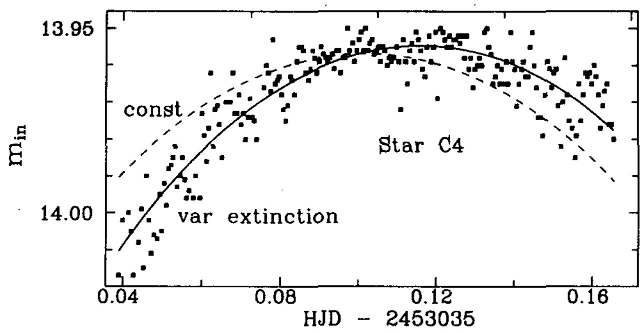

Figure 5. Instrumental magnitude of the comparison star $C4$. The dashed line shows the best fit assuming the model of constant extinction coefficient. Solid line represents the fit with linear variability of the extinction coefficient.

Table 4. The estimated extinction coefficients and airmasses for the adopted comparison stars and artificial star.

Star	1	2	3	4	5	6	0
m_{00}	15.178	13.706	13.999	13.502	15.549	14.521	-0.467
$\pm$	0.033	0.033	0.039	0.036	0.044	0.039	0.036
k_{00}	0.4190	0.4159	0.4141	0.4081	0.3572	0.4031	0.4083
$\pm$	0.0287	0.0293	0.0345	0.0316	0.0381	0.0338	0.0317
σ_{00}	0.0102	0.0105	0.0123	0.0113	0.0136	0.0121	0.0113
m_{10}	15.135	13.661	13.940	13.449	15.484	14.464	-0.520
$\pm$	0.022	0.021	0.021	0.020	0.023	0.021	0.020
k_{10}	0.4555	0.4556	0.4655	0.4540	0.4139	0.4533	0.4545
$\pm$	0.0196	0.0185	0.0181	0.0175	0.0203	0.0180	0.0176
k_{11}	0.2679	0.2835	0.3418	0.3128	0.3570	0.3331	0.3147
$\pm$	0.0125	0.0118	0.0115	0.0111	0.0129	0.0114	0.0112
σ_{10}	0.0070	0.0066	0.0064	0.0062	0.0072	0.0064	0.0062

the artificial star: $\tilde{\sigma}_\alpha^2 = \sigma_\alpha^2 - \langle\sigma_{0k}^2\rangle$.

These characteristics are listed in the Table 5 and these results are shown in Fig. 6.

Assuming that the observational noise is caused by Poisson noise, one may write that intensity I is $I = 10^{-0.4 \cdot m} = \beta n$, where m is magnitude, n is photon count rate for the star (without background) and β is coefficient of proportionality. From the Poisson statistics, $\sigma_n^2 = n + n_b$, where the index b corresponds to the background count rate. Thus

$$\left(\frac{\sigma_n}{n}\right)^2 = \left(\frac{\sigma_I}{I}\right)^2 = \frac{n + n_b}{n^2} = \frac{\beta}{I} + \frac{\gamma}{I^2} \tag{8}$$

where $\gamma = \beta^2 n_b = \beta I_b$. For $\sigma_i = \sigma_I/I \ll 1$, one may approximately write $\sigma_I/I = \epsilon\sigma_m$, $\epsilon = 0.4 \ln 10 = 0.9210$.

As the accuracy estimate of σ is approximately proportional to it's value. One has to take this into account, thus finally the equation to be solved using least squares is

$$\lg \sigma_m = \eta - \frac{1}{2} \lg I + \frac{1}{2} \lg \left(1 + \frac{I_b}{I}\right), \tag{9}$$

where $I_b = \gamma/\beta$ is intensity of background in the same arbitrary units, as the intensity of star. The best fit values for this non-linear least-squares fit are $\eta = -5.661 \pm 0.058$ and $m_b = -2.5 \lg(I_b) =$

Table 5. Characteristics of comparison stars: mean instrumental magnitude, unbiased and biased r.m.s. error estimates.

Star	53035			53036		
α	$\bar{m}_\alpha$	$\tilde{\sigma}_\alpha$	σ_α	$\bar{m}_\alpha$	$\tilde{\sigma}_\alpha$	Remark
0	14.3141	0.00072		14.6055	0.00114	Artificial
1	15.6563	0.0034	0.0033	15.9486	0.0043	
2	14.1817	0.0020	0.0019	14.4741	0.0025	
3	14.4722	0.0019	0.0018	14.7670	0.0032	
4	13.9683	0.0010	0.0006	14.2590	0.0025	comparison
5	15.9575	0.0042	0.0041	16.2482	0.0057	
6	14.9818	0.0021	0.0020	15.2703	0.0035	
7	20.8349	0.1245	0.1245	21.0092	0.2299	
8	20.2482	0.0884	0.0884	20.4530	0.2891	
9	20.3019	0.1269	0.1269	20.4508	0.2209	
10	18.4743	0.0710	0.0710	18.6535	0.1371	
11	20.5105	0.1234	0.1235	20.6412	0.2005	
12	20.6549	0.1645	0.1645	20.7925	0.1873	
13	18.3014	0.0716	0.0716	18.4707	0.1041	
14	20.8253	0.1159	0.1159	21.0730	0.3866	
15	19.5334	0.0988	0.0988	19.7057	0.1720	
16	19.0307	0.0761	0.0761	19.2286	0.2502	
17	18.9184	0.1081	0.1082	19.0745	0.1704	
18	16.5554	0.1754	0.1754	16.7590	0.1620	Variable

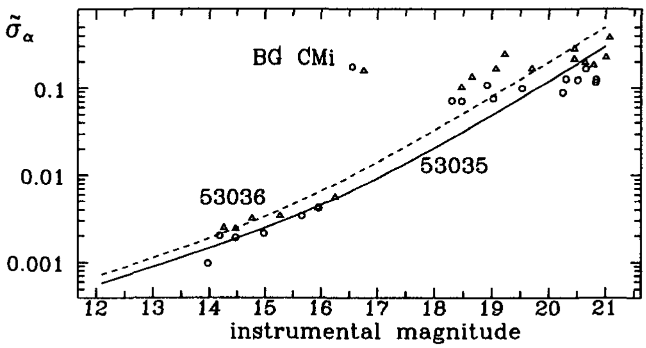

Figure 6. Dependence of unbiased error estimate for each star on the nightly mean instrumental magnitude. The data for "good" (53035) and "cloudy" (53036) nights are shown with open circles and triangles, respectively. The best fits assuming the Poisson statistics are shown with lines.

$16^{\rm m}.22^{+0.86}_{-1.03}$, so one may estimate $\lg\beta = 2(\eta + \lg\epsilon) = -11.39 \pm 0.12$ and the instrumental magnitude $m_1 = 2.5\lg\beta = -28^{\rm m}.48 \pm 0^{\rm m}.29$, which (from statistical noise) corresponds to one count.

For "cloudy" night, the statistical errors have significantly increased, e.g. for the "artificial" star "0" from $0^{\rm m}.00072$ to $0^{\rm m}.00114$, i.e. by a factor of 1.6. The corresponding best fit values are $\eta = -5.566$ and $m_b = 15.698$.

The value of η, which corresponds to the proportionality coefficient, is the same (within error estimates) with that obtained for the "good" night. This is a good coincidence, as this coefficient should not be dependent on atmospheric transparency. However, the effective brightness of the background for the cloudy night has increased by $0^{\rm m}.52$, the stars become fainter by $0^{\rm m}.29$, so the results are self-consistent. By using the fits, one may estimate statistic accuracy for the star of given amplitude. For the variable BG CMi, we got $0^{m}_{.}0067$ and $0^{m}_{.}0117$, respectively. These values are smaller than the r.m.s. amplitude of the variable by a factor of 26 and 14 times, respectively.

7. SUMMARY

We have used the observational data of BG CMi to check accuracy of the CCD observations obtained at the Korean 1.8 m telescope and to improve it using multiple variable stars. The light curve analysis of this intermediate polar will be presented elsewhere (Kim et al. 2004) Our results can be summarised as following:

- The accuracy estimates of the mean weighted "artificial" star reached $0^{m}_{.}00072$ for "good" and $0^{m}_{.}00114$ for "cloudy" night. From Fig.6, one may estimate the statistical error of the magnitude of the star of 13-th magnitude as $0^{m}_{.}002$
- Even for "good" night, the atmospheric transparency varied, so the method of "multiple comparison stars" leads to better accuracy than the time smoothing of the comparison star.
- The method of multiple comparison stars is an effective tool to decrease statistical errors by few dozens per cent and may be recommended for usage instead of one comparison star.

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