횡방향 하중을 받는 I형강 단순보의 비탄성 좌굴거동

Inelastic Buckling Behavior of Simply Supported I-Beam under Transverse Loading

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요 약 : 각각 집중하중과 등분포하중을 받는 단순보의 비탄성 횡-비틀림 좌굴에 대하여 연구하였다. 잔류응력을 단순형과 다항식형으로 하여 line-type 유한요소법으로 해석하였다. 잔류응력의 형태는 플랜지에서는 4차 곡선으로 웨브에서는 2차 곡선으로 가정하였다. 우리나라에서 생산되는 4종류의 I형강에 대하여 비탄성 횡-비틀림 좌굴에 대하여 해석한 후 결과를 강구조편람의 내용과 비교하였다. 해석 결과로부터 강구조편람에 의한 설계는 주보에 보조보가 있는 경우나 없는 경우 모두 전반적으로 과설계임을 알 수 있었다.

ABSTRACT : In this paper, the inelastic buckling behavior of the beam under uniform bending was investigated using the energy-based method, which can tackle problems in fourth order eigenvalue. The pattern of residual stress was not available to satisfy the I-sections manufactured in Korea, however; therefore, the well-known polynomial and simplified pattern of residual stress was adopted in this study. The inelastic lateral-distortional buckling behavior of the beam with I-sections manufactured in Korea was investigated. The study was then extended to the inelastic lateral-torsional buckling of the beam by minimizing the out-of-plane web distortion. The inelastic lateral-torsional buckling results obtained in this paper were compared with the prediction of allowable bending stress given in the Korean steel designers' manual (1995). Results showed that the importance of inelastic lateral-distortional buckling did not arise for beams under uniform bending. Likewise, the design method in KSDM (1995) was proven to be too conservative for intermediate and short spans of beams without intermediate bracing.

핵 심 용 어 : 좌굴, 선요소, 하중높이, 잔류응력
KEYWORDS : buckling, line-element, load-height, residual stress.

1. Introduction

The buckling strength of the hot-rolled and the welded I-sections are strongly influence by the presence of residual stresses. The residual stresses are inherent during the manufacturing of the I-sections. The different pattern of the residual stresses has been observed for the hot-roll (to which this paper restricted) and the welded I-sections.It has been known that the compressive residual stress is formed at the flange tip and the tensile stress at the flange/web junction for the hot-rolled I-sections. The residual stress is depending upon the section geometry but not by the material properties. It appears to be that development of the residual stress model is date back to 1955. Ketter et al (1955) developed a simplified pattern of residual stress that assumed the bilinear distribution of residual stresses in the flange and a constant tensile stress in the web. Beedle and Tall (1960) conducted an experimental study of the residual stress in the hot-rolled I-sections and found that the pattern of the residual

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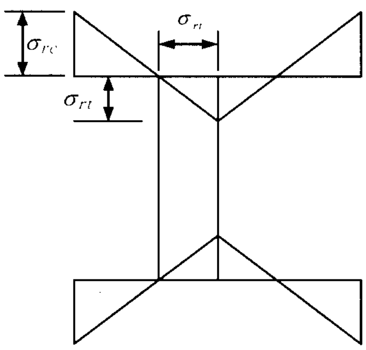
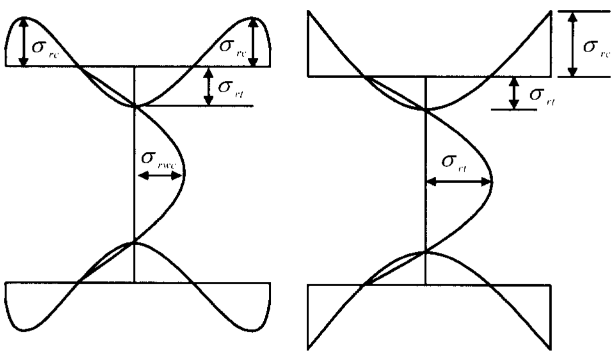
stress in the flange is similar but residual stress in the web is variable. Lee et al (1967) recommended the fourth order polynomial distribution of residual stress in the flange and web but this pattern of residual stress is only suited the compact I-sections. Young (1975) found that North American I-sections are suited the simplified pattern of residual stress whereas Australian and British I-sections have shown the compression residual in the web, furthermore he proposed the approximate formula for maximum stresses at the flange tip and flange/web junction, and the mid height of web. Experimental study conducted by Kitipornchai and Trahair (1975) found that the compressive residual stress is relatively small in the flange tip and high tensile residual stress in the flange/web junction. The development of the residual stress model is discussed in this paper because the pattern of residual stress with the maximum stress at the flange tip and flange/web junction is far more important factors to be considered in buckling analysis.



1 (a) Simplified residual stresses



(b) Slender I-section (c) Compact I-section

Polynomial residual stresses

Figure 1. Model of residual stresses

Figure 1 shows the pattern of residual stresses used in this study. The simplified pattern of stress is shown in Fig. 1(a) with a bilinear distribution in the flange and a constant tensile stress in the web. The maximum stress at the flange tip and flange/web can be determined as

$$\sigma_{rc} = 0.3\sigma_y \tag{1}$$

and

$$\sigma_{rt} = \left[\frac{BT}{BT + t_w(D-T)}\right]\sigma_y \tag{2}$$

where B and T are the flange width and thickness respectively, D is the overall depth of I-section and t_w is thickness of the web.

The other pattern of residual stress used in study shows in Fig. 1 (b) and (c) which assumes the polynomial distribution of residual stress in the flange and the parabolic distribution in the web. This pattern of residual stress in the flange and the web satisfies the slender I-sections (Fig. 1b) and the compact I-sections (Fig. 1c). The quartic distribution of the residual stresses in the flanges assumes

$$\frac{\sigma_{rf}}{\sigma_y} = a_1x^4 + a_2x^2 + a_3 \tag{3}$$

and in the web

$$\frac{\sigma_{rw}}{\sigma_y} = a_4y^2 + a_5 \tag{4}$$

where σ_y is the yield stress of the steel. The maximum residual stress at the flange tip and the flange/web junction is determined according to Bradford and Trahair (1985). The coefficients $a_1, \ldots, a_5$ can be determined from the static equilibrium conditions of axial force and moment about the x and y axis, and axial torque and is given as;

$$\int_A \sigma_r dA = \int_A x\sigma_r dA = \int_A y\sigma_r dA = \int_A \left(x^2 + y^2\right)\sigma_r dA = 0 \qquad (5)$$

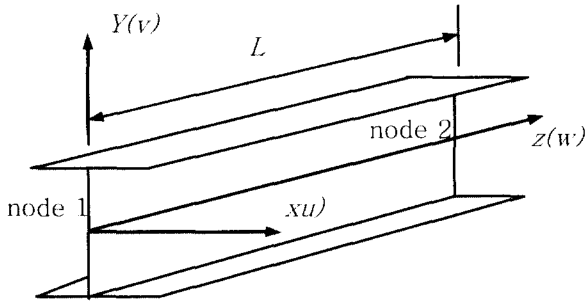
σ_r represents the residual stress and A is the area of cross-section. Equation (5) ensures the static equilibrium conditions of vanishing axial force, moment and torque. The simplified residual stress satisfies the first three static equilibrium conditions but not that due to axial torque equilibrium.

The research on inelastic buckling of simply supported beam under a uniform bending is plentiful. The uniform bending is not true representation of realistic loading condition of I-sections that can be experienced by the steel structures. The buckling analysis of the simply supported beam subjected to transverse loading is more complicated than those of the beam under uniform bending due to the non-uniformity of the beam. Owning to the complexity of analysis, limit number of studies (Kitipornchai and Trahair 1975, Nethercot 1975, and Yoshida and Imoto 1973) has conducted on the inelastic lateral-torsional buckling behavior of simply supported beam under a transverse loading. The inelastic lateral-torsional buckling of beam under a central concentrated load and a uniformly distributed load applied at the shear center and the top flange is studied in this paper. A line-type finite element method is employed to analyze the beam under transverse loading. The method is incorporated with inelasticity and the monosymmetric effects caused by combination of the residual stresses and the applied load. The tangent modulus is adopted in this study, which assumes E_t being equal to the elastic modulus for the elastic regions and the strain hardening modulus for yielded and strain hardened regions of cross-section. Firstly, this study investigates the inelastic lateral-torsional buckling behavior of beam with four different I-sections manufactured in Korea and these results are then compared it with design method in KSDM (1995).

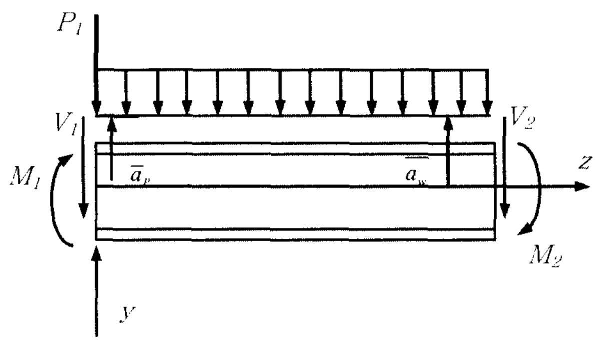
2. Theory

2.1 General

A line-type finite element method developed by Lee (2001) is used in this study to analyze the simply supported beam under a transverse loading. Figure 2a shows the beam element with an arbitrary reference system located at the mid height of the web. The concept of an arbitrary axis system (Bradford and Cuk 1988) is adopted in order to eliminate the complication arises when the shear center and centroid do not coincided due to the yielding of a cross-section.



2(a) Beam element



2(b) Point load and UDL on beam

Figure 2. Beam element and loading

Figure 2(b) and 2(c) show the applied bending moments M_1 and M_2 and shear force V_1 and V_2 at the element nodes. The subscript 1 and 2 is denoted as z=0 and z=L respectively. Since the reference axis system is positioned at the mid height of the web, the height of load is measured from a mid height of the web rather than the shear center of the cross-section. The beam element is subjected to a concentrated load P_1 at node 1 and to a uniformly distributed load w applied at a distance from reference axis is denoted as $\bar{a}_P$, $\bar{a}_w$ respectively. The in-plane analysis is performed in order to establish the applied curvature and elastic and inelastic regions of the cross-section, and the out-of-plane analysis are then performed based on the finite element method that requires the stiffness and stability matrices to be developed. The tangent modulus theory based on dislocation model of yielding (Bradford and Trahair 1985) is used in this study, which is equal to the elastic modulus for the elastic regions and the strain hardening modulus for yielded and strain hardened regions.

2.2 In-plane bending analysis

The structures considered in this study are the simply supported beam subjected to a central concentrated load and to a uniform distributed load, which is statically determinate member. Therefore the applied bending moment and the shear force along the member can be determined easily by employing simple statics;

For a central concentrated load

$$M = \frac{Pz}{2} \qquad 0 \le z \le L/2 \tag{6}$$

and a for uniformly distributed load

$$M = \frac{wLz}{2} - \frac{wz^2}{2} \tag{7}$$

where L is overall length of the beam. The applied curvature and elastic, inelastic regions of the cross-section along the beam can be determine by using non-linear moment-curvature relationship (Lee 2001) with predetermined the distribution of moment in Eqn 6 and 7 for a central concentrated load and a uniformly distributed loads respectively.

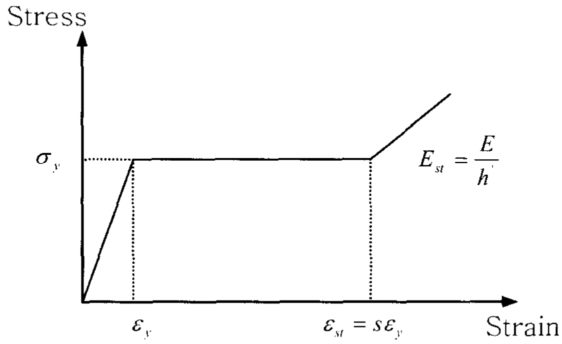


Figure 3. Constitutive relationship

2.3 Out-of-plane bending analysis

Figure 4 shows the buckling displacement of the cross-section. The lateral displacements u_T, u_B and twists ϕ_T, ϕ_B assume as cubic polynomial in z direction. The subscript T and B are denoted as the top and the bottom flange respectively. The out-of-plane buckling deformation of the flange may therefore be written as

$$\{u\} = [M]\{q\} \tag{8}$$

where

$$\{u\} = \langle u_T, u_B, \phi_T, \phi_B \rangle^T \tag{9}$$

$$\{q\} = \langle q_1, q_2, \ldots, q_{16} \rangle^T \tag{10}$$

The interpolation matrix $[M]$ of the flange is given as

$$[M] = \begin{bmatrix} LM_1 & 0 & 0 & 0 \\ 0 & LM_1 & 0 & 0 \\ 0 & 0 & M_1 & 0 \\ 0 & 0 & 0 & M_1 \end{bmatrix} \tag{11}$$

where $M_1 = \left\langle 1, \frac{z}{L}, \frac{z^2}{L^2}, \frac{z^3}{L^3} \right\rangle$

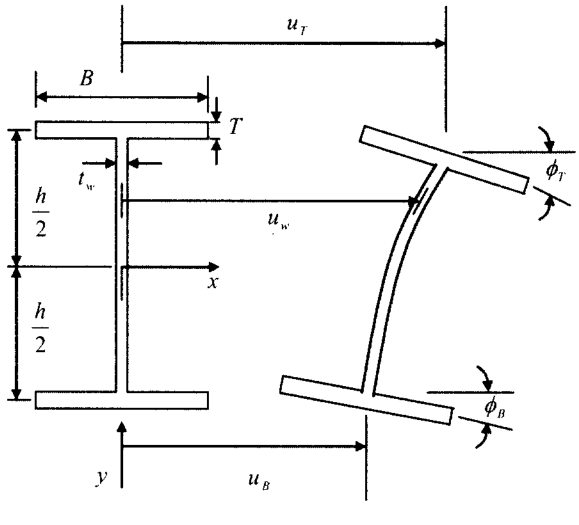


Figure 4. Buckling deformations in the plane of the cross-section

The buckling displacement of element can be express as vector $\{\delta\}$ and is given as

$$\{\delta\}=\begin{Bmatrix} u_{T1},u'_{T1},u_{B1},u'_{B1},\phi_{T1},\phi'_{T1},\phi_{B1},\phi'_{B1}, \\ u_{T2},u'_{T2},u_{B2},u'_{B2},\phi_{T2},\phi'_{T2},\phi_{B2},\phi'_{B2} \end{Bmatrix}^T \quad (12)$$

The subscript 1 and 2 are freedom at z=0 and L respectively, and the prime denoted as differentiation with respect to z. The buckling freedom of element may be written as

$$\{\delta\}=[C]\{q\} \quad (13)$$

So substitute Eqn. 13 into 8, the lateral displacement of flanges may be written as

$$\{u\}=[M][C^{-1}]\{\delta\} \quad (14)$$

where the transformation matrix $[C^{-1}]$ is given by Lee (2001).

The buckling deformation of the web assumes as cubic curve, as was by Bradford and Trahair (1981).

$$u_w=\langle M_w\rangle\{q_w\}f_3(z) \quad (15)$$

where $\{q_w\}=\langle q_{17},q_{18},q_{19},q_{20}\rangle^T$ is polynomial coefficient and $\langle M_w\rangle=h\left\langle 1,\frac{y}{h},\frac{y^2}{h^2},\frac{y^3}{h^3}\right\rangle$ is interpolation matrix of the web respectively, and $f_3(z)$ is a function of z. The polynomial coefficient $q_{17},q_{18},q_{19},q_{20}$ may be determined from geometric compatibility condition between the flange/web junction and is given as

$$u_T=(u_w)_{y=\frac{h}{2}},\ u_B=(u_w)_{y=-\frac{h}{2}}$$
$$\phi_T=-\left(\frac{\partial u_w}{\partial y}\right)_{y=\frac{h}{2}},\ \phi_B=-\left(\frac{\partial u_w}{\partial y}\right)_{y=-\frac{h}{2}} \quad (16)$$

So substitute equation 16 into 8, the flange displacement can be expressed as

$$\{u\}=[C_w]\{q_w\} \quad (17)$$

Hence,

$$u_w=\langle M_w\rangle[C_w]^{-1}\{u\}=\langle N\rangle[C]^{-1}\{\delta\} \quad (18)$$

where the web displacement interpolation matrix $\langle N\rangle=\langle M_w\rangle[C_w]^{-1}[M]$ is given by Lee(2001).

2.4 Strain energy stored during buckling

The strain energy stored in the beam element during buckling is the sum of the strain energy stored in the flange U_F and the strain energy stored in the web U_w, and is expressed as

$$U=U_F+U_w \quad (19)$$

The strain energy stored in the flange is determined from well-known beam theory (Timoshenko and Gere 1961), and the tangent modulus theory is employed to determine the minor axis flexural as was done by Trahair and Kitipornchai (1972). The torsional rigidities are determined according to Booker and

Kitipornchai(1971). Isotropic plate theory (Thimoskenko and Woinowsky-Kriegr 1959) for the elastic regions and orthotropic plate theory based on flow theory (Haaijer 1975, Dawe and Kulak 1984) is used for the web

Therefore, the strain energy stored during buckling of the flange and the web can be expressed as

$$U_F = \frac{1}{2}\int_0^L \langle \varepsilon_F \rangle^T [D_F]\{\varepsilon_F\} dz \qquad (20)$$

$$U_w = \frac{1}{2}\int_0^L \int_{h/2}^{-h/2} \{\varepsilon_w\}^T [D_w]\{\varepsilon_w\} dy dz \qquad (21)$$

where

$$\{\varepsilon_F\} = \left\langle \frac{\partial^2 u_T}{\partial z^2}, \frac{\partial^2 u_B}{\partial z^2}, \frac{\partial \phi_T}{\partial z}, \frac{\partial \phi_B}{\partial z} \right\rangle^T \qquad (22)$$

$$\{\varepsilon_w\} = \left\langle \frac{\partial^2 u_w}{\partial y^2}, \frac{\partial^2 u_w}{\partial z^2}, -2\frac{\partial^2 u_w}{\partial y \partial z} \right\rangle^T \qquad (23)$$

where $\{\varepsilon_F\}$ and $\{\varepsilon_w\}$ is the buckling strain of the flange and the web respectively. The property matrix of the flange and the web is denoted as $[D_F]$ and $[D_w]$ respectively and is given by Lee(2001).

The buckling strain of the flange and the web can be determined after appropriate differentiation of Eqn. 12 for the flange, and Eqn. 18 for the web and can be expressed as matrix format

$$\{\varepsilon_F\} = [B_F]\{q\} \qquad (24)$$

$$\{\varepsilon_{Fw}\} = [B_w]\{q\} \qquad (25)$$

The patterns of residual stress used in this study are the simplified and the polynomial. As noted earlier that the polynomial pattern of residual stresses satisfy the static and torsional equilibrium but the simplified pattern of residual stresses does not satisfied the axial torque equilibrium condition and it is therefore necessary to changes the torsional rigidity of the cross-section to

$$\left((GJ)_t - \int_A \sigma_r (x^2 + y^2) dA \right).$$

The strain energy stored in the flanges and the web can be deduced to in term of stiffness matrix $[k_F]$ and $[k_w]$ as

$$U_F = \frac{1}{2}\{\delta\}^T [k_F]\{\delta\} \qquad (26)$$

$$U_w = \frac{1}{2}\{\delta\}^T [k_w]\{\delta\} \qquad (27)$$

where

$$[k_F] = [C^{-1}]^T \left(\int_0^L [B_F]^T [D_F][B_F] dz \right) [C^{-1}] \qquad (28)$$

$$[k_w] = [C^{-1}]^T \left(\int_0^L \int_{-h/2}^{h/2} [B_w]^T [D_w][B_w] dy dz \right) [C^{-1}] \qquad (29)$$

It should be point out that the stiffness matrices of the flange $[k_F]$ and the web $[k_w]$ are depended on the applied curvature.

The stiffness matrix of the element may be expressed as

$$[k] = [k_F] + [k_w] \qquad (30)$$

2.5 Work done during buckling

The loss of potential energy due to the applied moment and the shear, and off shear center loading can be express as

$$V = V_F + V_w + V_{\bar{a}} \qquad (31)$$

The loss of potential energy in the flanges and the web is given as

$$V_F = \frac{1}{2}\int_{A_F}\sigma_F\int_0^L\left\{\begin{aligned}&\left(\frac{\partial u_T}{\partial z}\right)^2+\left(\frac{\partial u_B}{\partial z}\right)^2+\\&x^2\left(\frac{\partial \phi_T}{\partial z}\right)^2+x^2\left(\frac{\partial \phi_B}{\partial z}\right)^2\end{aligned}\right\}dzdA_F \quad (32)$$

$$V_w = \frac{1}{2}\int_{A_w}\int_0^L\begin{Bmatrix}\frac{\partial u_W}{\partial z}\\ \frac{\partial u_W}{\partial y}\end{Bmatrix}^T\begin{bmatrix}\sigma_w & \tau\\ \tau & 0\end{bmatrix}\begin{Bmatrix}\frac{\partial u_W}{\partial z}\\ \frac{\partial u_W}{\partial y}\end{Bmatrix}dzdA_w \quad (33)$$

where A_F and A_w is the cross-sectional area of the flange and the web respectively. σ_F and σ_w is the stress in the flange and the web respectively, and the shear stress is denoted as τ which can be calculated from thin-walled expression (Trahair and Bradford 1998)

The loss of potential energy in the flanges and the web can be deduced to in term of stability matrix after appropriate differentiation and may be expresses as matrix format

$$V_F = \frac{1}{2}\{\delta\}^T[g_F]\{\delta\} \quad (34)$$

$$V_w = \frac{1}{2}\{\delta\}^T[g_w]\{\delta\} \quad (35)$$

where

$$[g_F] = [C^{-1}]^T\left(\int_A\int_0^L[s_F]^T\sigma[s_F]dzdA_F\right)[C^{-1}] \quad (36)$$

$$[g_w] = [C^{-1}]^T\left(\int_A\int_0^L[s_w]^T\begin{bmatrix}\sigma & \tau\\ \tau & 0\end{bmatrix}\sigma[s_w]dzdA_w\right)[C^{-1}] \quad (37)$$

Again, the stability matrices of the flange and the web $[g_F]$ and $[g_w]$ are depend on the applied curvature.

The buckling resistance of the beam is decreased as the height of load is increased from the reference axis and is a well-known destabilizing phenomenon (Bradford and Cuk (1988), Trahair (1993)). When the load is applied within the web, the buckling resistance is decreased by $\frac{1}{2}\int_0^{\bar{a}_P}\left(\frac{\partial u_w}{\partial y}\right)^2_{z=0} dy$ due to the web flexure. When load is applied above the top flange, the twist of the flange is increased by $\phi_{T1}\left(\bar{a}_P - \frac{h}{2}\right)$. Therefore, the loss of the potential energy due to the load height of central concentrated load P_1 at node 1 (z=0) acts at a distance $\bar{a}_P$ above reference axis can be express as

$$V_{\bar{a}_P} = \frac{P_1}{2}\int_0^{\bar{a}_P}\left(\frac{\partial u_w}{\partial y}\right)^2_{z=0} dy + P_1\phi_{T1}\left(\bar{a}_P - \frac{h}{2}\right) \quad (38)$$

Similarly, the loss of potential energy due to load height of uniformly distributed load w applied $\bar{a}_w$ above the reference axis can be express as

$$V_{\bar{a}_w} = \frac{P_1}{2}\int_0^L\int_0^{\bar{a}_w}\left(\frac{\partial u_w}{\partial y}\right)^2 dydz + w\left(\bar{a}_w - \frac{h}{2}\right)\int_0^L\phi_T dz \quad (39)$$

The integrals with respect to y in equation 37 and 38 may be expresses using

$$\frac{\partial u_w}{\partial y} = [L][C^{-1}]\{\delta\} \quad (40)$$

The loss of potential energy due to load height may be expresses as

$$V_a = \frac{1}{2}\{\delta\}^T[g_{\bar{a}}]\{\delta\} \quad (41)$$

where for a central concentrated load

$$[g_{\bar{a}}] = \{\delta\}^T[C^{-1}]^T\left(\int_0^{\bar{a}_P}[L]^T P_1[L]dy\right)[C^{-1}]\{\delta\} \quad (42)$$

for a uniformly distributed load

$$[g_{\bar{a}}]=\{\delta\}^T[C^{-1}]^T\left(\int_0^L\int_0^{a_w}[L]^T W[L]dydz\right)[C^{-1}]\{\delta\} \qquad (43)$$

Finally, the loss of total potential energy in element can be expressed as

$$V=\frac{1}{2}\{\delta\}^T[g]\{\delta\} \qquad (44)$$

where $[g]=[g_F]+[g_w]+[g_{\bar{a}}]$

2.6 Buckling solution

The element stiffness matrices $[k_F]$, $[k_w]$ and the element stability matrices $[g_F]$, $[g_w]$ and $[g_{\bar{a}}]$ may be assembled into the global stiffness matrix $[K]$ and the global stability matrix $[G]$ respectively. The buckling solution of the beam can be express as

$$([K(\lambda)]-[S(\lambda)])\{\Delta\}=[A(\lambda)]\{\Delta\}=\{0\} \qquad (45)$$

where $\{\Delta\}$ is the vector of buckling deformations (eigenvector). The detemination of buckling solution is more complicated than those of elastic buckling analysis due to the non-linearity of the cross-section. Since the stiffness and stability matrix is function of the applied curvature, the value of applied curvature is adjusted until equation 44 vanishes. The appropriate iterative method employed in this method is the method of bisection.

3. Accuracy of solution

The current method to predict the inelastic buckling load is compared with experimental and independent theoretical results. These results were inelasticlateral -torsional buckling. The finite element method presented in this paper is lateral-distortional buckling and therefore it is necessary to suppress the strain energy due to out-of-plane plate flexure of the web then the lateral-distortional buckling mode becomes lateral-torsional one as was done by Bradford and Trahair (1982) and is given as

$$U_{wp}=\frac{1}{2}\gamma_r\int_0^L\int_{-\frac{h}{2}}^{\frac{h}{2}}D_w\left(\frac{\partial^2 U_w}{\partial y^2}\right)dydz \qquad (46)$$

where D_w is plate rigidity factor and γ_r is set to a large value (say 106)

Kitipornchai and Trahair (1975)conducted the experimental study of simply supported beam subjected to a central concentrated load applied above the top flange (8.64 inches (219.456mm) from mid height of the web). The material property and geometry of the cross-section are same as their experimental data. The number of element used in this comparison is 8 elements. Table 1 shows the comparison between inelastic lateral-torsional buckling results obtained by independent studies and the inelastic lateral-torsional results obtained from this study.

Table 1. Inelastic lateral-torsional buckling of simply supported beam

	8ft	10ft	12ft
Experimental Results (Kitiporncahi and Trahair 1975)	1267	1248	1174
Finite Integral Method (Kitiporchai and Trahair 1975)	1366	1311	1252
Transfer Matrix (Yoshida and Imoto 1973)	1344	1317	1170
Finite Element Method (Nethercot 1975)	1367	1350	1217
This Study	1378	1317	1215

Table 2. Inelastic lateral-torsional buckling of simply supported beam

	8ft	10ft	12ft
Experimental Results (Kitiporncahi and Trahair 1975)	1267	1248	1174
This Study	1320	1264	1117

It can be seen in Table 1 that the current method to predict the inelastic lateral-torsional buckling solution are agreed very well with independent studies. It can also be observed from table 1 that the inelastic lateral-torsional buckling solutions are slightly higher than those of the experimental results. This study is then extended to the inelastic lateral-distortional buckling by allowing the web distortion and these results are compared it with experimental results. It can be seen in table 2 that the inelastic lateral-distortional buckling results agreed very well with experimental results than those of inelastic lateral-torsional buckling results. This is due to the twist of the flange, which is accentuated as the load height is increased. But when the load is applied within the web the effects of web distortion is very small. Since the load height considered in this study is the shear center and the top flange, thus this study has not been considered the effects of web distortion.

4. Inelastic lateral-torsional buckling of beam

The inelastic lateral-torsional buckling of simply supported beam under a central concentrated load and a uniformly distributed load is considered in this section. Since the accurate model of residual stresses has not been available to suit the I-sections manufactured in Korea, the "well-known" simplified model and polynomial residual stresses is used in this study. The cross-sections used in this study are 200×150, 300×175, 400×400 and 800×300 that represent the I-sections manufactured in Korea. The material property is dopted in this study is E(elastic modulus) = 2.1×10^6kg/cm^2 (205.926×10^3MPa (g=9.806m/sec^2)), σ_y (yield stress)=2400kg/cm^2 (235 MPa(g=9.806m/sec^2)), ν (poisson's ratio)= 0.3, $\varepsilon st = 10\,\varepsilon_y$, and E/E_{st} = 40.

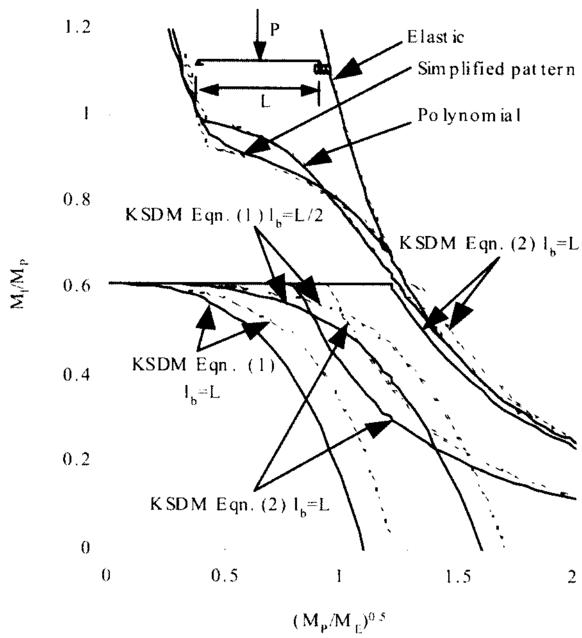


Figure 5. Inelastic buckling of simply supported beam 200×150 under a central concentrated load

The inelastic lateral-torsional buckling results for the simply supported beams under a central concentrated load applied at the shear center and the top flange is shown in Figs. 5 to 8, while Figs. 9 to 12 are for a uniformly distributed load applied at the shear center and the top flange. The dash and dot lines in the figures indicate the load applied at the shear center and the top flange respectively.

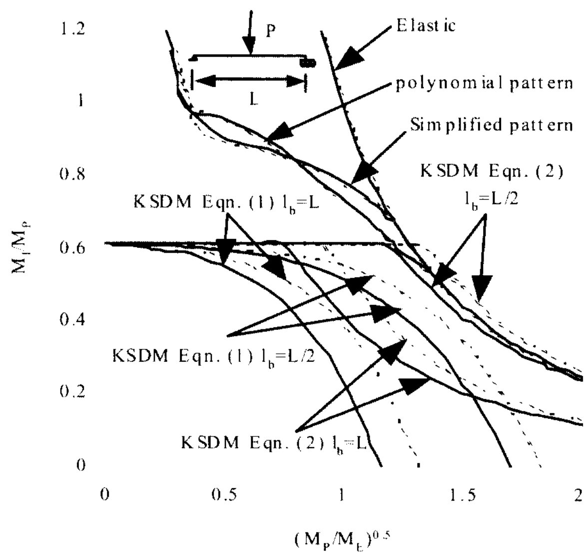


Figure 6. Inelastic buckling of simply supported beam 300×175 under a central concentrated load

The inelastic lateral-torsional buckling moment M_I is non-dimensionlized with respect to the plastic moment M_P, while dimensionless slenderness $\sqrt{M_P/M_E}$ is used, where M_E is the elastic lateral buckling load which is obtained with current method by using very high value of yield stress f_y and neglecting the residual stress. The bending stress curved derived from KSDM (1995) is plotted in the figures. These curves are derived from

$$\sigma_b = \left[1 - 0.4\frac{\left(\frac{l_b}{i_b}\right)^2}{C\lambda_p^2} \right]\frac{\sigma_y}{1.5} \le \frac{\sigma_y}{1.5} \qquad (47)$$

$$\sigma_b = \frac{900}{\frac{l_b h}{A_f}} \le \frac{\sigma_y}{1.5} \qquad (48)$$

where l_b is the distance between compressive flanges

$$C = 1.75 - 1.05\left(\frac{M_1}{M_2}\right) + 0.3\left(\frac{M_1}{M_2}\right)^2 \le 2.3, \qquad M_1 > M_2$$

$$i_b = \sqrt{\frac{I_f}{A_f + \frac{1}{6}A_w}}, \qquad \lambda_p = \sqrt{\frac{\pi^2 E}{0.6\sigma_y}}$$

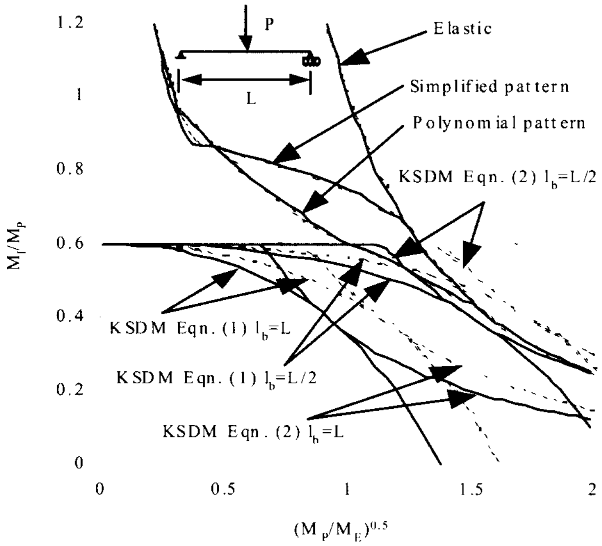


Figure 7. Inelastic buckling of simply supported beam 400×400 under a central concentrated load

KSDM Eq. (1) and (2) in the figures is referred equation 46 and 47 respectively. KSDM (1995) provides two different equations to determine the allowable bending stress. It should be point out that Eqns. 46 and 47 are depended on the geometric property of the cross-section, but not that with the restraint condition at the support and the effect of load height has not been included. Furthermore, Eqn. 47 has not been allowed for loading conditions whereas Eqn. 46 used the value of C to allow for the different types of loading conditions. The use of this value is very restricted, and furthermore the value of C does not distinguish between a concentrated load and a uniformly distributed load. The type of loading considered in this study is a central concentrated and a uniformly distributed, which produces the positive (sagging) moment and therefore the value of C is equal to 1. At a given I-section, a single design curve can be derived from design method in KSDM (1995) regardless of restraint condition at the support and height of load, and different types of loading. The length l_b is play important role to increase the allowable bending stress.

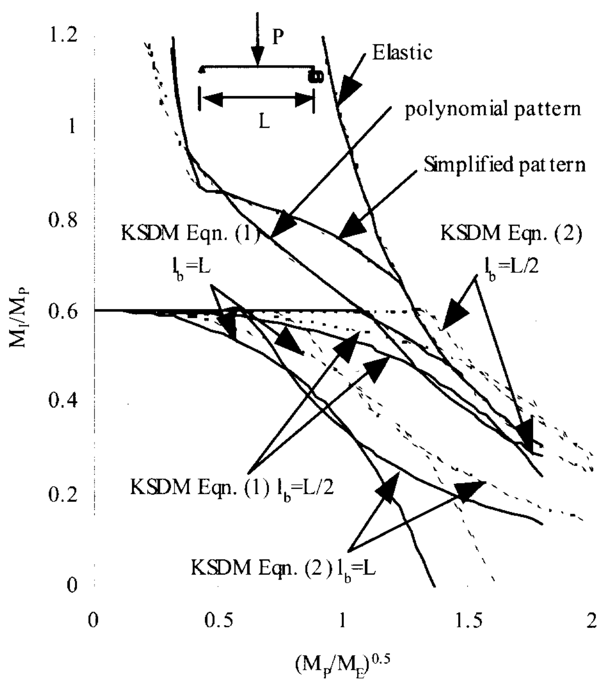


Figure 8. Inelastic buckling of simply supported beam 800×300 under a central concentrated load

The effective length l_b can be increase by introducing intermediate bracing (number of sub-beams can be attached to the main-beam) but the bending stress cannot be exceeded $\sigma_y/1.5$. Therefore this study is adopted two different effective length l_b. Firstly, this study considered the effective length is span length (without intermediate bracing) of beam and the effective length is then halved by assuming that beam is restraint at mid-span of beam.

Generally speaking, the prediction of KSDM (1995) is very conservative without and with intermediate bracing at mid-span of the beam except when the load is applied at the top flange with intermediate bracing at mid-span of beam. The finite element method presented in this paper is unrestrained beam and therefore it is not relevant to compared results with the braced beam but the brace beam at the mid span considered in this comparison study is to examine the conservativeness of design method.

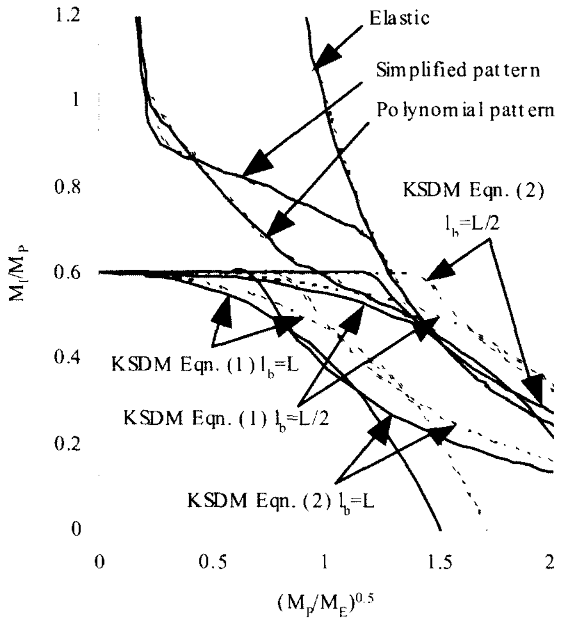


Figure 10. Inelastic buckling of simply supported beam 300×175 under a uniformly distributed load

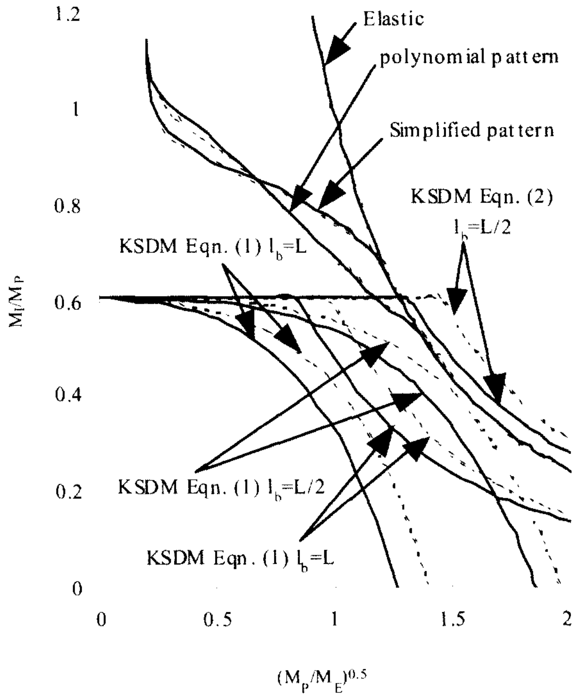


Figure 9. Inelastic buckling of simply supported beam 200×150 under a uniformly distributed loa d

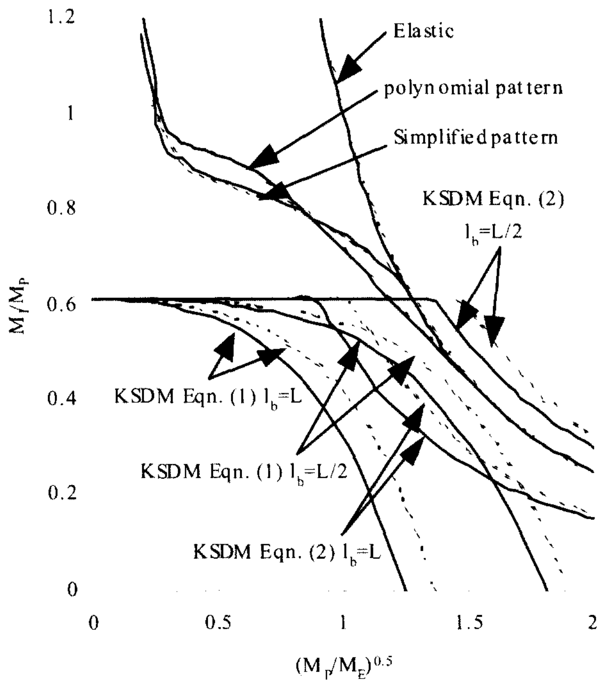


Figure 11. Inelastic buckling of simply supported beam 400×400 under a uniformly distributed load

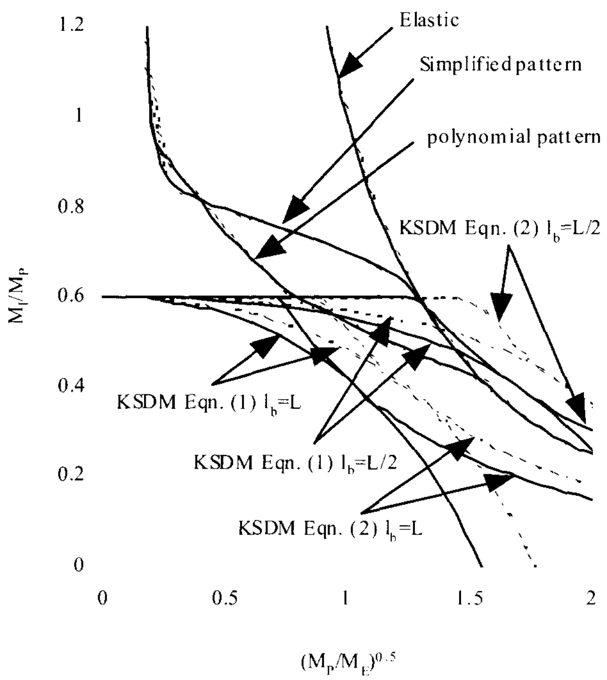


Figure 12. Inelastic buckling of simply supported beam 800×300 under a uniformly distributed load

The discrepancy between the design method and buckling results obtained for inelastic lateral-torsional buckling is increased as the length of beam is decreased. As would be expected that onset of the yielding of the cross-section occurs at lower dimensionless length for the polynomial pattern residual stress then the simplified pattern. This is due to the facts that the maximum residual stress at the flange tip and flange-web junction is much higher for the polynomial residual stress than those of the simplified pattern. The different types of pattern of residual stresses have little affect on the inelastic lateral-torsional buckling behavior of the beam as the dimensionless length of the beam $\sqrt{M_P/M_E}$ is decreased.

5. Conclusion

The inelastic buckling behavior of the beam subjected to a central concentrated load and to a uniformly distributed load is investigated by employing the finite element method with the two conceptually different patterns residual stress. The patterns of residual stress used in this study are the polynomial and the simplified pattern. The accuracy of the method is demonstrated with experimental and independent studies. When load is applied above the top flange the buckling mode of beam become a lateral-distortional one due to the twist of the flange, which is increased as the load height is increased. The inelastic lateral-torsional buckling results obtained from this study are compared with the bending stress obtained from KSDM (1995). As would expected that there is disparity between the bending stress obtained from KSDM (1995) and the inelastic lateral-torsional buckling moments obtained from this study which has not accounts for geometric imperfection and the considered the only bifurcation buckling. Generally, the design method is conservative without and with intermediate bracing at mid-span of beam. This study have used the residual stresses model that suited North American, Britain and Australia but the I-sections manufactured in Korea might be different to western country. Therefore, it is necessary to develop the pattern of residual stresses that suited the I-sections manufactured in Korea in order to estimate the buckling capacity of the I-sections that manufactured in Korea.

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