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CONTROLLABILITY OF NONLINEAR DELAY INTEGRODIFFERENTIAL EQUATION WITH NONLOCAL CONDITIONS IN BANACH SPACES

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Abstract. In this paper, sufficient conditions for the controllability of integrodifferential systems are established. The results are obtained by using the Schauder fixed point theorem and the resolvent operator. Example is provided to illustrate the theory.

I. Introduction

Byszewski([3]) has studied the existence and uniqueness of mild, strong and classical solutions of the following nonlocal Cauchy problem :

$$\begin{cases} \dfrac{du(t)}{dt} & +Au(t) = f(t, u(t)), \quad t \in [0, a] \\ u(t_0) & +g(t_1, t_2, \cdots, t_p, u(\cdot)) = u_0 \end{cases}$$

where $0 \le t_0 < t_1 < \cdots, t_p \le a$, $-A$ is the infinitesimal generator of a C_0-semigroup in a Banach space X, $u_0 \in X$ and $f : [0, a] \times X \to X$, $g : [0, a]^p \times X \to X$ are given functions. Subsequently, he has investigated the same type of problem for different type of evolution equations in Banach spaces ([4,5]). Ntouyas and Tsamatos ([11]) has established the global existence of semilinear evolution equations with nonlocal conditions. Balachandran and Ilamaran ([2]), Dauer and Balachandran

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([1]) have studied the nonlocal Cauchy problem for various classes of integrodifferential equations. In Desh ([7]), he consider the following integrodifferential equation

$$\frac{dx(t)}{dt} = A[x(t) + \int_0^t F(t-s)x(s)]ds + f(t, x(t)), t \geq 0$$
$$x(0) = x_0.$$

These type of equation also occurs in the study of viscoelastic beams and thermo-viscoelasticity.

In this paper we consider the following integrodifferential equation with nonlocal condition of the form :

(E)

$$\begin{cases} \dfrac{dx(t)}{dt} & = A[x(t) + \displaystyle\int_0^t F(t-s)x(s)ds] + Bu(t) \\ & +f(t, x_t) + \displaystyle\int_0^t g(t, s, x_s, \int_0^s k(s, \tau, x_\tau)d\tau)ds, \quad t \in [0, T] = J \\ x(t) & +h(x_{t_1}, x_{t_2}, \cdots, x_{t_p})(t) = \phi(t), \ t \in [-r, 0] \end{cases}$$

where A generates a strongly continuous semigroup in a Banach space X, $F(t)$ is a bounded operator for $t \in J$

$$f : J \times C([-r, 0] : X) \to X,$$
$$g : J \times J \times C([-r, 0] : X) \times X \to X$$
$$k : J \times J \times C([-r, 0] : X) \to X$$
$$h : C([-r, 0] : X)^p \to X$$

Further sufficient conditions for the controllability of integrodifferential systems are established. The results are obtained by using the Schauder fixed point theorem and the resolvent operator. Example is provided to illustrate the theory.

II. Controllability result

In this paper, we consider the following integrodifferential equation with nonlocal condition of the form :

$$(2.1)\quad \begin{cases} \dfrac{dx(t)}{dt} & = A[x(t) + \displaystyle\int_0^t F(t-s)x(s)ds] + f(t, x_t) \\ & \quad + \displaystyle\int_0^t g(t, s, x_s, \int_0^s k(s, \tau, x_\tau)d\tau)ds, \quad t \in [0, T] = J \\ x(t) & + h(x_{t_1}, x_{t_2}, \cdots, x_{t_p})(t) = \phi(t) \end{cases}$$

where A generates a strongly continuous semigroup in a Banach space X, $F(t)$ is a bounded operator for $t \in J$,

$$f : J \times C([-r, 0] : X) \to X,$$

$$g : J \times J \times C([-r, 0] : X) \times X \to X,$$

$$k : J \times J \times C([-r, 0] : X) \to X,$$

$$h : C([-r, 0] : X)^p \to X$$

are given functions. $F(t) : Y \to Y$ and for $x(\cdot)$ continuous in Y, $AF(\cdot)x(\cdot) \in L^1(J, X)$. Y is the Banach space formed from $D(A)$, the domain of A, endowed with the graph norm. For $x \in X$, $F'(t)x$ is continuous in $t \in J$. Then there exists a unique resolvent operator for the equation.

$$(2.2)\quad \frac{dx(t)}{dt} = A[x(t) + \int_0^t F(t-s)x(s)ds]$$

The resolvent operator $R(t) \in B(X)$ for $t \in J$ satisfies the following conditions ([8]) :

(a) $R(0) = I$ (the identity operator on X),

(b) for all $x \in X$, $R(t)x$ is continuous for $t \in J$,

(c) $R(t) \in B(Y)$, $t \in J$. For $y \in Y$, $R(t)y \in C^1([0, T], X) \cap C([0, T], X)$ and

$$(2.3)\quad \begin{aligned} \frac{d}{dt}R(t)y &= AR(t)y + \int_0^t F(t-s)R(s)yds \\ &= R(t)Ay + \int_0^t R(t-s)F(s)yds, \quad t \in J. \end{aligned}$$

In Grimmer ([8]), we have studied the existence and uniqueness of solutions via variation of constants formula and other properties of resolvent operators. In this paper we shall study the existence of mild solution and strong solution of the integrodifferential equation by utilizing the techniques developed by Pazy ([12]) and Byszewski ([3]).

Let $Y = C(J, X)$ and define the sets $X_r = \{x \in X : ||x|| \le r\}$ and $Y_r = \{y \in Y : ||y|| \le r\}$ where the constant r is defined below. Assume the following conditions :

(H1) The resolvent operator $R(t)$ is compact and there exists a constant $M_1 > 0$ such that $||R(t)|| \le M_1$.

(H2) Nonlinear operator $f : J \times C([-r, 0] : X) \to X$, $k : J \times J \times C([-r, 0] : X) \to X$ are continuous and there exist constants $M_2 > 0$, $M_3 > 0$ such that

$$||f(t, x_t)|| \le M_2, \quad t \in J, \quad x_t \in X_r,$$

$$||g(t, s, x_s, y(s))|| \le M_3, \quad (t, s) \in J \times J, \quad x_s,\ y \in X_r,$$

$$||k(t, s, x_s)|| \le N, \quad (t, s) \in J \times J, \quad x_s \in X_r.$$

(H3) $h : C([-r, 0] : X)^p \to X$ is continuous and there exist a constant $H > 0$ such that

$$||h(x_{t_1}, \cdots, x_{t_p})(t)|| \le H, \quad x_{t_i} \in Y_r \ (i = 1, \cdots, p)$$

and

$$\begin{aligned} h(\alpha x_{t_1} &+ (1-\alpha) y_{t_1}, \cdots, \alpha x_{t_p} + (1-\alpha) y_{t_p})(t) \\ &= \alpha h(x_{t_1}, \cdots, x_{t_p})(t) + (1-\alpha) h(y_{t_1}, \cdots, y_{t_p})(t), \\ &\qquad\qquad x_{t_i}, y_{t_i} \in Y_r \ (i = 1, \cdots, p). \end{aligned}$$

(H4) The set $\{y(0) : y \in Y_r,\ y(0) = x_0 - h(y_{t_1}, \cdots, y_{t_p})(0)\}$ is precompact in X.

To simplify the notation, let us take

$$Q(t) = \int_0^t k(t, s, x_s) ds.$$

Definition 2.1. A continuous solution $x(t)$ of the integral equation

$$x(t) = x_t(0) = R(t)[\phi(0) - h(x_{t_1}, \cdots, x_{t_p})(0)] + \int_0^t R(t-s)f(s, x_s)ds + \int_0^t R(t-s)\int_0^s g(s, \tau, x_\tau, Q(\tau))d\tau ds$$

is called a mild solution of the problem (2.1).

We will study a new type of controllability problem for integrodifferential systems in Banach spaces. With the help of fixed point argument several authors have investigated the problem of controllability of nonlinesr systems in Banach spaces ([1,7,11,12]). In particular, the Schauder fixed point theorem is used to study the controllability of Volterra systems in [8,9]. Now we shall establish a set of sufficient conditions for the controllability of semilinesr integrodifferential system with nonlocal condition. Consider the semilinear integrodifferential system with control parameter as

$$(2.4)\quad \begin{cases} \dfrac{dx}{dt} &= A[x(t) + \displaystyle\int_0^t F(t-s)x(s)ds] + Bu(t) \\ & \qquad + f(t, x_t) + \displaystyle\int_0^t g(t, s, x_s, Q(s))ds, \\ x(t) & + h(x_{t_1}, \cdots, x_{t_p})(t) = \phi(t), \quad t \in [-r, 0] \end{cases}$$

where the state $x(\cdot)$ takes values in the Banach space X and the control function $u(\cdot)$ is given in $L^2(J, U)$, a Banach space of admissible control functions with U as a Banach space. Here B is a bounded linear operator from U into X. Then for the system (2.4),

$$(2.5)\quad \begin{aligned} x(t) = x_t(0) &= R(t)[\phi(0) - h(x_{t_1}, \cdots, x_{t_p})(0)] \\ &+ \int_0^t R(t-s)[Bu(s) + f(s, x_s) + \int_0^s g(s, \tau, x_\tau, Q(\tau))d\tau]ds \end{aligned}$$

where the resolvent operator $R(t) \in B(X)$ for $t \in J$ and functions f, g, k and h satisfy the conditions stated in the assumptions (H2), (H.3).

Definition 2.2. The system (4.1) is said to be controllable with nonlocal condition on the interval J, if for every $\phi(0)$, $x^1 \in X$, there exists a control $u \in L^2(J, U)$ such that the mild solution $x(\cdot)$ of (4.1) satisfies

$$x(0) + h(x_{t_1}, \cdots, x_{t_p})(0) = \phi(0)$$

and

$$x_T = x^1$$

where x^1 is the target. To establish the result we need the following additional hypothesis.

(H5) The linear operator W from U into X, defined by

$$Wu = \int_0^T R(T-s)Bu(s)ds$$

has an invertible operator W^{-1} defined on $L^2([0,T] : U)/kerW$, and there exists a constant $M_6 > 0$ such that

$$||BW^{-1}|| \leq M_6.$$

Theorem 2.1. If the hypothesis $(H1) \sim (H5)$ are satisfied, then the system (2.4) is controllable on J.

Proof. Using the hypothesis $H5$, for an arbitrary function $x(\cdot)$, define the control term

$$\begin{aligned} u(t) &= W^{-1}[x^1 - R(T)(\phi(0) - h(x_{t_1}, \cdots, x_{t_p})(0)) \\ &\quad - \int_0^T R(T-s)(f(s, x_s) + \int_0^s g(s, \tau, x_\tau, Q(\tau))d\tau)ds](t) \end{aligned}$$

The control term is substituted into (2.4) in the case $t = T$, we have

$$x(T) = x^1.$$

Now we shall show that, when using this control, the operator defined by

$$(\Phi x_t)(0) = (\Phi x)(t) = R(t)[\phi(0) - h(x_{t_1}, \cdots, x_{t_p})(0)]$$
$$+ \int_0^t R(t-\eta)BW^{-1}[x^1 - R(T)(\phi(0) - h(x_{t_1}, \cdots, x_{t_p})(0))$$
$$- \int_0^T R(T-s)(f(s,x_s) + \int_0^s g(s,\tau,x_\tau,Q(\tau))d\tau)ds](\eta)d\eta$$
$$+ \int_0^t R(t-s)(f(s,x_s) + \int_0^s g(s,\tau,x_\tau,Q(\tau))d\tau)ds$$

has a fixed point. This fixed point is then a solution of equation (4.1).

Clearly, $(\Phi x)_T(0) = (\Phi x)(T) = x^1$, which means that the control u steers the semilinear integrodifferential system from the initial state $\phi(0) - h(x_{t_1}, \cdots, x_{t_p})(0)$ to x^1 in time T, provided we can obtain a fixed point of the nonlinear operator Φ. Let

$$Z_0 = \{x_t \in C([-r,0]:Y) |\ x(0) + h(x_{t_1}, \cdots, x_{t_p})(0)$$
$$= \phi(0),\ ||x_t||_C \leq r,\ t \in J\}$$

where the positive constant r is given by

$$r = M_1(||\phi||_C + H)(1 + TM_1M_6) + TM_1(||x^1||M_6 + (1 + M_1M_6T)(M_2 + M_3\frac{T}{2}))$$

Then Z_0 is clearly a bounded, closed and convex subset of $C([-r,0]:Y)$. For $x_t \in Z_0$, we obtain

$$||(\Phi x_t)(\theta)|| = ||(\Phi x)(t+\theta)|| = ||R(t+\theta)[\phi(0) - h(x_{t_1}, \cdots, x_{t_p})(0)]$$
$$+ \int_0^{t+\theta} R(t+\theta-\eta)BW^{-1}[x^1 - R(T)(\phi(0) - h(x_{t_1}, \cdots, x_{t_p})(0))$$
$$- \int_0^T R(T-s)(f(s,x_s) + \int_0^s g(s,\tau,x_\tau,Q(\tau))d\tau)ds](\eta)d\eta$$
$$+ \int_0^{t+\theta} R(t+\theta-s)(f(s,x_s) + \int_0^s g(s,\tau,x_\tau,Q(\tau))d\tau)ds||$$
$$\leq M_1(||\phi||_C + H) + M_1M_6[||x^1|| + M_1(||\phi||_C + H)^2$$
$$+ M_1M_2T + M_1M_3\frac{T^2}{2}](t+\theta) + M_1M_2(t+\theta) + M_1M_3\frac{1}{2}(t+\theta)^2.$$

Therefore, we have

$$\begin{aligned}
||(\Phi x_t)||_C &= \sup_{-r\le\theta\le 0} ||(\Phi x)(t+\theta)|| \\
&\le M_1(||\phi||_C + H) + TM_1M_6[||x^1||_C + M_1(||\phi||_C + H) \\
&+ M_1M_2T + M_1M_3\frac{T^2}{2}] + M_1M_2T + M_1M_3\frac{1}{2}T^2 \\
&= M_1(||\phi||_C + H)(1 + TM_1M_6) \\
&+ TM_1(||x^1||_C M_6 + (M_1M_6T + 1)(M_2 + M_3\frac{T}{2})) \\
&= r
\end{aligned}$$

It follows that Φ is continuous and maps Z_0 into itself. Moreover, Φ maps Z_0 into precompact subset of Z_0. To prove this, we first show that for every fixed $t \in [-r, T]$, the set

$$Z_0(t) = \{(\Phi x_t) : x_t \in Z_0\}$$

is precompact in X. This is clear for $t = 0$, since $Z_0(0)$ is precompact by assumption $(H4)$. Let $t > 0$ be fixed and $0 < \epsilon < t$, define

$$\begin{aligned}
(\Phi_\epsilon x_t)(0) = R(t)[\phi(0) - h(x_{t_1}, \cdots, x_{t_p})(0)] \\
+ \int_0^{t-\epsilon} R(t-\eta)BW^{-1}[x^1 - R(T)(\phi(0) - h(x_{t_1}, \cdots, x_{t_p})(0)) \\
- \int_0^T R(T-s)(f(s, x_s) + \int_0^s g(s, \tau, x_\tau, Q(\tau))d\tau)ds](\eta)d\eta \\
+ \int_0^{t-\epsilon} R(t-s)(f(s, x_s) + \int_0^s g(s, \tau, x_\tau, Q(\tau))d\tau)ds
\end{aligned}$$

Since $R(t)$ is compact for every $t > 0$, the set

$$Z_\epsilon(t) = \{(\Phi_\epsilon x_t) | x_t \in Z_0\}$$

is precompact on $c([-r,T]:X)$ for every ϵ, $0<\epsilon<t$. Furthermore for $x_t \in Z_0$, we have

$$\begin{aligned}
&||(\Phi x)(t+\theta)-(\Phi_\epsilon x)(t+\theta)|| \\
&\leq \epsilon M_1 M_6(||x^1||+M_1(||\phi||+H)+M_1M_2T+M_1M_3\frac{T^2}{2}) \\
&\quad+\epsilon M_1(M_2+M_3\frac{\epsilon}{2})
\end{aligned}$$

and

$$\begin{aligned}
&||(\Phi x)-(\Phi_\epsilon x_t)||_C \\
&\quad= sup_{-r\leq\theta\leq 0}||(\Phi x)(t+\theta)-(\Phi_\epsilon x)(t+\theta)|| \\
&\quad\leq \epsilon M_1 M_6(||x^1||+M_1(||\phi||_C+H)+M_1M_2T \\
&\qquad+M_1M_3\frac{T^2}{2})+\epsilon M_1(M_2+M_3\frac{\epsilon}{2}).
\end{aligned}$$

Which implies that $Z_0(t)$ is totally bounded, that is, precompact in X. We want show that

$$\Phi(Z_0)=\{\Phi x_t | x_t\in Z_0\}$$

is an equicontinuous family of functions. For that, let $t_2>t_1>0$. Then we have

$$\begin{aligned}
&||(\Phi x)(t_1+\theta)-(\Phi x)(t_2+\theta)|| \leq ||R(t_1+\theta)-R(t_2+\theta)||(||\phi||_C+H) \\
&+\int_0^{t_1+\theta}||R(t_1+\theta-\eta)-R(t_2+\theta-\eta)||M_6(||x^1||-M_1(||\phi||_C+H) \\
&+M_1T(M_2+M_3\frac{T}{2}))d\eta \\
&+|t_2-T_1|M_1M_6(||x^1||+M_1(||\phi||_C+H)+M_1T(M_2+M_3\frac{T}{2})) \\
&+\int_0^{t_1+\theta}||R(t_1+\theta-s)-R(t_2+\theta-s)||(M_2+M_3 s)ds \\
&+|t_2-t_1|M_1(M_2+M_3\frac{|t_2-t_1|}{2})
\end{aligned}$$

The compactness of $R(t)$, the right side of above inequality tends to zero as $t_2 - t_1 \to 0$. And we have

$$\begin{aligned}&||(\Phi x_{t_1}) - (\Phi x_{t_2})||_C \\ &= sup_{-r\leq\theta\leq 0}||(\Phi x)(t_1+\theta) - (\Phi x)(t_2+\theta)|| \to 0.\end{aligned}$$

So $\Phi(Z_0)$ is an equicontinuous family of functions.

Also, $\Phi(Z_0)$ is bounded in Z_0, and so by the Arzela-Ascoli theorem, $\Phi(Z_0)$ is precompact. Hence, from the Schauder fixed point theorem, Φ has a fixed point in Z_0. Any fixed point of Φ is a mild solution of (2.1) satisfying $(\Phi x_t) = x_t \in C([-r,T] : X)$. Thus the system (2.4) is controllable on $[-r,T]$.

III. Example

Let us give some examples of nonlocal conditions. Let $p \in N$ and $t_1, \cdots, t_p$ be given real numbers such that $0 < t_1 < \cdots < t_p < T$. We can be applied for h defined by the formulae

$$h(y) = \sum_{i=0}^{p} c_i y_{t_i} \text{ for } y \in X$$

or

$$h(y) = \sum_{i=0}^{p} \frac{c_i}{\epsilon_i} \int_{t_i-\epsilon_i}^{t_i} y(s)ds \text{ for } y \in X$$

where $c_i, \epsilon_i (i = 1, \cdots, p)$ are given positive constants such that $0 < t_1 - \epsilon_1$ and $t_{i-1} < t_i - \epsilon_i (i = 1, \cdots, p)$. For more examples on various types of nonlocal conditions one can refer ([5,6,10]).

Consider the following simplified classical heat equation for material with memory

(3.1)

$$\frac{\partial z(t,x)}{\partial t} = \frac{\partial^2}{\partial x^2}[z(t,x) + \int_0^t b(t-s)z(s,x)ds] + Bu(t) + p(t, z(t+\theta, x)) + \int_0^t q(t, s, z(s+\theta, x))) \int_0^s e(s, \tau, z(\tau+\theta, x))d\tau)ds, \quad \theta \in [-r, 0]$$

and given nonlocal initial and boundary conditions

$$(3.2) \qquad \begin{aligned} &z(0,t) = z(1,t) = 0,\ x \in (0,1), \quad t \in J \\ &z(x,t) + h(z(x,\cdot)) = \phi(x,t), \quad t \in J \end{aligned}$$

where b is continuous and bounded and h satisfies appropriate condition. Here $B : U \to X$ with $U \subset J$ is a linear operator such that there exists an inverse operator W^{-1} on $L^2(J; U)/kerW$, where W is defined by

$$Wu = \int_0^T R(T-s)Bu(s)ds.$$

The resolvent operator $R(t)$ is compact and $p : J \times X \to X$, $e : J \times J \times X \to X$, $q : J \times J \times X \times X \to X$ are all continuous and uniformly bounded. The problem (3.1) can be brought to the form of (2.4) by marking suitable choices of A, B, f, k and g as follows. Let $X = L^2(J, R)$, $A\omega = \omega_{xx}$ and $D(A) = \{\omega \in X : \omega_{xx} \in X, \omega(0) = \omega(1) = 0\}$. Let

$$\begin{aligned} f(t, \omega_t)(x) &= p(t, \omega(t+\theta, x)), \quad (t, \omega) \in J \times X \\ k(t, s, \omega_t)(x) &= e(t, s, \omega(s+\theta, x)) \\ g(t, s, \omega_s, \sigma)(x) &= q(t, s, \omega(s+\theta, x), \sigma(x)), \quad x \in I \end{aligned}$$

be such that the condition in hypothesis $(H2)$ is satisfied. Then the system (3.1) be comes an abstract formulation of (2.4). Also by Theorem 3 of [9], the solutions are all bounded. Further, all the conditions stated in the above theoem are satisfied. Hence the system (3.1) is controllable on J.

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