Construction of Three-Strut Tension Systems

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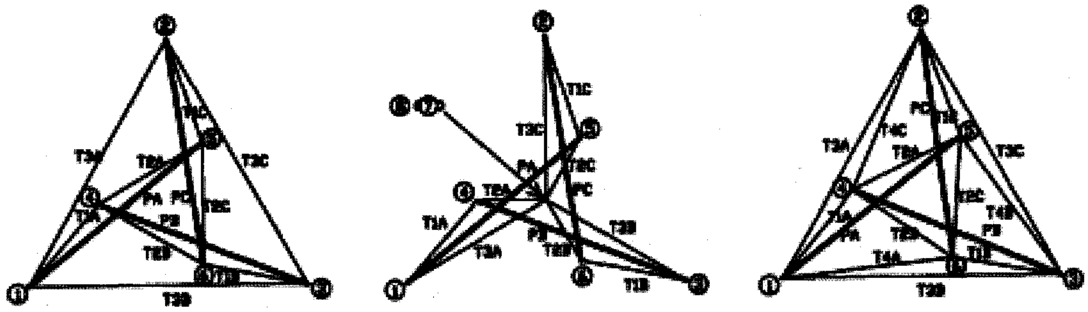
INTRODUCTION

Tensegrity structures have been appealing to many architects and researchers with their peculiar appearance. However even a very typical model have never been applied for structural elements of buildings. Space grids assembled with tensegrity elements have been proposed by some researchers but never been constructed for practical use [5]. Cable domes and hyper tensegrity domes have already been constructed in large scales. However they are rather cable nets with struts in rigid compression rings and are somewhat different from the general image of the tensegrities. One of the authors have proposed and constructed "truss structure stabilized by cable tension". This system was found as a result of the exploration between space trusses and tensegrities [6].

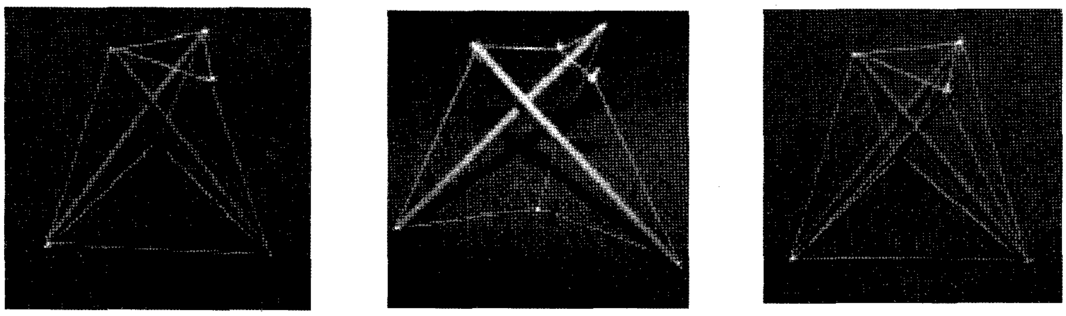
In the paper the results of a numerical study, which was carried out for the pre-design of a structure, on structural behavior of three-strut tension systems are briefly discussed. Then the full-scale test and the construction of a pair of three-strut tension systems as supporting structures for a membrane roof, especially their tension introduction process, is reported.

THREE-STRUT TENSION SYSTEM

We firstly compare the structural behavior of three variations of the three-strut tension system, shown in fig.1, by means of numerical analysis. Since we had planned to use three-strut tension system to support a membrane roof the numerical models are in a special trapezoidal proportion. Model 1, fig.1(a), is a trapezoidal version of the most well known three-strut tension system, sometimes called as a "simplex", a triangular prism module of three-strut tension system. This model has six joints, three struts and nine tension members. Model 2, fig.1(b), is a variation of model 1. Two triangles, at the top and the bottom, are replaced by three-pronged tension members. New two joints appear at the center of upper and bottom triangles, where three tension members meet together. The numbers of struts and tension members remain same. Since only two tension members gather at each end of struts and they are arranged in one plane the joint details may become simpler than model 1. Model 3, fig.1(c), is also a variation of model 1 and has other three more tension members.



(a) Plan of Model 1 (b) Plan of Model 2 (c) Plan of Model 3



(d) Model 1 (e) Model 2 (f) Model 3

Figure 1. Trapezoidal three-strut tension systems

Table 1. Member properties for 3 models

Model 1			
Member	Area (cm^2)	Initial Force (kN)	Yielding Stress(MPa)
T1	4.9	78.40	274
T2	4.9	59.31	274
T3	4.9	25.42	274
P	40.4	-93.58	323
Model 2			
Member	Area	Initial Force	Yield Stress
T1	4.9	78.40	274
T2	4.9	102.72	274
T3	4.9	44.02	274
P	40.4	-93.58	323
Model 3			
Member	Area	Initial Force	Yield Stress
T1	4.9	77.91	274
T2	4.9	62.07	274
T3	4.9	91.22	274
T4	4.9	26.61	274
P	40.4	-116.47	323
Young's modulus : 210 Gpa Poisson's Ratio : 0.3			

Table 2. Joint coordinates for the 3 models

Model 1			
Joint No.	x (cm)	y(cm)	z(cm)
1	-606.2	-350.0	0.00
2	0.00	700.0	0.00
3	606.2	-350.0	0.00
4	-300.0	0.00	800.0
5	150.0	259.8	800.0
6	150.0	-259.8	800.0
Model 2			
Joint No.	x (cm)	y(cm)	z(cm)
1	-606.2	-350.0	0.00
2	0.00	700.0	0.00
3	606.2	-350.0	0.00
4	-300.0	0.00	800.0
5	150.0	259.8	800.0
6	150.0	-259.8	800.0
7	0.00	0.00	0.00
8	0.00	0.00	0.00
Model 3			
Joint No.	x (cm)	y(cm)	z(cm)
1	-606.3	-350.0	0.00
2	0.00	700.1	0.00
3	606.3	-350.0	0.00
4	-299.3	0.00	790.6
5	168.1	212.7	790.6
6	131.2	-248.6	790.6

These three models have the common dimension. The height is 800cm, the side length of the triangle at the bottom is 1212cm and the side length of the triangle at the top is 520cm.

R.Motro carried out a research including loading test for a triangular prism model, similar to the model l, and reported its great flexibility and geometrical nonlinearity [2]. Model 1 is statically and kinematically indeterminate structure of order one. Its inextensional displacement mode is twisting motion between the top and the bottom triangles and is stiffened only by introduction of initial tension.

Model 2 has same number of members as model 1 while it has another two joints. Therefore it is supposed to have more kinematic indeterminacy than model 1. Further since one strut and two tension members are arranged in one plane at the end of each strut, it is easily imagined that stiffness for out of plane motions is supposed to be low. The selfstress mode for model 2 is easily derived from the equilibrium conditions at each joint.

Model 3 is statically indeterminate and kinematically determinate structure since it has three more members than model 1. For the practical application a certain statically indeterminacy is preferable just in case of member failure. The number of tension members meeting at the end of each strut is now four, which makes joint design more complicated than other models.

NUMERICAL ANALYSIS

In this section the results of geometrically nonlinear analyses for models shown in fig.1 with parameters indicated in table 1 and 2 are shown. Joints 4,5 and 6 were loaded vertically and equally and joints 1, 2 and 3 are constrained in z-direction and freely supported in xy-plain.
The initial axial forces in T1 members are set to be equal for all models.

Load-displacement relationships in z-direction at joint 4 under vertical load are plotted in fig.2. Each calculation was terminated when one of the members reached yielding stress level. Slackening of tension members was also considered.

Initial behavior of models 1 and 2 are similar. Model 1 gradually shows nonlinear behavior and T1 members reach yielding level about 100kN. T3 members of model 2 reach yielding level around 30 kN, which is much earlier than model 1.

Model 3 exhibits very high rigidity until about 50kN where T4 members are slackened. This high rigidity is due to the action of T4 members as compressive members until their slackening. After the slackening of these members the load-displacement relationship of model 3 is similar to those of model 1 and 2 and finally T3 members yield at around 150kN.
The high rigidity of the model 3 is remarkable and different from the other two models.

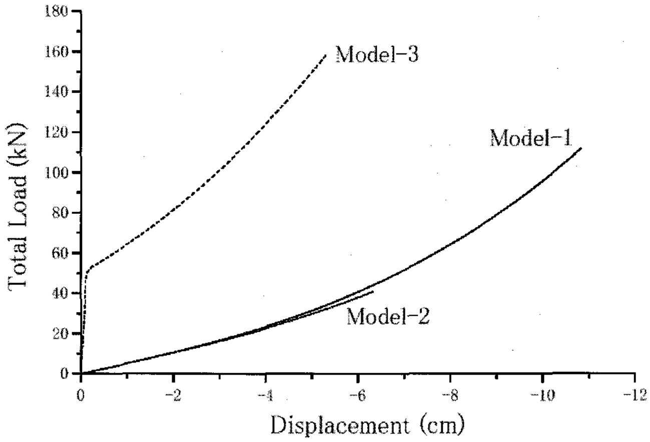


Figure 2. Load-displacement relationship

According to the results of eigenvalue analyses [7] model 1 and 2 have "zero" eigenvalue in the elastic stiffness. Their stiffness is provided only by geometric stiffness under the existence of initial stress while the stiffness of model 3 is sufficiently provided by the elastic stiffness until the member T4s' slackening.

Table 5. Initial Joint Coordinate of the Full-Scale Model

Joint No.	Designed joint coordinate (cm)			Measured joint coordinate (cm)		
	x-direction	y-direction	z-direction	x-direction	y-direction	z-direction
1	-1528.3	0.0	300.0	-1528.3	0.0	300.0
2	-886.8	-374.8	400.0	-887.3	-374.2	400.2
3	-886.8	374.8	400.0	-887.9	374.6	400.5
4	-1180.5	-170.9	879.6	-1182.8	-171.2	879.1
5	-1326.7	85.4	856.8	-1325.4	87.6	857.3
6	-1034.3	85.4	902.4	-1033.2	83.0	902.2

Table 6. Member Properties and Designed Axial Force

Member No.	Member Properties				Axial Force (kN)		
	Diameter (mm)	Sectional Area (cm^2)	Length (cm)	Yielding stress (Mpa)	Step 1	Step 2	Step 3
T1A	32	8.04	598.8	440	29.4	40.0	156.6
T1B	32	8.04	598.8	440	29.4	39.3	155.8
T1C	32	8.04	598.8	440	29.4	40.8	157.3
T2A	28	6.16	296.2	440	21.3	30.6	121.3
T2B	28	6.16	296.2	440	21.3	31.1	121.7
T2C	28	6.16	296.2	440	21.3	31.7	122.4
T3A	25	8.04	750	440	8.2	13.7	49.5
T3B	25	8.04	750	440	8.2	14.2	49.9
T3C	25	8.04	750	440	8.2	13.2	48.9
T4A	25	4.91	681.3	440			18.5
T4B	25	4.91	681.3	440			18.5
T4C	25	4.91	681.3	440			18.5
PA	216.3x8.2	53.61	783.4	323	-38.5	-56.8	-230.5
PB	216.3x8.2	53.61	783.4	323	-38.5	-58.8	-232.5
PC	216.3x8.2	53.61	783.4	323	-38.5	-57.8	-231.5

TENSION INTRODUCTION TEST

According to the results of numerical analyses, we found that model 3 had high rigidity and high strength, which was enough for the practical use. Before going to the real design and construction we had an opportunity to carry out full scale assembling test.

Tension Introduction

For the realization of designed structural performance of tensegrities, introduction of initial tension is one of the most important issues. Therefore the tension introducing process should be carefully planned.

Prior to the construction, we had an opportunity to carry out a full-scale and tried to ensure our tension introduction scheme. Since model 1 in fig.1 has just one state of selfstress the initial prestressed state can be easily realized by shortening any of the tension members. On the other hand, model 3 has three independent selfstress states [7]. In order to realize the designed initial stress states the selfstress states should be appropriately adjusted. Further if we desire to introduce the tension through manual operation of turnbuckles inserted in the tension members, without any electric or hydraulic devices, we should consider the limit of the torque that can be induced by human power at the construction site.

In our scheme we firstly prepare model 1 with certain initial tension and then set three more members, T4's, and shorten them with low torque by long lengths. The objective tension was 155kN in T1 members and 18kN in T4 members.

The tension introduction scheme was as follows:

Step 1 : Support the joints 4, 5 and 6, vertically. Assemble model 1 and introduce 30kN into T1 members by shortening their lengths simultaneously.

Step 2 : Remove the supports.

Step 3 : Set T4 members and introduce 155kN into T1 members by shortening T4 members simultaneously until T4 members' tension become 18kN.

Wire strain gauges were mounted on the all of the members and their strain was monitored at real time so that the readings can be immediately fed back to the tension introduction process.

Test Results of the tension introduction test

Before start the test one of the T1 members was shortened during model 1 state and we confirmed that satisfactory tension distribution could be realized by just one member operation. In order to avoid uneven member length change we operated the three T1 members simultaneously. Even after the setting of T4 members tension introduction process was not difficult than we expected because of the effective feedback scheme from the real time reading of the member strain.

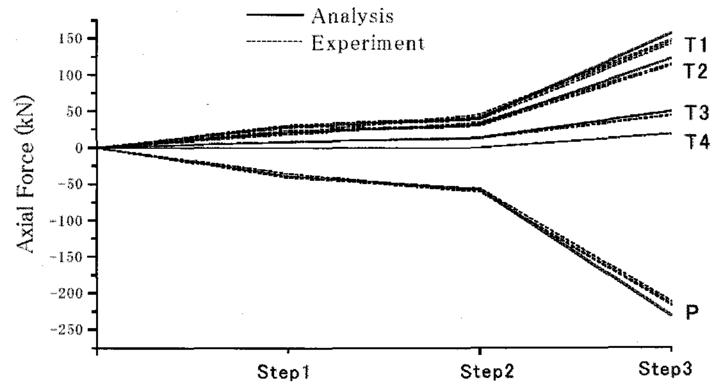
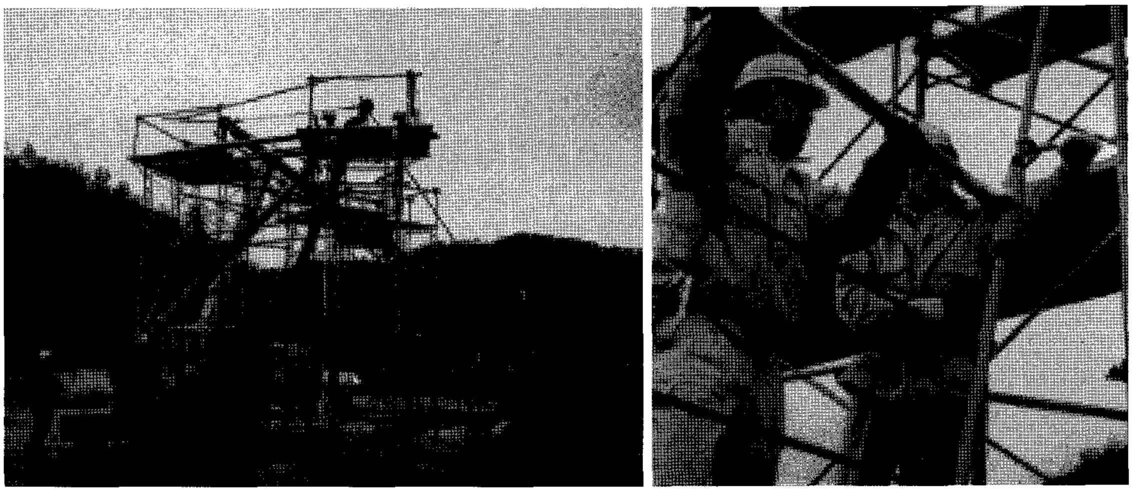


Figure 3. The results of the test: Tension distribution



(a) General view of the test (b) Tension introduction

Figure 4. Full-scale tension introduction test

The results of tension introduction test are plotted in fig.3. The test results agreed very well with the designed quantities. In order to increase the tension in T1 members from 30kN to 155kN, T4 members were shortened by about 8 cm.

CONSTRUCTION

Gaining confidence from the results of a number of numerical analyses and a full-scale tension introducing test we designed a pair of supporting structures for a membrane roof employing three-strut tension systems of model 3 (figs.5 and 6). One post member was added for each tension-strut system, hung from three joints of top triangle by tension rods, to push up the membrane roof (fig.5). Joint at the bottom end of the post is flexibly connected to the tension rods so that the large displacement of the membrane roof can be absorbed by the inclining action of the post. The bigger tension-strut system was called tensegrity A and the other was called tensegrity B (fig.6). The height and the side length of the bottom triangle of the tensegrity A are 9m and 12m, respectively, and 6m and 9m for the tensegrity B.

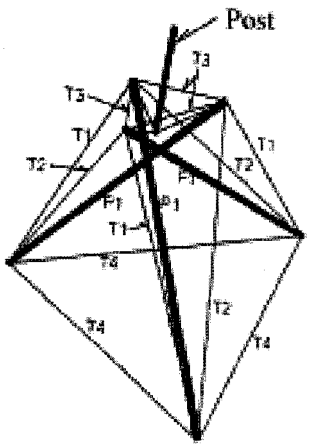


Figure 5. The supporting structure

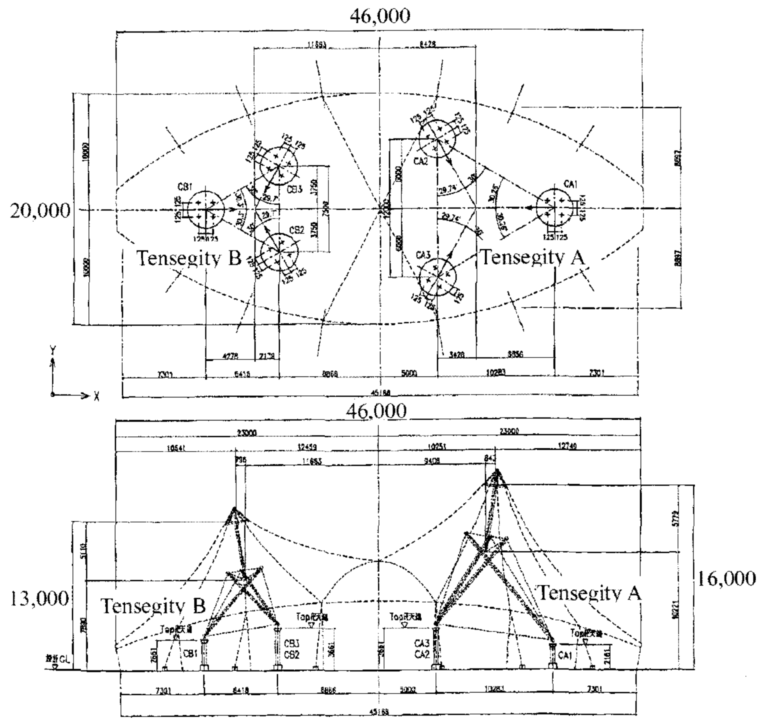
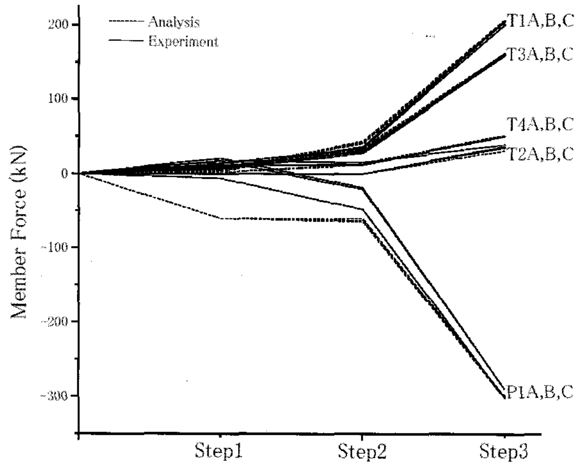


Figure 6. Plan and section of the structure

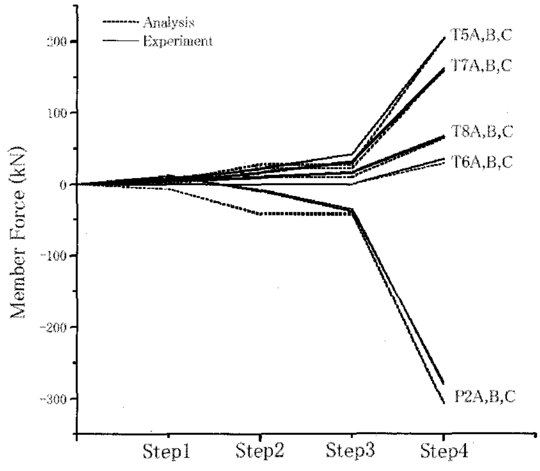
Tension Introduction

After the foundation was prepared two strut tension systems were assembled one by one. Joints of the top triangle were set on the jacks on the support columns and members were assembled in the form of model 1 in fig.1. Three T1, T5 for tensegrity B, members were equally shortened until the 42kN self-equilibrated axial force was introduced to them. Then the jacks were removed since the structure could stand by itself. Three more rods, T2's for the tensegrity A and T6's for the tensegrity B, were then set in and the system had the form of model 3 in fig.1. These new tension rods were carefully and equally shortened until their axial force reach 30kN and the tension introduction process was completed. It should be noted that the structure experiences large deformation during the tension introduction scheme. The joint details should be designed that they can accommodate to this large deformation.

All the tension introduction works were manually done just by human power (fig.4). A pair of wire strain gauges was mounted on the every member, the tension rod and the compression pipe, and its strain was always monitored at real time at the site. The readings were immediately fed back to the tension introduction work. Members for the tensegrity A was newly prepared while the members used for the preceding tension introduction test were diverted to the tensegrity B.

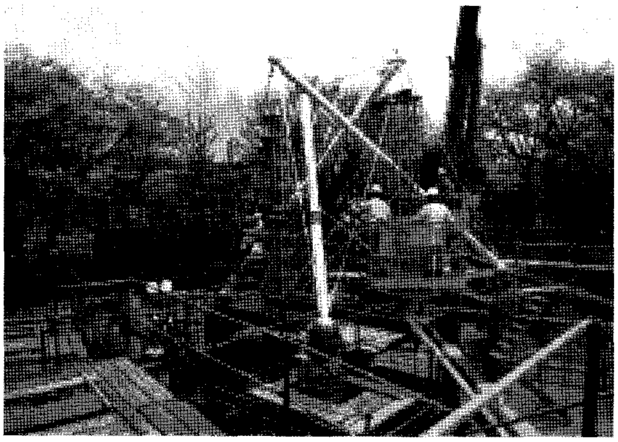


(a) Tensegiryt A

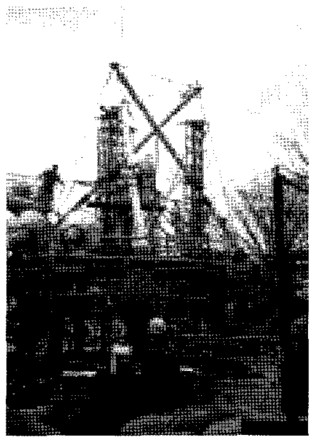


(b) Tensegrity B

Figure 7. The measured axial forces during the tension introducion work



(a) Tension introduction work



(b) Real time measuring at the site

(c) Two three-strut tension systems

Figure 8. Supporting structures under the construction

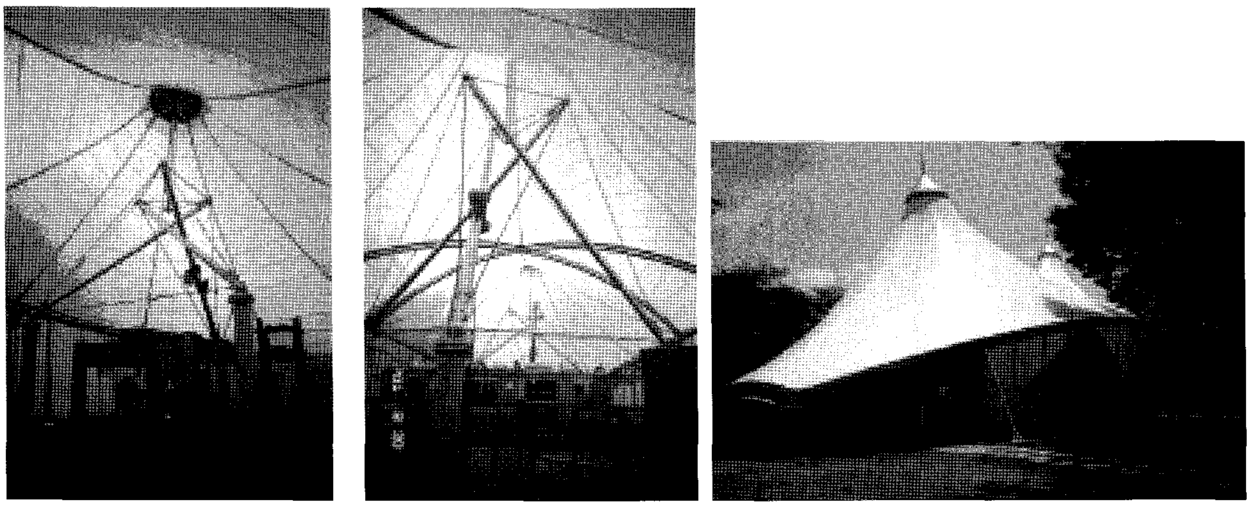
Results of the tension introduction at the site

The measured axial forces during tension introduction work are shown in fig.7. Just like the results of the preceding test in fig.3 the results agreed with the designed quantities well enough to expect the designed structural performance to the structures.

For tensegrity A the shortened length of the tension rods T2 during the operation from the model 1 stage to the model 3 stage was about 17cm and the displacements of the joints of the top triangle were about 16cm in twisting motion. For tensegrity B the the tension rods T2 were shortened during the same operation by about 14cm and the displacements of the joints of the top triangle were about 15cm also in twisting motion. These long distance changes of the rods enable us the manual work for the tension introduction although a big amount of pretension must be introduced in the structure.

CONCLUSION

A number of numerical analyses and a full-scale tension introduction test were carried out before the design and construction of the tension strut systems. A part of the numerical analyses was shown and the validity of the model 3 in the application to practical use was discussed in this paper. The remarkable improvement in structural performance of model 3, comparing to the model 1, is attributed to the effective reaction in compression of pretensioned members. The proposed tension introduction scheme was successful and effective so that just manual works were enough to complete the tension introduction process.



(a) Tensegrity B supporting the membrane roof (b) Interior view (c) Exterior view

Figure 9. General view of the completed structure

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