# 불균등 단부 모멘트를 받는 I형강의 비탄성 좌굴거동에 관한 연구

## Inelastic Buckling Behavior of I-Beam with Unequal End Moment

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요 약 : line-type 유한요소법을 이용하여 불균등 단부 모멘트를 받는 보의 비탄성 좌굴 거동에 대하여 연구하였다. 잔류응력은 단순형과 다항식형 모델을 채택하였으며 잔류응력으로 인해 발생하는 단면의 불균등 항복을 고려하였다. 본 연구에서 얻어진 비탄성 횡-비틀림 좌굴에 대한 결과는 강구조편람의 허용응력법에 의한 설계 경우와 비교하였다. 결과적으로, 강구조편람에 의한 설계는 중지간 보에서 중간 브레이싱이 있는 경우나 없는 경우 모두 과설계가 됨을 알 수 있었다.

ABSTRACT : The aim of this study is to investigate the inelastic buckling behavior of the beams under moment gradient using a line-type finite element method. The method is incorporated the non-uniform yielding of the cross-section caused by the presence of residual stress and accepted model of residual stress so called 'simplified' and 'polynomial' pattern is adopted in this study. The inelastic lateral-torsional buckling results obtained in this study is compared with the buckling results obtained from the design method based on the allowable stress method given in Korean Steel Designers' Manual (KSDM 1995). This study have found that the design method in KSDM (1995) is conservative without and with intermediate bracing applied at the mid span of the beam, and there is some scope for improving the provisions of KSDM (1995)

핵 심 용 어 : 좌굴, 비탄성, 선형 유한요소, 모멘트 변화도, 잔류응력

KEYWORDS : Buckling, Inelasticity, Line-type finite element, Moment gradient, Residual stress.

## 1. Introduction

The elastic lateral-torsional buckling of simply supported doubly symmetric I-beam under unequal end moment can be determined as

$$M_c = \alpha_m M_{yz} \qquad (1)$$

where $\alpha_m$ is the moment modification factor and $M_{yz}$ is the closed form solution of the elastic critical buckling load of doubly symmetric I-beam under uniform bending and is given as

$$M_{yz} = \sqrt{EI_y GJ \frac{\pi^2}{L^2} + EI_y EI_w \frac{\pi^4}{L^4}} \qquad (2)$$

The moment modification factor $\alpha_m$ is give as

$$\alpha_m = 1.75 + 1.05\beta + 0.3\beta^2 \qquad (3)$$

where $\beta$ $(-1 \le \beta \le 1)$ is end moment ratio.

It can be noted that the maximum moment modification $(\alpha_m)$ factor used in Korean and American steel structure standard is 2.3 but the maximum $\alpha_m$ in Australian and British steel structure

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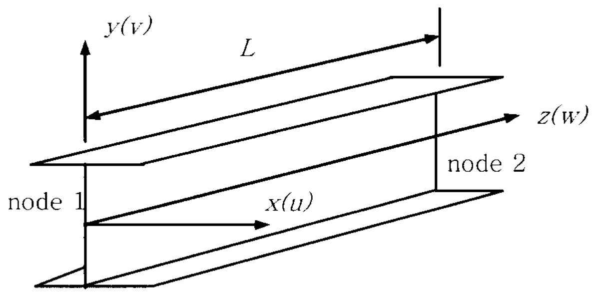
standard is 2.56. The design method given in KSDM (1995) has been developed from the closed form solution of the elastic lateral-torsional buckling of doubly symmetric I-beam, and the moment modification $(\alpha_m)$ can be used to determine the allowable bending stress of I-beam under moment gradient. Thus the aim of this study is to examine the accuracy of the design method given in KSDM (1995) with the inelastic lateral-torsional buckling results obtained in this study. The inelastic lateral-torsional buckling behavior of the I-beam under uniform bending is plentiful. Trahair and Kitipornchai (1972) studied the inelastic lateral-torsional buckling behavior of the beam under uniform bending. The mono symmetric effect caused by the combination of residual stress and applied load has been allowed in their study and the reduced inelastic rigidities $(EI_y)_i$, $(GJ)_i$, $(EI_w)_i$ were estimated by using tangent modulus theory. The non-uniformity of the cross-section along the beam does not occur when the I-beam is under uniform bending, but this is not case for a beam under unequal end moment. Therefore, the inelastic lateral-torsional buckling analysis of simply supported beam unequal end moment is more complicated then those of the beam under uniform bending due to the non-uniformity of the cross-section of the beam. Nethercot and Trahair (1976) proposed the design approximation of beam under unequal end moment using the finite element method of analysis developed by Nethercot (1973). Dux and Kitipornchai (1983) conducted the experimental and theoretical study of the simply supported beam under moment gradient with similar cross-section considered by Nethercot and Trahair (1976), and they found that couple of the experimental results were not agreed with the theoretical results obtained by Nethercot and Trahair (1976).

This study investigates the inelastic lateral-torsional buckling behavior of the simply supported I-beam under unequal end moment with I-sections manufactured in Korea using a finite element method. The line-type finite element employed in this study allows for non-uniformity of the cross-section along the beam caused by non-uniform yielding of the cross-section induced by the combination of residual stresses and moment gradient. The pattern of residual stress used in this study are the polynomial model that is generally accepted by Australian and British hot-rolled I-sections, and the simplified model that is generally accepted by American hot-rolled I-section. This study considers the inelastic lateral-torsional buckling of the beam under unequal ends moment with four different I-sections manufactured in Korea and these results are compared it with design method in KSDM (1995). This study has found that there are disparity between the inelastic lateral-torsional buckling results and KSDM (1995) that is based on the allowable stress method.

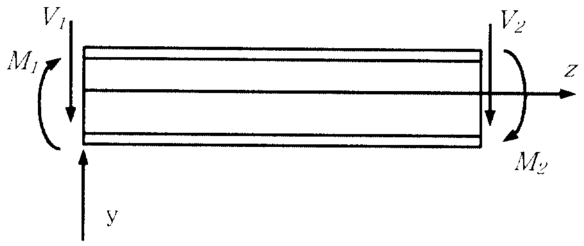
## 2.Theory

### 2.1 General

A line-type element developed by Lee (2001) is used in this study to analyze the inelastic lateral-torsional buckling behavior of the beams under moment gradient. Complete details of a line-type finite element method presented by Lee (2001), and Lee and Oh (2004) and a brief description of the method is given in this paper.




1(a) Beam element




1(b) End moments

Figure 1. Beam element and loading

Figure 1(a) shows a line-type element used in this study with reference axis located at mid height of the web. Figure 1(b) shows the simply supported beam is subjected to an unequal end moment. The strategy adopted in this study is that the in-plane analysis of the beam is firstly undertaken to determine the distribution of the moment along the beam by initially assuming applied moment and then corresponding applied curvatures are estimated by deploying moment-curvature relationship (Lee 2001, Lee and Oh 2004), and the out-of-plane buckling analysis can then be performed with predetermined curvature and new neutral axis along the beam. The applied moment is adjusted until buckling occurs. It should be noted that the stiffness and the stability matrices in the out-of-plane buckling analysis is depended on the combination of the applied load and the residual stress. This study considers simply supported beam under moment gradient and thus the applied bending moment and shear force along the beam can be determined from simple statics. The tangent modulus $(E_t)$ theory that is used in the buckling analysis is equal to the elastic modulus for the elastic regions and the strain hardening modulus for yielded and strain hardened regions of the cross-section as shows in Fig. 2.

The patterns of residual stress that may exist in hot-rolled I-section used in this study are the simplified and the polynomial model as was used by number of researchers as shown in Fig. 3. The simplified pattern of residual stress is shown in Fig. 3(a) and Fig. 3(b) and (c) is for polynomial pattern of residual stress for the slender I-section and compact I-section respectively. The distribution of residual stress in the flange and the web with the maximum residual stress at the flange tip and the flange/web junction is given in Lee (2001) and Lee and Oh (2004).

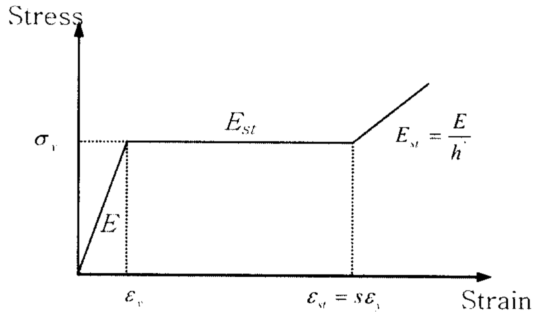
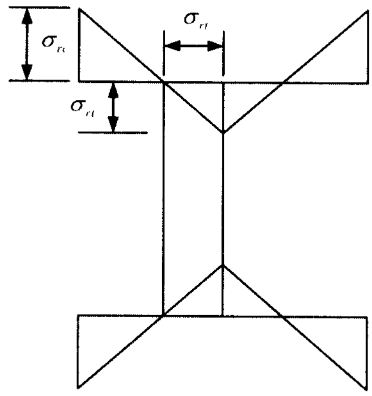
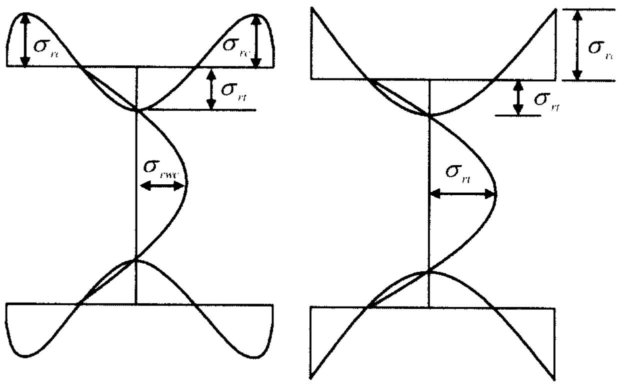



Figure 2. Constitutive relationship



3(a) Simplified residual stresses



(b) Slender I-section (c) Compact I-section

3(b) Polynomial residual stresses

Figure 3. Model of residual stresses

## 2.2 In-plane bending analysis

The distribution of the moment along the simply supported beam under moment gradient can be determine from simple statics

$$M_x = M\{1-(1+\beta)z/L\} \quad (4)$$

$\beta$ is end moment ratio range from -1 to 1. The negative value of $\beta$ indicates single-curvature whereas the positive value of $\beta$ indicates double- curvature. The applied curvature, and the elastic and inelastic regions of the cross-section along the beam can be determined with predetermined nonlinear moment-curvature relationship (Lee 2001, Lee and Oh 2004) for the cross-section at a given value of applied moment.

## 2.3 Out-of-plane buckling analysis

Figure 4 shows the buckling deformation of the cross-section. As the cross-section displaces, the lateral displacements of the top and the bottom flange is expressed as $u_T$ and $u_B$ respectively, while $\phi_T$ and $\phi_B$ represented the twist of the top and the bottom flange respectively. The buckling deformations $\{q\}=\langle u_T, u_B, \phi_T, \phi_B\rangle$ of the flanges for an element are assumed as cubic variations of z direction, while the deformation of the web are assumed as a cubic curve.

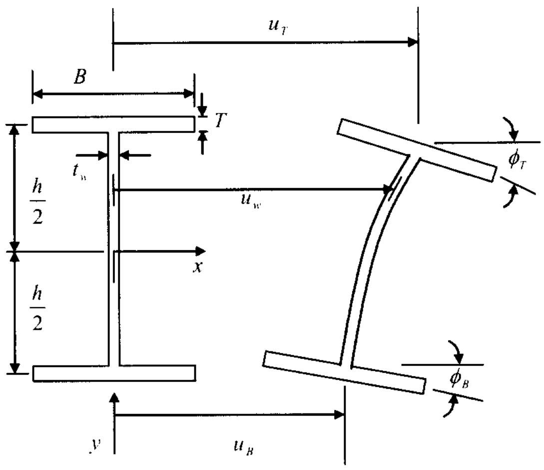



Figure 4. Buckling deformations in the plane of the cross-section

The beam theory (Timoshenko and Gere 1961) is used to derive the stiffness matrices of the flanges conjunction with tangent modulus $(E_t)$. The minor axis flexural and torsional rigidities are determined as was done by Trahair and Kitipornchai (1972). The plate theory is employed to determine the stiffness matrix of the web. Isotropic plate theory (Thimoskenko and Woinowsky-Kriegr 1959) and orthotrapic plate theory (Haaijer 1957, Dawe and Kulak 1984 and Bradford 1986) based on the flow theory of the inelasticity is used for elastic and inelastic regions of the web respectively. The simplified and the polynomial residual stresses are conceptually different. The distribution of the polynomial residual stress in the flange and the web is determined by satisfying the axial static and torque equilibrium condition of the residual stress. The simplified residual stress satisfies the static equilibrium condition but not with the axial torque equilibrium condition. Therefore, the torsional rigidity of the cross-section is altered $\left((GJ)_t - \int_A \sigma_r (x^2+y^2) dA\right)$ to eliminate the axial torque produced by residual stress.

The stiffness matrix $[k]$ of the beam element is sum of the flange $[k_F]$ and the web stiffness matrix $[k_w]$, which is derived from the beam and the plate theory respectively, while the stability matrix $[g]$ of the beam element is sum of the flange $[g_F]$ and the web stability matrix $[g_w]$. It must be noted that the stiffness and the stability matrices of the flange and the web are not constant due to the monosymmmtric effects caused by the applied load and the residual stress.

## 2.4 Buckling analysis

The stiffness and the stability matrix of the beam element can be assembled into the global stiffness and stability matrix. The inelastic buckling load can be determine as

$$([K]-[G])\{\Delta\}=0 \quad (5)$$

where $[K]$ and $[G]$ are the global stiffness and stability matrix respectively and $\{\Delta\}$ is eigenvector. The determination of inelastic buckling load is more complicated than the elastic buckling due to non-uniformity of the cross-section. The critical buckling solution can be obtained when determininant of equation 5 is vanishes. The iterative method has been employed because ill-behave nature of eigenvalue. the most appropate method iteration is bisection method.

## 3. Accuracy of method

The finite element method used in this paper to predict the inelastic lateral-torsional buckling moment of the beams under moment gradient is compared with the experimental study. Dux and Kitipornchai (1983) conducted the experimental study of the beams under unequal end moment. The measured mean material properties were $E$(elastic modulus)=209.9GPa, $\sigma_y$ (yield stress)=285MPa (flange) and $\sigma_y$ (yield stress)=321MPa (web). The cross-sectional dimensions were $h$(distance between flange centroid)=245.1 mm, flange width ($b_f$)=146.4 mm, thickness of web ($t_w$)= 6.4 mm and thickness of flange ($t_f$)= 10.9 mm. The pattern of residual stress used in this comparison study is the polynomial pattern with a parabolic distribution of residual stress in the flange and a quartic distribution in the web. The number of element used in this comparison is 16 because the yielding of cross-section is confined towards the beam ends and the bending moment in the mid span region is small, and 6 elements with equal length of beam near the supports and the remaining 4 elements of equal length the mid span region is used in this study. Fig. 5 shows the comparison between this study and the experimental results of Dux and Kitipornchai (1983). The end moment ratio $(\beta)$ used in experimental study was -1, 0.7 and 0 which produces a single-curvature bending. It can be seen in the figure that the inelastic lateral-torsional buckling results obtained in this study is agreed very well with experimental results for end moment ratio $(\beta)$ of -1, 0.7 and 0.

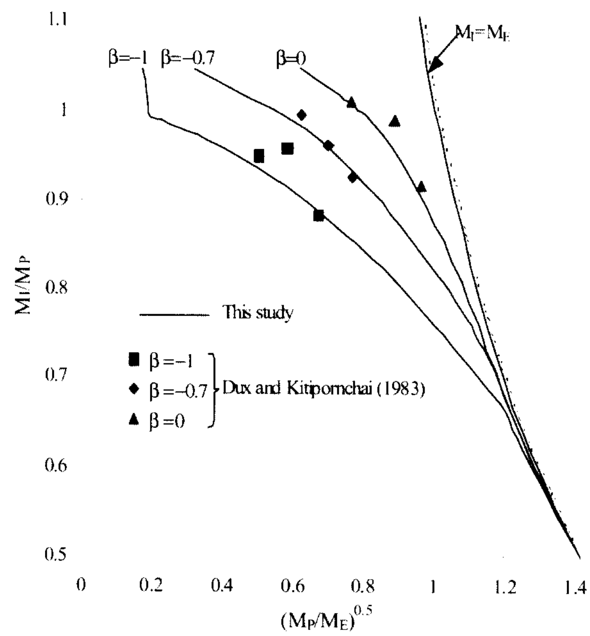



Figure 5. Comparison with experimental study

## 4. Inelastic lateral-torsional buckling of beam

The inelastic lateral-torsional buckling of beam under a moment gradient is investigated in this section with four I-section members manufactured in Korea. The polynomial and the simplified pattern of residual stress are used in this study. The considered cross-sections are 200×150, 300×175, 400×400 and 800×300. The material properties are $E$(Elastic modulus)=2.1×10$^6$ kg/cm$^2$ (205.926 ×10$^3$MPa), $\sigma_y$ (yield stress)=2400 kg/cm$^2$ (235 MPa), $\nu$(Poisson's ratio)= 0.3, $\varepsilon_{st}$ = $10\varepsilon_y$, and $E/E_{st}$ = 40. Figures from 6 to 9 show the inelastic lateral-torsional buckling results for $\beta=0$ that is subjected a single curvature bending, while 10 to 13 are for I-beams with $\beta=1$ that is subjected to a double (reverse)-curvature bending. The inelastic lateral-torsional buckling moment $M_I$ is non-dimensionlized with respect to the plastic moment $M_P$, while dimensionless slenderness $\sqrt{M_P/M_E}$ is used, where $M_E$ is the elastic lateral buckling load.

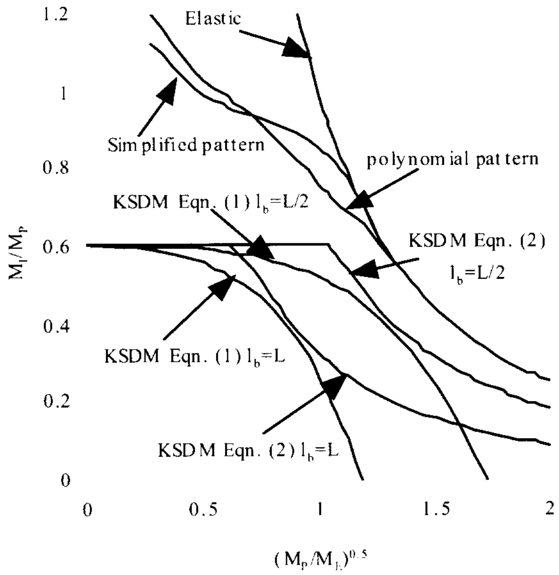



Figure 6. Inelastic buckling of simply supported beam 200×150 with $\beta$=0.

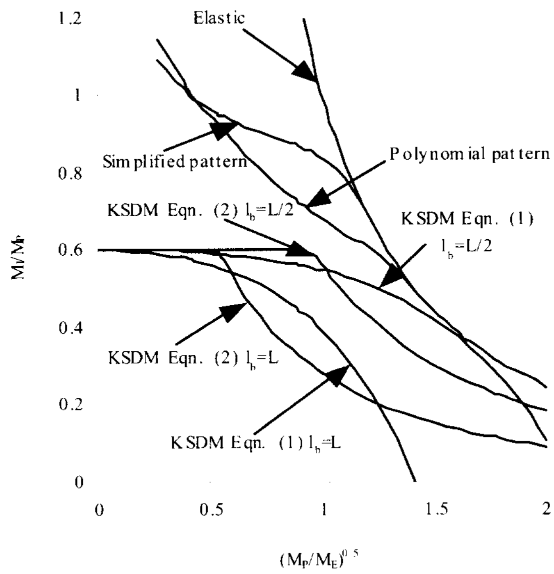



Figure 7. Inelastic buckling of simply supported beam 300×175 with $\beta$=0.

Also shown in these figures are the allowable bending strength curves in the KSDM (1995) and uses;

$$\sigma_b = \left[1 - 0.4\frac{\left(\frac{l_b}{i_b}\right)^2}{C\lambda_p^2}\right]\frac{\sigma_y}{1.5} \le \frac{\sigma_y}{1.5} \qquad (6)$$

$$\sigma_b = \frac{900}{\frac{l_b h}{A_f}} \le \frac{\sigma_y}{1.5} \qquad (7)$$

where $l$b = distance between the compressive flanges

$$C = 1.75 - 1.05\left(\frac{M_1}{M_2}\right) + 0.3\left(\frac{M_1}{M_2}\right)^2 \le 2.3, \qquad M_1 > M_2$$

$$i_b = \sqrt{\frac{I_f}{A_f + \frac{1}{6}A_w}}, \quad \lambda_p = \sqrt{\frac{\pi^2 E}{0.6\sigma_y}}$$

KSDM Eqn. (1) and KSDM Eqn. (2) in the figures are referring to the equation 6 and 7 respectively.

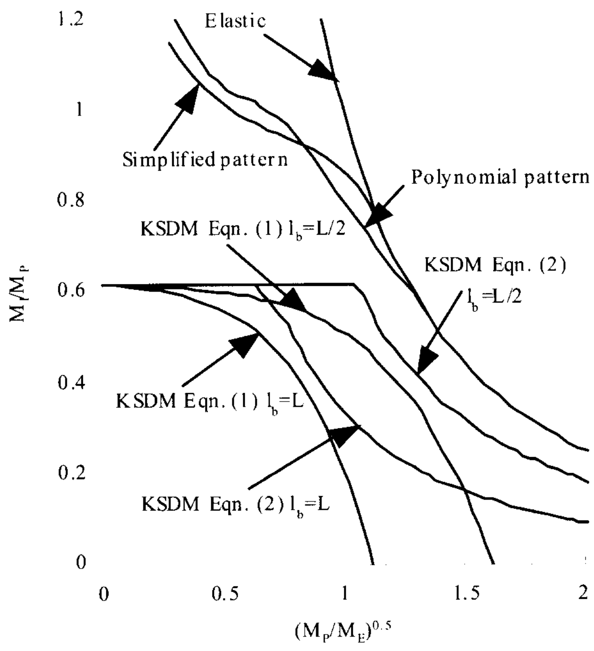



Figure 8. Inelastic buckling of simply supported beam 400×400 with $\beta$=0.

Two different effective lengths are used to determine the allowable bending stress in this study. This study considers the effective length is the span length of the beam and the effective length is then halved by introducing intermediate bracing at mid-span of beam.

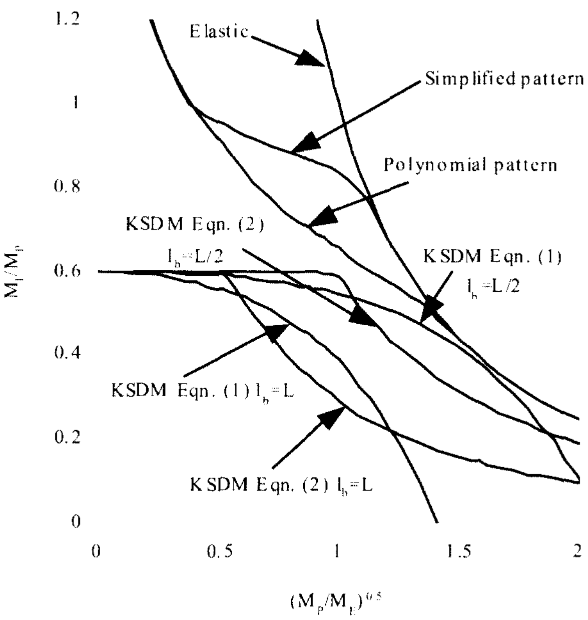



Figure 9. Inelastic buckling of simply supported beam 800x300 with $\beta$=0.

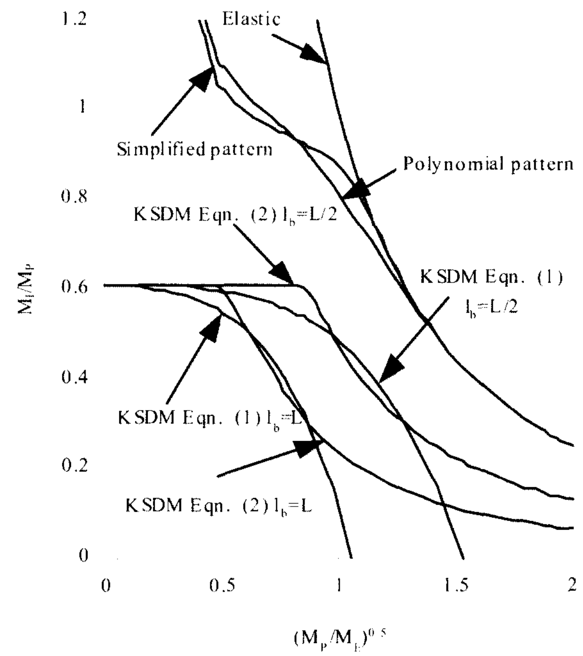



Figure 10. Inelastic buckling of simply supported beam 200x150 with $\beta$=1

It would be expected that KSDM (1995) is conservative for unrestrained beams, but not with retrained beams. It can be seen in the figures that the design method in KSDM (1995) is excessively conservative with and without intermediate bracing at the mid span of the beam.

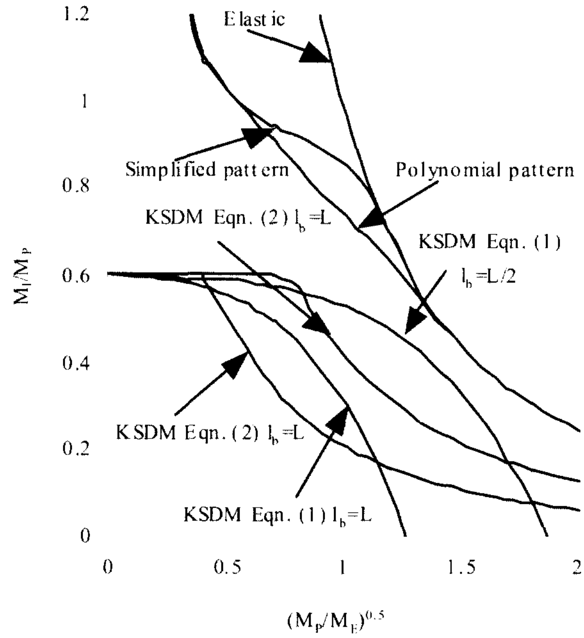



Figure 11. Inelastic buckling of simply supported beam 300x175 with $\beta$=1.

The buckling results of the beam with $\beta=0$ and 1 are shown that the compact I-section (200×150 and 400×400) is more conservative than the slender I-sections (300×175 and 800×300). The allowable bending stress in KSDM (1995) can be determined from Eqn. either 6 or 7 and selected whichever provides the highest value. The buckling results of the compact I-sections are shown that the allowable bending stress determined from Eqn. 7 is higher than Eqn. 6, but the slender I-section are depended on the length of the beam. The design method KSDM (1995) is more conservative as the dimensionless length $\sqrt{M_p/M_E}$ is decreased, which is due to the maximum allowable bending stress used in KSDM that is $\sigma_y/1.5$.

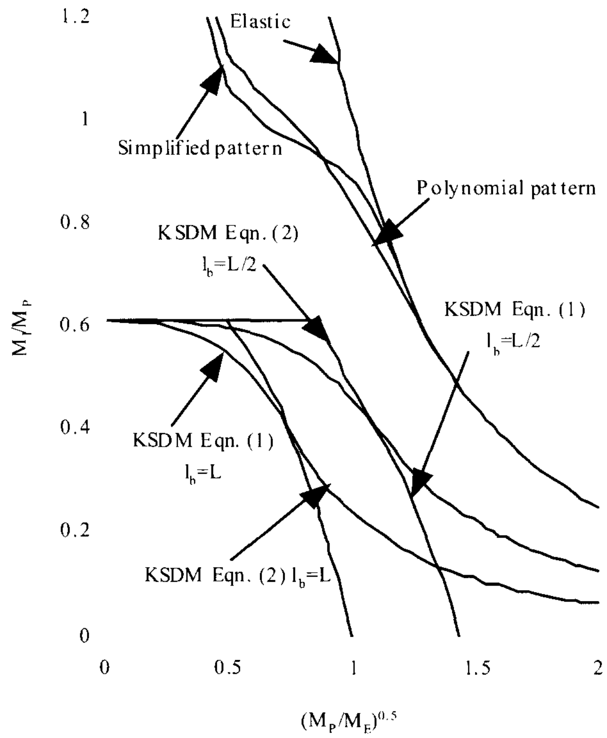



Figure 12. Inelastic buckling of simply supported beam 400x400 with $\beta=1$.

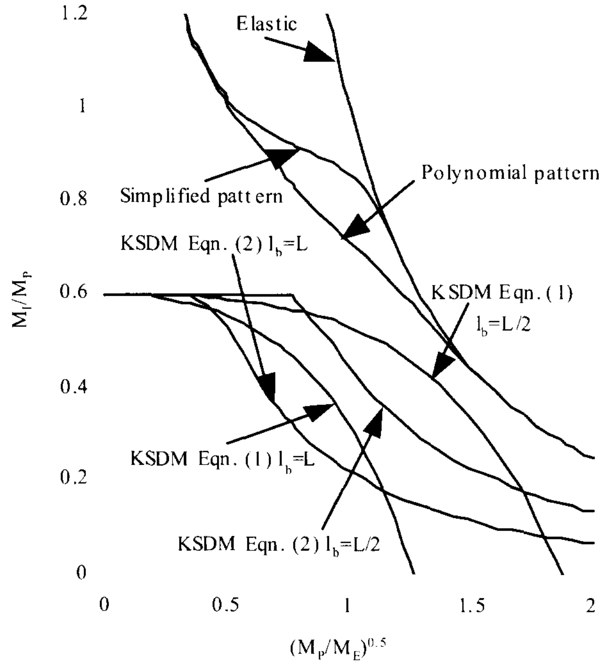



Figure 13. Inelastic buckling of simply supported beam 800x300 with $\beta=1$.

Different pattern of residual stresses have little influence on the inelastic lateral-torsional buckling behavior of the beams for compact I-sections but the buckling loads of the slender I-sections are strongly influenced by assumed pattern of the residual stress, especially in the regions of mid-slenderness. The inelastic lateral-torsional buckling results obtained from the polynomial residual stress is tended to be lower than the simplified pattern. This is due to the maximum residual stress at the flange tip and the flange/web junction. The maximum residual stress in the polynomial pattern of residual stress is high than those of the simplified pattern of residual stress.

## 5. Conclusions

This paper investigates the inelastic buckling behavior of the simply supported beams under unequal end moment using a line-type finite element method. The finite element method is incorporated the well-known simplified pattern and the polynomial pattern of residual stresses. This study has considered four different I-sections manufactured in Korea to analyze the I-beams under unequal end moment. The accuracy of KSDM (1995) is examined by comparing the inelastic lateral-torsional buckling results obtained in this study. This study has found that the KSDM (1995)is generally conservative with and without intermediate bracing at the mid span of the beam and there is some scope for improvement.

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