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A Combined Approach of Pricing and (S-1, S) Inventory Policy in a Two-Echelon Supply Chain with Lost Sales Allowed

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다단계 SCM 환경에서 품절을 고려한 최적의 제품가격 및 재고정책 결정

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This paper considers a continuous-review two-echelon inventory control problem with one-to-one replenishment policy incorporated and with lost sales allowed where demand arrives in a stationary Poisson process. The problem is formulated using METRIC-approximation in a combined approach of pricing and (S-1, S) inventory policy, for which a heuristic solution algorithm is derived with respect to the corresponding one-warehouse multi-retailer supply chain. Specifically, decisions on retail pricing and warehouse inventory policies are made in integration to maximize total profit in the supply chain. The objective function of the model consists of sub-functions of revenue and cost (holding cost and penalty cost). To test the effectiveness and efficiency of the proposed algorithm, numerical experiments are performed with two cases. The first case deals with identical retailers and the second case deals with different retailers with different market sizes. The computational results show that the proposed algorithm is efficient and derives quite good decisions.

Keywords: inventory, pricing, two-echelon supply chain, lost sales, heuristic

1. Introduction

Traditional inventory planning models for supply chains assume that all associated demand processes and revenue streams are exogenously determined. As a consequence, such models focus on operation cost minimization in the associated supply chains based on demand forecasts which are usually determined by marketing models. On the other hand, marketing models focus on determining pricing strategies and analyzing their impact on sales volumes and revenues, typically by rudimentary and simplistic treatment of operation cost in their supply chains(Chen, Federgruen and Zheng, 2001). However, it has been noticed that simultaneous decision on pricing and inventory policy leads to more profit in a single-echelon inventory system(Kunreuther and Richard, 1971; Whitin, 1955). Motivated by this notice, the combined approach of pricing and inventory policy has been intensively investigated for many single-echelon systems(Abad, 2001; Karlin and Carr, 1962; Pekelman, 1974). It has also been shown that the combined approach works better than any separated approach of treating the two problems individually. However, there are few studies about the combined approach for multi-echelon inventory systems. This provides the motivation for this paper to integrate the pricing and inventory

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control issues together in a two-echelon inventory system with stochastic demand processes incorporated. The inventory system consists of a central warehouse and multiple retailers, where the warehouse distributes a single type of products to multiple retailers who will sell it to consumers. The retailers serve geographically dispersed and heterogeneous markets. Demands at each retail market arrive continuously but in a fashion of forming a non-linearly decreasing function of retail price in the market. The warehouse replenishes its inventory from an external supplier with ample capacity. Each retailer and the warehouse use the same (S-1, S) policy.

Generally, the (S-1, S) inventory policy is applied to situations where demand loss is not allowed or penalty changes are severe as in military service. Accordingly, the approach of combining the pricing issue and the issue of (S-1, S) inventory policy adaptation with lost sales allowed may be applied to dealing with expensive goods like jewelry or deluxe cars whose demand rates are subject to their price changes.

It has been reported in the literature that if the price and multi-echelon inventory decisions are made together at the same time, then the associated supply chain will get more profits due to increased sales and decreased operation cost, and so make the supply chain customers satisfied. Thereupon, this paper will focus on constructing a model that combines both the multi-echelon inventory decision and the pricing decision in the associated supply chain.

The exact cost for a single-echelon lost sales inventory system having Poisson-distributed demands and fixed leadtime has been derived in Hadley and Whitin (1963). Sung and Yang (1988) have considered (s, S) inventory policy with limited backlogging and stochastic leadtime. Smith (1997) has demonstrated how to evaluate and find the optimal (S-1, S) inventory policy for an inventory system with lost sales allowed but without any replenishment cost allowed and with generally distributed stochastic leadtime allowed. A METRIC-model has been suggested as one of the most widely known multi-echelon inventory models in Sherbrooke (1986). Nahmias and Smith (1994) have considered a lost sales case for a multi-echelon system via the METRIC-model which has specifically considered periodic review batch order policies with partial lost sales allowed.

There are some marketing literatures studied on supply chain coordination between retailer and supplier which focuses on pricing. For example, Jeuland and Shugan (1983) have considered a simple pricing issue for a single-supplier and single-retailer system. Their model did not consider any inventory replenishment. The single-retailer part has been extended to multi-retailer settings by Ingene and Parry (1995). Monahan (1984) has determined prices subject to the restriction that both supplier and retailer use identical order intervals. Lal and Staelin (1984) have considered a pricing problem with non-identical retailers, under the assumption that all demand processes are not exogenously given and inventory replenishment is made infrequently.

The approach of integrating inventory control and pricing issues together was first advocated by Whitin (1955). Both Whitin (1955) and Mills (1959) have addressed a single-period, single-location model to determine the associated single-price and supply quantity. Karlin and Carr (1962) have considered an in finite horizon model for a single item, under the assumption that a single price needs to be specified at the beginning of the planning horizon. Chen *et al*. (2001) have considered both coordination (power-of-two) mechanism and non-coordination mechanism for multi-retailer systems under a periodic review inventory policy. Lee and Hong (2002) have integrated the pricing issue and the (r, Q) policy adaptation issue in a single-warehouse and single-retailer system with stochastic demand processes incorporated.

The organization of this paper is briefed as follows. Section 2 presents the problem description and formulation. Section 3 analyzes the solution properties and proposes a solution algorithm based on the solution properties. Section 4 gives the computational results of some numerical examples, and Section 5 states conclusions.

2. Problem Description

The proposed problem considers a two-echelon inventory system with single central warehouse and multiple retailers as depicted in <Figure 1>. The retailers, which have different market sizes, serve to satisfy customer demands and replenish the associated stocks from the central warehouse. The warehouse, in turn, replenishes its stock from an outside supplier. The customer demand rate decreases exponentially as the price increases, while the retail price at each market is the same. The objective of the problem is to find the combined policy for inventory replenishment and pricing that maximizes long-run average profit in the associated two-echelon supply chain. There is a central planner who makes the pricing and inventory replenishment decisions. That is, the central planner makes both decisions simultaneously to maximize the total average profit of the two-echelon supply chain.

Demand process at each retailer follows a stationary Poisson process with constant arrival rate. When a retailer is out of stock, any arriving demand at the retailer will be lost. However, when a stockout occurs at the warehouse, all demands from the retailers are fully backlogged and the backorders are filled according to a FIFO-policy. Each retailer and the warehouse use the same (S-1, S) policy.

The transportation time from the warehouse to any retailer is assumed to be constant. The transportation time from the external supplier to the warehouse is also constant. The external supplier is assumed to have infinite capacity, which means that the replenishment leadtime for the central warehouse is constant. The replenishment and backorder costs are assumed to be negligible, compared to the holding and stockout costs. Any units held in stock at the warehouse and the retailers incur holding costs per unit per time. A fixed shortage cost per lost customer is incurred at the retailers.

The objective of the problem is to maximize the long-run total average profit of the associated supply chain, which is defined as the difference between the associated total revenue and cost. The total revenue function can be easily defined by multiplying total sales by retail price. However, the cost function is more complex, so that some detailed analysis on the warehouse and the retailers should be made before deriving the associated cost function. The total cost consists of inventory holding costs at the warehouse and all the retailers, and penalty costs at all the retailers. Let us introduce the following notation:

N : Number of retailers
λ_i : Demand intensity at retailer i, $i=1,2,\cdots,N$
λ_0 : Sum of customer demands arrived at the retailers $=\sum_{i=1}^{N}\lambda_i$
Λ : Demand intensity at the warehouse
L_i : Transportation time for delivery from the warehouse to retailer i, $i=1,2,\cdots,N$
L_0 : Transportation time for delivery from the external supplier to the warehouse
S_0 : Order-up-to level at warehouse
S_i : Order-up-to level at retailer i, $i=1,2,\cdots,N$ $(\vec{S}=S_1, S_2, \ldots S_N)$
h_0 : Unit holding cost per unit time at the warehouse
h_i : Unit holding cost per unit time at retailer i, $i=1,2,\cdots,N$
π_i : Unit penalty cost for lost sale at retailer i, $i=1,2,\cdots,N$
c : Unit purchasing cost at the warehouse
p : Unit retail price charged by retailer i, $i=1,2,\cdots,N$
A_i : Market size of retailer i, $i=1,2,\cdots,N$
α : Elastic coefficient of retail price p
D_i : Costumer demand in the market served by retailer i, $i=1,2,\cdots,N=A_ie^{-\alpha p}$
$G_i(\cdot)$: Sales revenue function at retailer i, $i=1,2,\cdots,N$
$C_0(\cdot)$: Long-run cost function per unit time for the warehouse in steady state
$C_i(\cdot)$: Long-run cost function per unit time for retailer i in steady state, $i=1,2,\cdots,N$
$C_i^{\min}\equiv\min_{S_i} C_i(S_i, L_i)$: minimum cost per unit time for retailer i in steady state when the leadtime, L_i, is substituted for $\overline{L_i}$, $i=1,2,\cdots,N$, given fixed p
$TC(\cdot)$: Long-run total cost function for the inventory system per time unit in steady state $=C_0(\cdot)+\sum_{i=0}^{N}C_i(\cdot)$

A queueing system analogy will be used when evaluating costs at the retailers, which has been adapted successfully in the analysis of inventory systems as in Sherbrooke (1986). It is noted that the demand process at the retailers follows a stationary Poisson process and the replenishment leadtime is

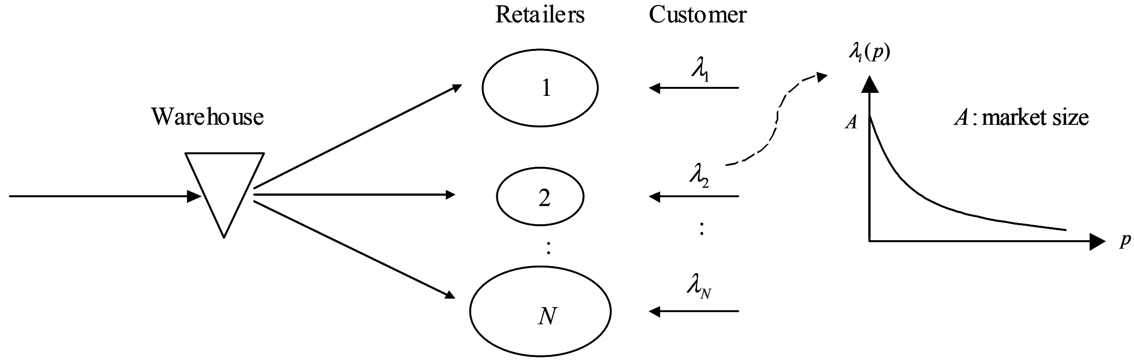

Figure 1. Two-echelon inventory system (1:1:N).

stochastic, since orders from the retailers can be delayed at the central warehouse due to stochastic stockouts. According to Palm (1938), it is also noted that the steady state occupancy level is Poisson distributed with mean λL, where λ is the mean arrival rate and L is the mean service time, which holds for i.i.d. service times. However, the stochastic leadtimes in the proposed problem are evidently not independent to each other. By the way, by disregarding any associated correlation, the number of outstanding orders will be approximated in this paper as to follow a Poisson distribution, as adapted in the METRIC-approximation in Sherbrooke (1986).

In the situation where lost sales are allowed, the corresponding queueing system of interest can be modeled as an M/G/S/S queue with S servers, each with generally distributed service time and no queueing allowed. If the service times are independent random variables with mean $\overline{L}$, then the associated Erlang's loss formula can be viewed as stating the steady-state distribution of occupancy level at the retailer as $q^s(j)=\dfrac{(A_i e^{-\alpha p}\overline{L})^j/j!}{\sum_{n=0}^{S}(A_i e^{-\alpha p}\overline{L})^n/n!}$ $0\le j\le S$, where $q^s(j)$ is the probability that j servers (out of S) are occupied in steady state. Based on the METRIC approximation explained above, the number of outstanding orders at the retailer can be modeled as $q^s(j)$.

Let the mean replenishment leadtime at retailer i be $\overline{L_i}$ and let $q_i^s(j)$ be the steady-state probability of j outstanding orders, given a desired-stock level S_i. Then, the expected number of lost sales per unit time is derived as $\lambda_i q_i^{s_i}(S_i)=A_i e^{-\alpha p} q_i^{s_i}(S_i)$ and the expected number of units in stock is derived as $\sum_{j=0}^{S_i}(S_i-j)q_i^{S_i}(j)=S_i-[1-q_i^{S_i}(S_i)]A_i e^{-\alpha p}\overline{L_i}$. The demand rate from retailer i without loss allowed is derived as $(1-q_i^{S_i}(S_i))A_i e^{-\alpha p}$. Therefrom, the associated total cost function at retailer i can be derived as

$$C_i(S_i,\overline{L_i},p)=A_i e^{-\alpha p}\pi_i q_i^{Si}(S_i)+h_i(S_i-[1-q_i^{Si}(S_i)]A_i e^{-\alpha p}\overline{L_i}) \quad (1)$$

In the backorder case, the demand process at the warehouse follows the same stationary Poisson process as at the retailers. However, in the lost sales case, the warehouse may not have the same Poisson process, because any demand from customers may be lost during leadtime intervals for the orders of the retailer placed to the warehouse. For example, if the basestock level at a retailer is one, then the retailer leadtime will be included in the inter-arrival time interval between two successive demand arrivals at the warehouse from the retailer so that all the demands from customers arrived during the leadtime will be lost due to stockout at the retailer. Therefore, the associated demand process at the warehouse does not remain as the original Poisson process any longer. The remaining demand process will be rather complex to characterize. Therefore, in this paper the demand process at the warehouse will be approximated as a stationary Poisson process but with adjusted arrival rate. The arrival rate is assumed to be Λ where Λ depends on how much demand is lost at all the retailers, which is determined as

$$\Lambda=\sum_{i=1}^{N}A_i e^{-\alpha p}(1-q_i^{S_i}(S_i)) \quad (2)$$

Now, given a fixed deterministic leadtime L_0, we can find the average holding cost incurred at the warehouse as a function of Λ and S_0.

$$C_0(S_0,\Lambda)=h_0\sum_{j=0}^{S_0}(S_0-j)\frac{(\Lambda L_0)^j}{j!}\exp(-\Lambda L_0) \quad (3)$$

where Λ is the function of S_i and p, so that Eq. (2) can be incorporated as

$$C_0(S_0,\overline{S},p)=h_0\sum_{j=0}^{S_0}(S_0-j)\frac{(\sum_{i=1}^{N}A_i e^{-\alpha p}(1-q_i^{S_i}(S_i))L_0)^j}{j!}\exp(-\sum_{i=1}^{N}A_i e^{-\alpha p}(1-q_i^{S_i}(S_i))L_0) \quad (4)$$

The mean delivery delay can also be derived in consideration of stockout at the warehouse by using B_0, the average number of backorders at the warehouse, which can be calculated as

$$B_0=\sum_{j=S_0+1}^{\infty}(j-S_0)\frac{(\Lambda L_0)^j}{j!}\exp(-\Lambda L_0) \quad (5)$$

Then, the Little's formula is applied to obtain the average delivery delay, B_0/Λ. Therewith, the mean leadtime for retailer i is derived as

$$\overline{L_i}=L_i+B_0/\Lambda \quad (6)$$

The total revenue function can be represented as

$$G_i(S_i,p)=A_i e^{-\alpha p}(1-q_i^{S_i}(S_i))(p-c) \quad (7)$$

The total cost function is obtained in the associated supply chain as

$$TC(\cdot) = C_0(S_0, \overline{S}, p) + \sum_{i=1}^{N} C_i(S_i, \overline{L}_i, p) \qquad (8)$$

As mentioned above, a central planner makes the pricing and inventory replenishment decisions together so as to maximize the long-run average profit in the two-echelon supply chain, which is equal to the total revenue minus cost, expressed as

$$\begin{aligned} TP(S_0, \overline{S}, p) &= \sum_{i=1}^{N} G_i(S_i, p) \\ &\quad - \left[C_0(S_0, \overline{S}, p) + \sum_{i=1}^{N} C_i(S_i, \overline{L}_i, p) \right] \end{aligned} \qquad (9)$$

It can be represented as the function of S_0, S_i, and p which is given as

$$\begin{aligned} TP(S_0, \overline{S}, p) &= \sum_{i=1}^{N} A_i e^{-\alpha p}(1 - q_i^{S_i}(S_i))(p - c) \\ &\quad - h_0 \sum_{j=0}^{S_0} (S_0 - j) \frac{(\sum_{i=1}^{N} A_i e^{-\alpha p}(1 - q_i^{S_i}(S_i))L_0)^j}{j!} \\ &\quad \exp(-\sum_{i=1}^{N} A_i e^{-\alpha p}(1 - q_i^{S_i}(S_i))L_0) \qquad (10) \\ &\quad - \sum_{i=1}^{N} A_i e^{-\alpha p} \pi_i q_i^{S_i}(S_i) + h_i(S_i - [1 - q_i^{S_i}(S_i)] A_i e^{-\alpha p} \overline{L}_i) \end{aligned}$$

3. Solution Procedure

The objective of the proposed problem is to find the optimal inventory positions S_0^* and and $\overrightarrow{S^*}$ the optimal price p^* together which maximize the total supply chain profit. The total profit is calculated by subtracting the total cost at the warehouse and all the retailers from the total revenue of the retailers. Inventory holding costs at the warehouse and all the retailers and penalty costs at all the retailers for lost sales cases are considered. To maximize the total profit, the total revenue must be maximized, while the total cost is minimized. As shown in Eq. (10), the total profit function is non-linear so that each involved decision variable is hard to clearly find its mathematical feature. Therefore, the near-optimal solution will be found by using a heuristic search algorithm. For the search, several solution properties will be characterized in this chapter.

Lemma 1

$C_i(S_i, \overline{L}_i, p)$ is convex with S_i, given fixed $\overline{L}_i$ and p.

Proof. $C_i(S_i, \overline{L}_i, p)$ can be proved to be convex in S_i with $\overline{L}_i$ fixed, referring to Smith (1997), and the value of p is not subject to S_i's. This implies that $C_i(S_i, \overline{L}_i, p)$ is convex too, given fixed p.
This completes the proof.

Given $\overline{L}_i$ and p, the optimal order-up-to-level S_i, which minimizes the cost of the retailers, is obtained by a local search procedure. $\overline{L}_i$ is a function of Λ and S_0 as shown in Eqs. (5) and (6). This implies that the optimal order-up-to-level S_i can be found based on Lemma 1. The retail price p and order-up-to-level at the warehouse S_0 are decision variables and Λ is a value calculated from Eq. (2). For the given random variable Λ, S_i is calculated, with which Λ is updated as in Eq. (2). Therewith, given p and S_0, the optimal value of S_i can be calculated.

Lemma 2

Given p fixed, $\min_{s_i} C_i(S_i, L_i) \le \min_{s_i} C_i(S_i, \overline{L}_i)$

Proof. Let $C_i^{\min}$ be the minimum cost per unit time for retailer i in steady state when the leadtime, L_i, is substituted for $\overline{L}_i$, $i = 1, 2, \cdots, N$, given fixed p. It is represented as $\min_{s_i} C_i(S_i, L_i)$. Then, it is needed to show that $\min_{s_i} C_i(S_i, L_i) \le \min_{s_i} C_i(S_i, \overline{L}_i)$ *for* $L_i \le \overline{L}_i$. Let l_i be an arbitrarily chosen leadtime, where $L_i \le l_i \le \overline{L}_i$. Consider the cost $C_i(S_i, l_i)$, where S_i is set at its optimal value for each l_i. Start with $l_i = \overline{L}_i$ and let l_i be continuously lowered until it reaches the level L_i as $l_i = L_i$. Since $C_i(S_i, l_i)$ is a continuous function of l_i, for fixed S_i, it is also a continuous function of l_i. Moreover, the fact that S_i minimizes the cost $C_i(S_i, l_i)$ implies the relation $C_i(S_i, l_i) \le C_i(S_i - 1, l_i)$. For notational simplicity, the index i will be omitted from all the variables. Thereupon, let $\lambda = Ae^{-\alpha p}$.

It can then be shown that

$$\begin{aligned} \frac{\partial C(S,l)}{\partial l} &= -h\lambda(1 - q^s(S)) + \lambda(h\lambda l + \pi\lambda)(q^s(S-1) \\ &\quad - q^s(S) + q^s(S)^2). \end{aligned} \qquad ①$$

Moreover, the relation $C(S,l) \le C(S-1,l)$ implies that

$$\frac{h}{h\lambda l + \lambda\pi} \le q^{s-1}(S-1) - q^{s}(S). \qquad ②$$

Let $K = \frac{1}{\lambda(h\lambda l + \pi\lambda)}\frac{\partial C(S,l)}{\partial l}$. From Eq. ①, it holds that

$$K = \frac{h}{h\lambda l + \pi\lambda}(q^{s}(S) - 1) + q^{s}(S-1) - q^{s}(S) + q^{s}(S)^{2}.$$

Multiplying the right-hand side of Eq. ② by K gives (recalling that $q^{s}(S) < 1$)

$$\begin{aligned} K &\ge (q^{s-1}(S-1) - q^{s}(S))(q^{s}(S) - 1) + q^{s}(S-1) \\ &\quad - q^{s}(S) + q^{s}(S)^{2} \\ &= q^{s-1}(S-1)q^{s}(S) - q^{s-1}(S-1) + q^{s}(S-1) \\ &= q^{s-1}(S-1)q^{s}(S) - (q^{s-1}(S-1)q^{s}(S)) \\ &= 0. \end{aligned} \qquad ③$$

(Detail expression of ③)

From the definition of $q^{S}(S-1)$, we have $q^{S}(S-1) = q^{S}(S)\left(\frac{S}{\lambda\bar{L}}\right)$. It can be shown following equation:

$$q^{S-1}(s-1)\left\{\sum_{n=0}^{s-1}\frac{(\lambda\bar{L})^{n}}{n!}\right\} = q^{S}(s-1)\left\{\sum_{n=0}^{s}\frac{(\lambda\bar{L})^{n}}{n!}\right\},$$

where $q^{S}(s-1) = \dfrac{\dfrac{(\lambda\bar{L})^{s-1}}{(s-1)!}}{\sum_{n=0}^{s}\dfrac{(\lambda\bar{L})^{n}}{n!}}$.

Let $U = \sum_{n=0}^{s-1}\frac{(\lambda\bar{L})^{n}}{n!}$,

$$q^{S-1}(s-1)\,U = q^{S}(s-1)\left\{U + \frac{(\lambda\bar{L})^{S}}{S!}\right\}$$

$$\begin{aligned} q^{S-1}(s-1) - q^{S}(s-1) &= q^{S}(s-1)\frac{(\lambda\bar{L})^{S}}{S!}\frac{1}{U} \\ &= q^{S}(s-1)\frac{(\lambda\bar{L})}{S!}\,q^{S-1}(s-1) \\ &= q^{S}(s)q^{S-1}(s-1) \end{aligned}$$

Then we have $q^{S-1}(s-1) - q^{S}(s-1) = q^{S}(s)q^{S-1}(s-1)$.

Consequently, $K \ge 0$, so that $C_i(S_i,\overline{L_i})$ is a non-decreasing function for $L_i \le \overline{L_i}$. Let $S(\overline{L_i}) = \arg\min_{s_i} C_i(S_i,\overline{L_i})$. Then, the relation $C_i(S(\overline{L_i}),L_i) \le C_i(S(\overline{L_i}),\overline{L_i})$ holds. Let $S(L_i) = \arg\min_{s_i} C_i(S_i,L_i)$. Then, the relation $C_i(S(L_i),L_i) \le C_i(S(\overline{L_i}),L_i)$ holds. Therefore, the relation $\min_{s_i} C_i(S_i,L_i) \le \min_{s_i} C_i(S_i,\overline{L_i})$ for $L_i \le \overline{L_i}$ holds.

This completes the proof.

Lemma 3

Given p fixed, $C_0(S_0,\lambda_0) \le C_0(S_0,\Lambda)$ *holds with* S_0

Proof. λ_0 is the sum of customer demands arrived at all the retailers, so that $\Lambda \le \lambda_0$. Then, it is only needed to show that $\frac{\partial C_0(S_o,\Lambda)}{\partial\Lambda} \le 0$, which is shown as

$$\begin{aligned} \frac{\partial C_0(S_o,\Lambda)}{\partial\Lambda} &= -h_0 L_0\exp(-\Lambda L_0) \\ &\quad \left(S_0 + \sum_{j=1}^{S_0}(S_0 - j)\left(\frac{(\Lambda L_0)^{j}}{j!} - \frac{(\Lambda L_0)^{j-1}}{(j-1)!}\right)\right) \\ &= -hL_0\exp(-\Lambda L_0)\sum_{j=0}^{S_0-1}\frac{(\Lambda L_0)^{j}}{j!} \\ &\le 0. \end{aligned}$$

This completes the proof.

Theorem 1

Given p fixed, $TC_{lb}(S_0)$ *is a lower bound for the cost function*,

$$TC_{lb}(S_0) \equiv C_0(S_0,\lambda_0) + \sum_{i=0}^{N} C_i^{\min}.$$

Proof. Axsater (1993) has proved the convexity of $C_0(S_0,\lambda_0)$ in S_0. By Lemma 1, 2, and 3, given p fixed, it is found that $TC_{lb}(S_0)$ is a lower bound for the cost function $TC_{lb}(S_0) = C_0(S_0,\lambda_0) + \sum_{i=0}^{N} C_i^{\min}$. Moreover, the cost function $TC_{lb}(S_0)$ is convex in S_0.

This completes the proof.

Theorem 2

If the probability of lost sales at the retailers $q_i^{S_i}(S_i)$ is zero, then $G_i(S_i, p)$ has the maximum value at $p = \frac{1}{\alpha} + c$.

Proof. It is easy to show that $\frac{\partial G_i(S_i, p)}{\partial p} = Ae^{-\alpha p}$ $(\alpha c + 1 - \alpha p)$. If the probability of lost sales at the retailer $q_i^{S_i}(S_i)$ is zero, then $G_i(S_i, p)$ is differentiable in p, so as to find its maximum value at $p = \frac{1}{\alpha} + c$.

This completes the proof.

Theorem 3

If the probability of lost sales at the retailers $q_i^{S_i}(S_i)$ is zero, and p is allowed to vary over a finite interval $[c, \frac{2}{\alpha} + c]$ only, then $G_i(S_i, p)$ is concave.

Proof. If the probability of lost sales at the retailer $q_i^{S_i}(S_i)$ is zero, then $G_i(S_i, p)$ is differentiable in p. It can be shown that $\frac{\partial^2 G_i(S_i, p)}{\partial p^2} = Ae^{-\alpha p}\alpha(\alpha(p-c)-2)$. It holds that $Ae^{-\alpha p}$ and α are positive value. If p have the value at $c \le p \le \frac{2}{\alpha} + c$, then $\frac{\partial^2 G_i(S_i, p)}{\partial p^2} \le 0$.

This completes the proof.

Let notations with superscript * denote the maximum supply chain profits under the optimal pricing and inventory policy and notations with superscript 0 denote the maximum supply chain profits with the probability of lost sales being zero. Then, the total revenue of the supply chain has the maximum value at p which is equal to the reciprocal number of elastic coefficient plus the purchasing cost. At this point, p^0 denotes the current p. Given the retailer price p fixed, the optimal order-up-to-level at the warehouse S_0 and the optimal order-up-to-level at the retailers S_i are found as minimizing the total cost in the supply chain. However, it cannot be guaranteed that this value maximizes the total revenue. Therefore, to find the optimal S_0 and S_i with various retail prices p, it is needed to choose the optimal solutions which maximize the total profit.

Now, the solution procedure is derived. The procedure is designed to enumerate with all the possible p and S_0 values. It is natural that for a profit maximization problem, p and S_0 cannot be negative. Let $TC^*(S_0)$ be the minimum value of total cost (TC), given a fixed value of S_0. By Lemma 1, given $\overline{L_i}$ and p, the expected cost function for retailer i (C_i), being represented by the associated inventory cost is convex in S_i. Therefore, the optimal S_i which minimizes the retailer inventory cost (C_i) can be found, given $\overline{L_i}$ and p. By Theorem 1, the cost function $TC_{lb}(S_0)$ is convex in S_0, so that a search can be made for the optimal S_0 such that $S_0^{\max}$ satisfies $\min_{x \le S_0} TC^*(x) \le TC_{lb}(S_0^{\max})$. In conclusion, given p fixed, the required S_0 and S_i values, $i = 1, 2, \cdots, N$ can be calculated from $\min_{S_0, \vec{S}} TC(S_0, \vec{S})$, referring to <Figure 2>.

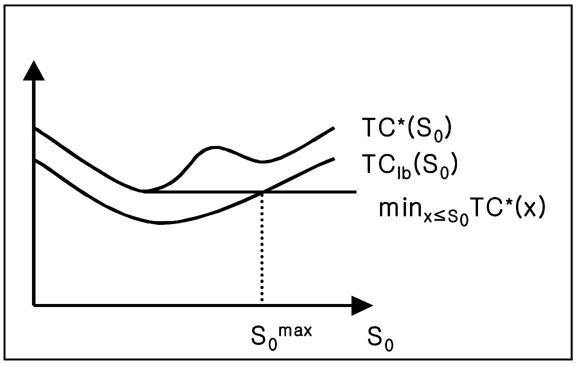

Figure 2. Illustration of the abortion criteria.

It is noted that each retail price has to be larger than the purchasing cost at the warehouse. Therefore, p is bounded from the purchasing cost. However, the upper bound of p is not easy to derive analytically due to the complexity of the total revenue function in Eq. (7). Therefore the upper bound will be derived based on experiments. Before describing the upper bound search procedure, it is noted that according to the results of Theorems 2 and 3, the total revenue reaches at the maximum at $p^0 = \left(\frac{1}{\alpha} + c\right)$, under the situation where the probability of lost sales is zero. Actually, if the probability of lost sales is zero, then S_0 and S_i values must be infinite. This is, however, not a reasonable solution. Rather, the optimal retail price p^*, which maximizes the total profit, can be considered to be near p^0, since lost sales may not appear under the (S-1, S) inventory policy. Therefore, the boundary of retail price p could be searched for within a reasonable range near p^0.

In the upper bound derivation, it is assumed that the upper bound is determined within several multiples of $\triangle$, based on the fact that the interval between the reasonable lower bound ($P_{LB} =: c$) and p^0 is $\triangle$. $\triangle$ is defined as the reciprocal number of elastic coefficient, since the elastic coefficient is represented as the change of demand subject to the change of price $\left(\Delta = \frac{1}{\alpha}\right)$. To determine a reasonable upper bound of p, through various cases the near-optimal solution is searched for within a range from $p^0 = \left(\frac{1}{\alpha} + c\right)$ to $p^0 + 1\Delta$, $p^0 + 2\Delta, \cdots$, $p^0 + 5\Delta$. After all the cases being searched for the near-optimal solution, the optimal is found within the $p^0 + 1\Delta$ range. Thereupon, it is recommended that p^0 plus one delta ($p^0 + 1\Delta$) is assigned as the upper bound of retail price, as shown in Fig. 3. It is noted that the upper bound is the same as the concavity range in Theorem 3. This implies that the upper bound derived through experiments is the worst case upper bound.

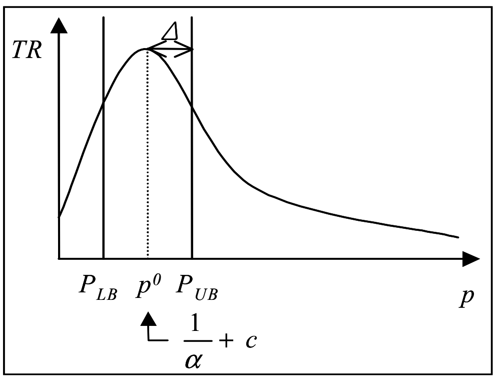

Figure 3. Illustration of bound of retail price.

Solution Procedure:

Step 1: Assign the lower bound and upper bound of retail price (P_{LB}, P_{UB}) and the step length (δ).

i) Set $p := P_{LB}$, P_{UB}, δ, goto Step2.

Step 2: Find the total cost function value, minTC^*, under given p.

i) Set $S_0^{\max} = 0$ and $TC_{lb}(S_0^{\max}) = \infty$.

ii) Set $S_0 = 0$ and $TC_{\min} = \infty$.

iii) Set $k = 0$ and $\Lambda = \lambda_0$.

iv) For each $i = 1, 2, \cdots, N$, calculate $\overline{L_i}$ by Eqs. (5) and (6) and set $S_i(k) = \arg\min_s(C_i(S, \overline{L_i}, p))$

v) If $k > 0$, and $S_i(k) = S_i(k-1)$ for all $i = 1$, $2, \cdots, N$, then goto vi), else calculate Λ by (2), set $k := k+1$ and goto iv).

vi) Set $TC^*(S_0) = C_0(S_0, S_i(k), p) + \sum_{i=0}^{N} C_i(S_i(k), \overline{L_i}, p)$.

If $TC^*(S_0) < TC_{\min}$, then $TC_{\min} = TC^*(S_0)$ and let $S_0^{opt} = S_0$ and $S_i^{opt} = S_i(k)$ for $i = 1$, $2, \cdots, N$. If $TC_{\min} \le TC_{lb}(S_0^{\max})$, then STOP, else set $S_o = S_o + 1$, and check if $S_0 \ge S_0^{\max}$, then set $S_0^{\max} = S_0^{\max} + 1$ goto ii), else goto iii).

Step 3: Calculate the total profit $TP^{\max}$

i) Set $TP = G - TC^*$, calculate TP.

If $p \ge P_{UB}$ then STOP. Else set $p = p + \delta$, goto Step2.

ii) Find $p^* = \arg\min_p(TP(p))$. Then $TP^{\max} = TP(S_0^{opt}, \overline{S^{opt}} \mid p^*)$.

To search within the lower and upper bounds of retail price p, the step length (δ) should be determined. To determine the step length of retail price, experiments with step lengths (δ)=0.2, 0.5, 1, 1.5 are performed with the parameter set given in Section 4. As the step length (δ) increases, the gap between the optimal solution and the solution of the proposed heuristic algorithm increases, and the calculation time decreases, as shown in <Figure 4> and <Figure 5>. Therefore, considering the trade-off between the solution gap and the calculation time, a reasonable value of the step length (δ) must be found. As shown in <Figure 4> and <Figure 5>, it is certain that the step length (δ) has a reasonable value at the value 1.

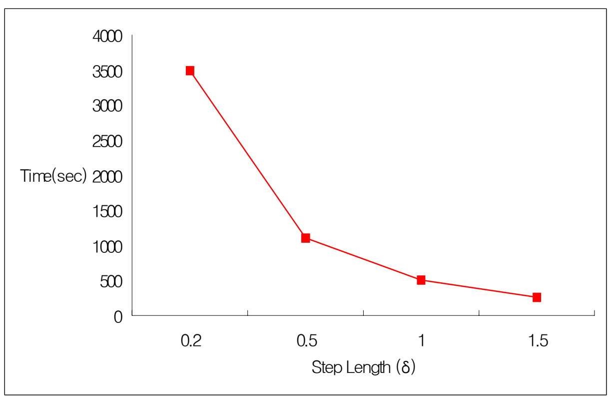

Figure 4. Illustration of the relation between calculation time and step length.

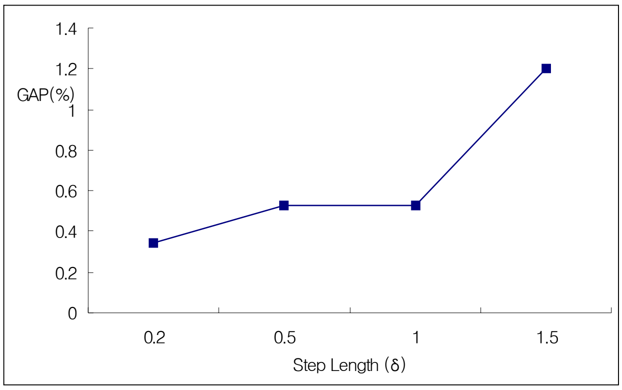

Figure 5. Illustration of the relation between solution gap and step length.

4. Computational Results

This section consists of three parts. The first part presents numerical results with the case of identical retailers with respect to market size. The second part presents numerical results with the case of different retailers. The last part shows the efficiency of the proposed heuristic algorithm in terms of calculation time and solution gap computed with various test instances in comparison with that of a full search algorithm. In order to examine the efficiency of the proposed heuristic algorithm, 6 different problems are considered and the number of retailers is 2 for each problem. For each test problem, the best order-up-to-level and retail price are obtained using the proposed heuristic algorithm. The experimental problem set provides 98% service level which means a very reliable set. The proposed algorithm is tested with various parameter settings to investigate solution sensitivities.

To test performance of the proposed algorithm, a full enumeration search is made iteratively at each unit increment of discrete p values. In the research, the boundary of the retail price p is set at the same range as in the proposed heuristic algorithm, and the ranges of S_0 and S_i are set at 4 times those of the proposed heuristic algorithm. The calculation time of the full search is set at the time for single unit iteration multiplied by the number of iterations. In this paper, the retail price is considered as a discrete function, since the retail price is determined by unit value on each customer arrival.

All the computational experiments are tested on IBM PC(Pentium IV processor / 1.7GHZ, 512MB memory). The computational results are given below.

4.1 The numerical Results with the Case of Identical Retailers

6 problems are considered when the retailers receive demands arrived at the same demand rate (i.e. same market size). It is assumed that the market size is 1000 for the first 3 problems. And the market size of the last 3 problems is set at the value 1200. On each case, the transportation time from the warehouse to each retailer is varied from 1 to 2. The average gap of total profit between the heuristic solution and full search solution is about 0.814303%. The average gap is calculated as

Table 1. The numerical results with two identical retailers case

#		A	L_i	Heuristic Algorithm				Full Search Algorithm				GAP(%)
				S_0^{heu}	S_i^{heu}	p^{heu}	TP^{heu}	S_0^{opt}	S_i^{opt}	p^{opt}	TP^{opt}	
1	R1	1000	1	5	18	50	114.99486	5	17	51	115.96577	0.837243
	R2				18				17			
2	R1	1000	2	4	21	52	111.81833	5	24	51	113.04188	1.082393
	R2				21				24			
3	R1	1000	1	5	17	51	113.34512	5	18	51	114.31446	0.847961
	R2		2		23				24			
4	R1	1200	1	5	19	51	140.78396	5	20	51	140.97873	0.138158
	R2				19				20			
5	R1	1200	2	5	27	51	136.21141	5	29	51	137.77732	1.136548
	R2				27				29			
6	R1	1200	1	5	19	51	138.1245	5	20	51	139.29952	0.843517
	R2		2		28				29			
AVG												0.814303

$(C = 40, \pi_i = 15, h_i = 1, h_0 = 0.5, \alpha = 0.1)$

$$\frac{\textit{full search solution} - \textit{heuristic solution}}{\textit{full search solution}} \times 100$$

The order-up-to-level of the retailer increases, while the total profit decreases, as the transportation time gets larger. This indicates that the possibility of lost sales grows as the transportation time becomes larger. Therefore, to prevent the increase of the cost, the stock level at the retailers must be larger. As the stock level at the retailers increases, the holding cost increases, which results in increase in the total cost and decrease in the total profit. The order-up-to-level of the retailers changes much due to the change of the market size and the transportation time. It can be interpreted that the change of the market size and the transportation time influences much more on the order-up-to-level of retailers than on the retail price. The problem data and results are presented as in <Table 1>.

The experiment is extended to consider a larger problem having 5 identical retailers. The market size of 2000 or 1000, and the retailer shortage costs of 25, 20, 15. The resulting average gap of total profit between the heuristic solution and full search solution is found at about 1.472063%, implying that the average gap increases as the number of retailers increases. Moreover, the order-up-to-level of the retailer increases as the retailer shortage cost gets larger. This implies that more inventories are needed to reduce lost sales.

4.2 The Numerical Results with the Case of Different Retailers

6 problems are considered with two different retailers. It is assumed that the market sizes of each market are 1000 and 1100(10% increasing), respectively, for the first 3 problems. Those of the last 3 problems are 1000 and 1200 (20% increasing), respectively. The variation of transportation time is the same as the problems of Section 5.1.

The average gap between the heuristic solution and full search solution is about at 0. 719457%. The order-up-to-level and the gap between the heuristic solution and full search solution increase as the transportation time gets larger. The possibility of lost sales increases as the transportation time increases, so that the gap with the optimal becomes larger. Between the different retailers, the order-up-to-level is larger for the retailer who considers larger market size. The problem data and results are presented as in <Table 3>. From <Tables 1, 2 and 3>, it can be stated that the proposed

Table 2. The numerical results with five identical retailers case

#		A	π_i	Heuristic Algorithm				Full Search Algorithm				GAP(%)
				S_0^{heu}	S_i^{heu}	p^{heu}	TP^{heu}	S_0^{opt}	S_i^{opt}	p^{opt}	TP^{opt}	
1	R1	2000	25	5	8	72	37.82067	5	8	73	38.3887	1.479667
	R2				8				8			
	R3				8				8			
	R4				8				8			
	R5				8				8			
2	R1	2000	20	5	7	73	39.53718	5	7	74	40.1375	1.495676
	R2				7				7			
	R3				7				7			
	R4				7				7			
	R5				7				7			
3	R1	1000	20	2	3	76	10.98872	3	3	77	11.13774	1.338009
	R2				3				3			
	R3				3				3			
	R4				3				3			
	R5				3				3			
4	R1	1000	15	2	3	75	12.65575	3	3	76	12.85825	1.5749
	R2				3				3			
	R3				3				3			
	R4				3				3			
	R5				3				3			
AVG												1.472063

$(C = 60,\ L_i = 2,\ h_i = 2,\ h_0 = 1,\ \alpha = 0.1)$

algorithm is effective.

<Table 4> table shows the numerical results on the change of the cost parameter. As shown in <Table 4>, the retail price increases and the order-up-to-level decreases, as the purchasing cost gets larger. The order-up-to- level increases as the retailer shortage cost increases. As seen in <Table 3>, the order-up-to-level decreases and the total profit decreases, as the retailer holding cost increases.

A sensitivity analysis is performed to study the effect of retail price p and order-up-to-level of the retailer S_i on the total profit. As shown in <Figure 6>, the total profit variation is steeper with retail price p than order-up-to-level of the retailer S_i. This implies that retail price has more influence on the variation of total profit than order-up-to-level of retailer.

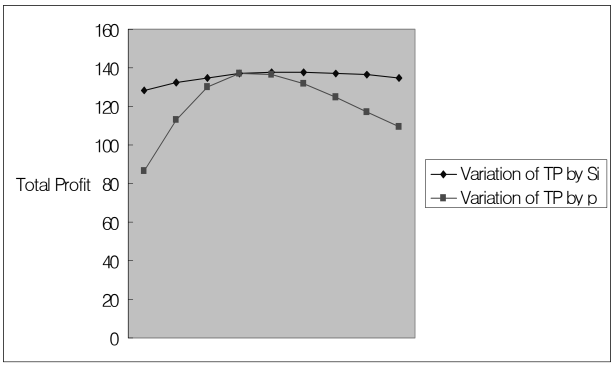

Figure 6. Illustration or the variation of total profit.

Table 4. The numerical results with various cost parameters

C	S_0^{heu}	S_i^{heu}	p^{heu}	TP^{heu}
20	15	68	35	905.258
		67		
30	10	39	41	350.073
		42		
40	5	16	51	121.5992
		17		
50	3	7	60	40.03397
		8		
60	1	3	73	11.92409
		3		
π	S_0^{heu}	S_i^{heu}	p^{heu}	TP^{heu}
10	5	15	51	121.5576
		17		
15	5	16	51	121.5992
		17		
30	5	17	51	120.3384
		17		
h_i	S_0^{heu}	S_i^{heu}	p^{heu}	TP^{heu}
1	5	16	51	121.5992
		17		
2	5	13	52	106.3685
		14		
5	5	11	52	70.39085
		12		

($N=2$, $A_i = 1000,\ 1100$, $L_i = 1$, $\alpha = 0.1$)

Table 3. The numerical results with different retailers case

#		A	L_i	Heuristic Algorithm				Full Search Algorithm				GAP(%)
				S_0^{heu}	S_i^{heu}	p^{heu}	TP^{heu}	S_0^{opt}	S_i^{opt}	p^{opt}	TP^{opt}	
1	R1	1000	1	5	16	51	121.59917	5	17	51	122.14703	0.448522
	R2	1100			17				19			
2	R1	1000	2	5	25	50	118.46084	5	24	51	119.20404	0.623472
	R2	1100			27				26			
3	R1	1000	1	5	16	51	120.02456	5	17	51	120.65188	0.519938
	R2	1100	2		27				26			
4	R1	1000	1	5	16	51	127.6682	5	17	51	128.41855	0.584299
	R2	1200			19				20			
5	R1	1000	2	5	23	51	123.96583	4	25	51	125.37256	1.122039
	R2	1200			27				29			
6	R1	1000	1	5	16	51	125.51272	5	17	51	126.80419	1.018472
	R2	1200	2		18				28			
AVG												0.719457

($C = 40, \pi_i = 15, h_i = 1, h_0 = 0.5, \alpha = 0.1$)

4.3 The Efficiency of the Proposed Algorithm

As shown in <Figure 7>, the optimal value calculation time in the full search highly increases, as the market size increases, while the calculation time of the proposed algorithm shows little increase. This is due to the exponential increase of iterations in the full search, as the demand increases. <Table 5> indicates that the heuristic algorithm takes about 3%~4% off from the full search calculation time. From computational point of view, the proposed algorithm is efficient and simple.

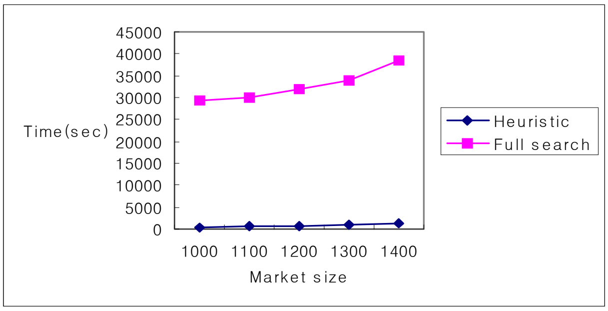

Figure 7. Illustration of the relation between calculation time and market size.

Table 5. Calculation time for two retailers case

INSTANCE	Retailers			
	Same		Different	
	Heuristic	Optimal	Heuristic	Optimal
1	425	29359	509	32545
2	1121	40083	1513	36025
3	822	32541	1241	34521
4	773	32956	590	32356
5	2589	39138	1857	34947
6	1622	36213	1426	33955
AVG	122.333	35048.33	1189.333	34058.17

5. Conclusions

This paper considers a combined model of pricing and (S-1, S) policy for a single-warehouse, multi-retailer inventory system with lost sales allowed. For the model evaluation, the well-known METRIC-approximation is used. Most of the multi-echelon models consider inventory policy only, while this paper integrates retailer pricing and inventory control issues together to maximize long-run total profit in the associated supply chain. The objective function of the integrated model consists of sub-functions of long-run total revenue and total cost (consisting of holding cost and penalty cost). Generally, the (S-1, S) inventory policy is applied to situations where demand loss is not allowed or penalty changes are severe as in military service. Accordingly, concerned with combining the pricing issues and the lost sale issue together, the proposed algorithm may be applied to dealing with expensive products like jewelry or deluxe cars whose demand rates are subject to their price changes.

A heuristic algorithm is derived to search for each approximate retail price. A guideline for the associated search boundary and step size determination is also provided as required in the search.

The effectiveness and efficiency of the proposed algorithm are also tested with some numerical problems in comparison with their full search results. From the computational results, it is found that the performance of the proposed algorithm is quite effective and simple to use.

For further study, it may be interested in the extensions of the proposed model to incorporate some other order policies including (T, Q) or (R, Q) and additional considerations including emergency lateral transshipment and multi-product. Another interesting research issue is to derive any better upper bound of the retail price.

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