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A STATISTICAL ANALYSIS METHOD FOR ESTIMATING GROUNDWATER CONTAMINANT CONCENTRATION

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Abstract. A practical estimation method for groundwater contaminant concentration is introduced. Using geostatistical techniques and symmetry, experimental variograms show significant improved correlation compared with those from conventional techniques.Numrical experiments are performed using a field data set.

1. Introduction

The punctual kriging for estimating groundwater contaminant concentrations at unmeasured locations consists of the following processes:

Step 1: Based on a given sample data set collected in a site, find an experimental semivariogram representing correlation between geological structure of the site and the variance of data distributions. This is the key step in kriging, and it is difficult. All available information of the site are incorporated. Concepts such as averaging process [3] and symmetry are considered in this step.

Step 2: Find a mathematical model representing the continuous information resided in the experimental semivariogram. This step requires development of a numerical algorithm for fitting the mathematical model to the experimental semivariogram.

Step 3: Solve the kriging system to obtain the optimal weight on the selected samples for kriging. For unmeasured location x_0, assume that

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the value at x_0 can be obtainded as a linear combination of the selected sample values; that is,

$$Z^*(x_0) = \sum_{i=1}^{n} \lambda_i Z(x_i), \tag{1.1}$$

where $Z^*(x_0)$ is the estimated value of the regionalized variable Z at x_0, $Z(x_i)$ is the measurement value at the point x_i and the λ_i's is weight, $i = 1, 2, \ldots, n$.

Boggs [2] and Cooper [4] didn't use symmetry in the kriging. We solve the following three problems by using symmetry in this paper.

Problem 1: How can we find a symmetric axis?

Problem 2: Where and how can we apply the symmetry?

Problem 3: What are advantages and disadvantages of symmetry, and how to overcome the disadvantages?

2. Kriging system

A *regionalized variables* is a numerical function with a spatial distribution which varies from one place to another with apparent continuity, but the changes of which cannot be represented by any workable function [5]. It contains two apparently contradictory characteristics: (i) a local, random, erratic aspect which calls to mind the notion of random variable and (ii) a general (or average) structured aspect which requires a certain functional representation. A random function $Z(x)$ defined on Ω is *stationary of order* 2 if (i) the mathematical expectation $E\{Z(x)\}$ exists and does not depend on x, i.e.,

$$E\{Z(x)\} = m \quad \text{for all} \quad x \in \Omega, \tag{2.1}$$

and (ii) for each pair of random variables $\{Z(x), Z(x+h)\}$ the covariance C exists and depends on the separation vector h with $x+h \in \Omega$, i.e.,

$$C(h) = E\{Z(x+h)Z(x)\} - m^2 \tag{2.2}$$

for all $x \in \Omega$.

The stationarity of the covariance implies the stationarity of the variance and the variogram. The following relation are immediately evident:

$$\begin{aligned} Var\{Z(\mathrm{x})\} &= E\{[Z(\mathrm{x}) - m]^2\} = C(0), \\ \gamma(h) &= \frac{1}{2}E\{[Z(\mathrm{x}+\mathrm{h}) - Z(\mathrm{x})]^2\} = C(0) - C(\mathrm{h}) \end{aligned} \tag{2.3}$$

for all $\mathrm{x} \in \Omega$, where Var is the variance of $Z(\mathrm{x})$ and $\gamma(\mathrm{h})$ is called the *semivariogram* or *intrinsic function.* Thus, under the hypothesis of second-order stationarity, the covariance and the variogram are two equivalent tools for $Z(\mathrm{x})$ separated by a vector h.

On the other hand, a random function $Z(\mathrm{x})$ satisfies the *intrinsic hypothesis* if (i) the mathematical expectation exists and does not depend on the support point x and (ii) for all vector h with $\mathrm{x}+\mathrm{h} \in \Omega$ the increment $[Z(\mathrm{x}+\mathrm{h}) - Z(\mathrm{x})]$ has a finite variance which does not depend on x, i.e.,

$$Var\{Z(\mathrm{x}+\mathrm{h}) - Z(\mathrm{x})\} = E\{[Z(\mathrm{x}+\mathrm{h}) - Z(\mathrm{x})]^2\} = 2\gamma(\mathrm{h}) \tag{2.4}$$

for all $\mathrm{x} \in \Omega$[5].

Let $Z(\mathrm{x})$ be a regionalized variable on a domain Ω in a three dimensional space R^3. Let $Z(\mathrm{x}_i)$ be the realization of the function

$Z(\mathrm{x})$ at $\mathrm{x}_i = (x_i, y_i, z_i) \in \Omega$, $i = 1, 2, \ldots, n$ i.e., Each $Z(\mathrm{x}_i)$ is the measured sample value of the regionalized variable $Z(\mathrm{x})$ at the location x_i. Then for any vector $\mathrm{h} = (h_x, h_y, h_z)$, *the classical estimator* or *conventional semivariogram* is defined by the arithmetic mean of the squared difference $[Z(\mathrm{x}_i) - Z(\mathrm{x}_i + \mathrm{h})]^2$, that is,

$$\gamma_c(\mathrm{h}) = \frac{1}{2N(\mathrm{h})} \sum_{i=1}^{N(\mathrm{h})} |Z(\mathrm{x}_i) - Z(\mathrm{x}_i + \mathrm{h})|^2, \tag{2.5}$$

where $N(\mathrm{h})$ is the number of data pairs separated by the vector h.

To obtain continuous informations and to solve the kriging system, mathematical variogram models are used. Moreover, in the process of constructing kriging system, we must calculate the variance for each lag h. Since experimental variogram can not give variance for every lag

h, finding the *mathematical variogram model* fitting the experimental semivariogram is necessary.

Averaging process is to obtain an improved relation between lag h and its variogram value. Let d be a lag average. Let H, $h_{\min}$ be

$$H = \{||\mathrm{x}_i - \mathrm{x}_j|| : i < j, 1 \le i, j \le n\},$$

$$h_{\min} = \min\{h \in H\}.$$

Let $h_1 = \text{mean}\{h \in H : h \in [h_{\min}, h_{\min} + d]\}$, $\gamma(h_1) = \text{mean}\{\gamma(h) : h \in H \text{ and } h \in [h_{\min}, h_{\min} + d]\}$, and k_1 is the number of $h \in H$ with $h \in [h_{\min}, h_{\min} + d]$.

Similarly, let $h_2 = \text{mean}\{h \in H : h \in [h_{k_1}, h_{k_1} + d]\}$, $\gamma(h_2) = \text{mean}\{\gamma(h) : h \in H \text{ and } h \in [h_{k_1}, h_{k_1} + d]\}$, and k_2 is the number of $h \in H$ with $h \in [h_{\min}, h_{k_1} + d]$.

By the same way, we can find the lag $h_j = \text{mean}\{h \in H : h \in [h_{k_{j-1}}, h_{k_{j-1}} + d]\}$, the corresponding averaged experimental variogram value $\gamma(h_j) = \text{mean}\{\gamma(h) : h \in H \text{ and } h \in [h_{k_{j-1}}, h_{k_{j-1}} + d]\}$ for lag h_j, and k_j is the number of $h \in H$ with $h \in [h_{\min}, h_{k_{j-1}} + d]$.

After fitting a mathematical model such as in (2.1) to the experimental semivariogram, kriging is performed to estimate the regionalized variable at unmeasured location. Kriging is an interpolation method for estimating the value at unmeasured locations. Kriging uses information from the semivariogram to find an optimal set of weights.

For an unmeasured location $\mathrm{x}_0 = (x_0, y_0, z_0)$, we want to estimate the value $Z(\mathrm{x}_0)$ based on selected samples $\{Z(\mathrm{x}_i), 1 \le \mathrm{x}_i \le n\}$. Assume that the value at $Z^*(\mathrm{x}_0)$ can be obtained as a linear combination of the selected sample values; that is,

$$Z^*(\mathrm{x}_0) = \sum_{i=1}^{n} \lambda_i Z(\mathrm{x}_i), \tag{2.6}$$

where $Z^*(\mathrm{x}_0)$ is the estimated value of Z at x_0 and the λ_i's are the weights to be determined. Optimization of the statistic will be performed by imposing the following two constraints:

$$E\{[Z^*(\mathrm{x}_0) - Z(\mathrm{x}_0)]\} = 0, \tag{2.7}$$

and

$$E\{[Z^*(\mathrm{x}_0) - Z(\mathrm{x}_0)]\}^2 \quad \text{is a minimum with respect to} \quad \lambda_i \tag{2.8}$$

where $Z(\mathrm{x}_0)$ is the value of the random function Z at x_0. Using (2.6) and (2.7), this can be written

$$E\{\sum_{i=1}^{n} \lambda_i Z(\mathrm{x}_i) - Z(\mathrm{x}_0)\} = 0$$

Taking the expectation of each value and equation it to the mean, m , which is assumed to be constant, yields

$$\sum_{i=1}^{n} \lambda_i E\{Z(\mathrm{x}_i)\} - E\{Z(\mathrm{x}_0)\} = \sum_{i=1}^{n} \lambda_i m - m = 0,$$

resulting in

$$\sum_{i=1}^{n} \lambda_i = 1$$

We assume that the mean value, m, is constant but unknown. We define $Z^*(\mathrm{x}_0)$ as a linear combination of the measured values such as in (2.6). Futhermore, the unbiasedness condition is satisfied only for weights whose sum is equal to 1. For punctual kriging, the minimization of (2.8) is carried out over $\lambda_1, \lambda_2, \ldots, \lambda_n$ subject to $\sum_{i=1}^{n} \lambda_i = 1$. Then minimize

$$E\left[\sum_{i=1}^{n} \lambda_i Z(\mathrm{x}_i) - Z(\mathrm{x}_0)\right]^2 - 2\mu\left(\sum_{i=1}^{n} \lambda_i - 1\right) \tag{2.9}$$

with respect to $\lambda_1, \lambda_2, \ldots, \lambda_n$ and μ.

The punctual kriging system is represented by the following matrix form

$$AW = B, \tag{2.10}$$

where

$$A = \begin{bmatrix} \gamma(x_1 - x_1) & \gamma(x_1 - x_2) & \dots & \gamma(x_1 - x_n) & 1 \\ \gamma(x_2 - x_1) & \gamma(x_2 - x_2) & \dots & \gamma(x_2 - x_n) & 1 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \gamma(x_n - x_1) & \gamma(x_n - x_2) & \dots & \gamma(x_n - x_n) & 1 \\ 1 & 1 & \dots & 1 & 0 \end{bmatrix}$$

$$W = [(\lambda_1, \lambda_2, \dots, \lambda_n,)\mu]^T,$$

$$B = [\gamma(x_0 - x_1), \gamma(x_0 - x_2), \cdots, \gamma(x_0 - x_n), 1]^T.$$

Note that A is an $(n+1) \times (n+1)$ nonsingular symmetric matrix. The optimal weight $\lambda_1, \lambda_2, \dots, \lambda_n$ can be obtained from $W = A^{-1}B$. The resulting estimation variance of punctual kriging becomes

$$\sigma^2 = \sum_{i=1}^{n} \lambda_i \gamma(x_0 - x_i) + \mu - \gamma(x_0 - x_0), \tag{2.11}$$

where $\gamma(x_0 - x_0)$ is the nugget effect.

3. Symmetry

We apply symmetry for punctual kriging. First, we consider theoretical aspects for using symmetry. Next, state overlaping averaging. Finally, questions regarding where and how symmetry can be applied are considered. Also, we study advantages and disadvantages of using symmetry in punctual kriging and consider how we can improve disadvantages.

(i) If recharge, chemical, biological, and other reactive processes are considered, the solute transport may be approximated, in Cartesian coordinates, by

$$\frac{\partial C}{\partial t} = \nabla \cdot (D \cdot \nabla C) - \nabla \cdot (VC) + f \tag{3.1}$$

with appropriate initial and boundary conditions, where $C = C(x, y, x; t)$ is the concentration of the solute, i.e., the mass of solute per unit volume of fluid, $D = D(x, y, z; t; C)$ is the dispersion tensor, $V = V(x, y, z; t; C)$

is the pore water velocity vector, $f = f(x, y, z; t; C)$ is a forcing function related to recharge, chemical, and biological activities. Chemical and biological activities can affect abvection/dispersion and complicate the transport description; nevertheless, these dispersion/diffusion and propagation processes are inherent properties of the evolution system governed by a parabolic partial differential equation [5]. As a simple case, assume that the porous medium is homogeneous, isotropic, saturated, the flow is stedy-state, and that there is no external source. Then the transport equation (3.1) can be simplified as

(3.2)
$$\frac{\partial C}{\partial t} = \left[D_x \frac{\partial^2 C}{\partial x^2} + D_y \frac{\partial^2 C}{\partial y^2} + D_z \frac{\partial^2 C}{\partial z^2}\right] - \left[\bar{v}_x \frac{\partial C}{\partial x} + \bar{v}_y \frac{\partial C}{\partial y} + \bar{v}_z \frac{\partial C}{\partial z}\right] + f,$$

where D_x, D_y and D_z are dispersion coefficients in the x-, y- and z-directions, $\bar{v}_x$, $\bar{v}_y$ and $\bar{v}_z$ are the average linear pore water velocities in each coordinate direction defined by $\bar{v}_x = v_x/\phi$, $\bar{v}_y = v_y/\phi$ and $\bar{v}_z = v_z/\phi$, in which v_x, v_y and v_z are specific discharge components, and ϕ is the porosity of the medium.

If a contaminant is released instantaneously at the origin $(x, y, z) = (0, 0, 0)$, the mass distribution of the contaminant at time t is given by

(3.3)
$$C(x, y, z; t) = \frac{M}{8(\pi t)^{3/2}\phi\sqrt{D_x D_y D_z}} exp\left(-\frac{(x - \bar{v}_x t)^2}{4D_x t} - \frac{(y - \bar{v}_y t)^2}{4D_y t} - \frac{(z - \bar{v}_z t)^2}{4D_z t}\right),$$

where M is the mass of contaminant introduced at the point sourec. The averged pore water velocites $\bar{v}_x$, $\bar{v}_y$ and $\bar{v}_z$ contribute movement of the center of mass of the contaminant plume (the propagation process), D_x, D_y and D_z contribute to the longitudinal and transverse spreading of the plume around the plume centroid (the dispersion/diffusion process).

(ii) Symmetry is one of typical characteristics of groundwater contaminant spatial distributions. Since the contaminant follows the groundwater flow path, for example, in y-direction, its distribution has certain

symmetric behaviors along the transverse directions, x- and z-directions, due to dispersion and diffusion processes.

Subdivide the interval I into n subintervals, and let

$$I_i = [x_{i-1}, x_i], \qquad i = 1, 2, \ldots, n$$

be the i-th interval of I. Let $n(I_i)$ be the number of sample included the i-th interval I_i and let

$$M_i = \frac{1}{n(I_i)} \sum_{j=1}^{n(I_i)} C_{ij}, \qquad i = 1, 2, \cdots, n,$$

be an average of groundwater contaminant concentration in the i-th interval I_i. C_{ij} is the j-th groundwater contaminant concentration included the i-th interval. Let

$$\left(\frac{I_i}{2}, M_i\right), i = 1, 2, \cdots, n,$$

be coordinates of the centers of interval I_i's and average M_i's. Symmetric axis can be obtained by interpolating the data set $\{(I_i/2, M_i) : i = 1, 2, \cdots, n\}$. By the same way, symmetric axes of y- and z- coordinates can be obtained. If the number of samples is small, for example, 2-3 samples in a large interval, bias can be involved.

To avoid it, for a given interval I, let

$$II_i' = [x_i, x_{i+2}], \qquad i = 1, 3, \ldots, n-3,$$
$$II_i = [x_{i-2}, x_i], \qquad i = 2, 4, \ldots, n,$$

and let $n(II_i'), n(II_i)$ be the number of samples in the i-th interval II_i' and II_i,respectively.

Let M_i' be the average of groundwater contaminant concentration in the interval II_i' and let M_i be the average of groundwater contaminant concentration in the interval II_i.

For a given interval I, let

$$\left(\frac{II_i'}{2}, M_i'\right), \qquad \left(\frac{II_i}{2}, M_i\right)$$

be the each coordinates of the centers of intervals II_i', II_i and averages M_i' and M_i, respectively. By interpolating each point M_i, M_i', symmetric axes can be obtained.

First of all, let us see where symmetry should be applied. It is easy to see that symmetry can be applied to Step1 and Step 3 in Section 1. That is, symmetry is applied to I) Step 1 procedure to find classical estimator, II) Step 3 procedure to find kriging matrix A and vector B.

In case I), $Z(\mathrm{x}_i) - Z(\mathrm{x}_i + h)$ in equation (2.1) is the difference of two samples located at x_i and x_j separated by the "distance" h. The distance between two samples is obtained by the formula

$$D_{ij} = \sqrt{(|x_i - x^0| - |x_j - x^0|)^2 + (|y_i - y^0| - |y_j - y^0|)^2} \sqrt{+(|z_i - z^0| - |z_j - z^0|)^2} \tag{3.4}$$

where x^0, y^0 and z^0 are symmetric axes for x-, y- and z-coordinates, respectively.

In case II), the distance using symmetry between $\mathrm{x}_j = (x_j, y_j, z_j)$ and $\mathrm{x}_0 = (x_0, y_0, z_0)$, the location for which the concentration to be estimated,is obtained by equation (3.4).

For instance, let c_0^* , β^* and α^* be optimal solutions obtained from fitting the experimental semivariogram by exponential mathematical model and let the distance h_{ij} between $\mathrm{x}_i = (x_i, y_i, z_i)$ and $\mathrm{x}_j = (x_j, y_j, z_j)$ be obtained from equation (3.4). Then we have the following semivariogram :

$$\gamma(h_{ij}) = c_0^* + \beta^* \left(1 - exp\left(-\frac{h_{ij}}{\alpha^*}\right)\right) \tag{3.5}$$

By applying symmetry the transition phenomena,a property of semivariogram, can be preserved: Any semivariogram or mathematical model value $\gamma(h)$ and lag h represent averaged semivariances. Semivariances are come from statistical consideration of variance of sample pairs, say, $Z(\mathrm{x}_i)$ and $Z(\mathrm{x}_j)$, separated by the distance $h = |h_i - h_j|$. Thus, if $h_1 < h_2$, the samples separated by the distance h_1 are more correlated than the one separated by the distance h_2.

If x_1 and x_n were located at the same distance from the symmetric axis then the components of the 1st and the n-th of vector B are same. Eliminate, for example, the n-th component of vector B while keeping the 1st component.

Kriging system before the elimination is as follows:

$$A_{(n+1)\times(n+1)}W_{(n+1)\times 1} = B_{(n+1)\times 1}. \tag{3.6}$$

From equation (3.6), weights are

$$A_{(n+1)\times 1} = A^{-1}_{(n+1)\times(n+1)}B_{(n+1)\times 1} \tag{3.7}$$

Kriging system after the elimination is as follows:

$$A_{n\times n}W_{n\times 1} = B_{n\times 1} \tag{3.8}$$

weights are

$$W_{n\times 1} = A^{-1}_{n\times n}B_{n\times 1} \tag{3.9}$$

For the given samples $x_i = (x_i, y_i, z_i), i = 1, 2, \cdots, n$, if the weights of equation (3.7) are substituted in equation (2.13) then the estimated value can be obtained at the unmeasured location $x_0 = (x_0, y_0, z_0)$.

Thus, the weights before elimination becomes

$$W_{(n+1)\times 1} = [\lambda_1, \lambda_2, \cdots, \lambda_{n-1}, \lambda_n, \mu]^T,$$

the estimated value at unmeasured location $x_0 = (x_0, y_0, z_0)$ is

$$Z^*(x_0) = \lambda_1 Z(x_1) + \lambda_2 Z(x_2) + \cdots + \lambda_{n-1}Z(x_{n-1}) + \lambda_n Z(x_n),$$

the weights after elimination becomes

$$W_{n\times 1} = [\lambda_1, \lambda_2, \cdots, \lambda_{n-2}, \lambda_{n-1}, \mu]^T,$$

and the estimate value at x_0 is

$$Z^*(x_0) = 2\lambda_1 Z(x_1) + \lambda_2 Z(x_2) + \cdots + \lambda_{n-1}Z(x_{n-1}),$$

where $\lambda_1 = \lambda_n$.

By the same way, when k samples are located at the symmmetric position, the corresponding kriging matrix and weights should be modified. Then the resulting kriging system becomes

$$A_{(n+1-k)\times(n+1-k)}W_{(n+1-k)\times 1} = B_{(n+1-k)\times 1}. \tag{3.10}$$

4. Application

The extensive tritium plume data collected during the MADE-2 experiment are used in this paper. After extracting macroscopic plume transport portion from groundwater contaminant data, geostatistical techniques and the classical estimator are applied. The use of linear geostatistics requires that the regionalized variable be additive, i.e., that linear combinations of the values of the variable have the same meaning.

The MADE-2 experiment was started on June 26, 1990 with a two-day pulse injection of $9.7m^3$ of tracer solution into the saturated zone approximately at $(x, y, z) = (0, 0, 57.8)$. The y-axis is the assumed groundwater flow direction and the x-axis crosses y-axis. The z-coordinate indicates the elevation above mean sea level. During the injection period, the maximum increase in hydraulic head in the nearby injection point was $0.45m$. Tracer concentration distributions were subsequently monitored at one to three month intervals over a period of 15 months in three dimensions using an extensive network of saturated zone multilevel samples. Tritium data derived from water samples collected on May 21, 1991, 60 days after injection are used in this analysis.

It spatial ranges are $-7 \leq x(m) \leq 6.8$, $-2.3 \leq y(m) \leq 15$ and $53.7 \leq z(m) \leq 61.18$ collected by the three-dimensional multilevel samples during a macrodispersion experiment conducted in a shallow unconfined alluvial aquifer. The y-axis is the assumed groundwater flow direction and the x-axis crosses the y-axis. The z-coordinate indicates the elevation above mean sea level. The aquifer is composed of poorly to well-sorted sandy gravel and gravelly sand with variable silt and clay content. Thus, due to the heterogeneity of the aquifer, the interaction

between injected tritiated water solution and the aquifer materials and the extraction of macroscopic portion, the selected data set for our analysis shows highly erratic behavior.

All computation were performed on a SUN SPARC Station 20 and the algorithms needed for our analysis were developed under the MATLAB environment. Symmetry behaviors were observed in the x- and z-direction. No clear symmetric structure was found in the y-direction. The symmetric axis in the y-direction was located at $y^0 = 2.1$m. And the symmetric axes were, approximately, $x^0 = 0.3$m and $z^0 = 57.92$m.

Figure 1 (a) and (c) did not use symmetry. Figure 1 (b) shows the experimental semivariogram obtained by the classical estimator with averaging process $d = 0.2m$. Figure 1 (c) shows the the experimental semivariogram obtained by the classical estimator with averaging distance $d = 0.2m$ and symmetric point $x^0 = 0.3m.y^0 = 2.1m$ and $z^0 = 57.92m$ in the $x-, y-$ and $z-$directions, respectively. Figure 1 (c), the experimental semivariogram obtained by classical estimator is improved with the one in Figure 1(a). Thus, by applying averaging process and symmetry, the experimental semivariograms can be improved significantly compared with those without any such consideration. Improvement of semivariogram indicates that the correlation between geological spatial structure and the corresponding data variances is improved. Also transition phenomena of semivariogram was satisfied by applying symmetry.

Using the exponential model fitted to experimental semivariogram obtainded by the classical estimator, kriging was used to estimate field data at unmeasured locations. We estimated field data at lattice points by $0.2m$ in the range of $-5 \leq x(m) \leq 5$ and $-0.3 \leq y(m) \leq 13.9$ by kriging.

Figure 2 (a) - (b) are the contour and the three-dimensional plots of the groundwater contaminant concentration distribution for the xy cut plane at $z = 58m$ without symmetry. Figure 2 (c) - (d) are the

contour and the three-dimensional plots of the groundwater contaminant concentration distribution for the xy cut plane at $z = 58m$ with symmetry.

Figure 3 (a) - (b) are the contour and the three-dimensional plots of the kriging variance for the xy cut plane $z = 58m$ without symmetry. Figure 3 (c) - (d) are the contour and the three-dimensional plots of the kriging variance for the xy cut plane $z = 58m$ with symmetric point $x^0 = 0.3$m, $y^0 = 2.1$m and $z^0 = 57.92$m in the x-, y- and z-directions, respectively. Figure 3 (a) shows kriging variance without symmetry. Figure 3 (b) shows kriging variance with symmetry. Thus, by applying symmetry, the kriging variance can be improved significantly compared with this any such consideration.

5. Conclusion

In this paper, we considered an estimation methd for groundwater contaminant concentrations using punctual kriging and symmetry. By applying symmetry we have the following observation:

1) It holds "transition phenomena" which are properties of semivariogram. (See Figure 1)

2) The estimated value were improved (See Figure 2)

3) The kriging variance were improved(See Figure 3)

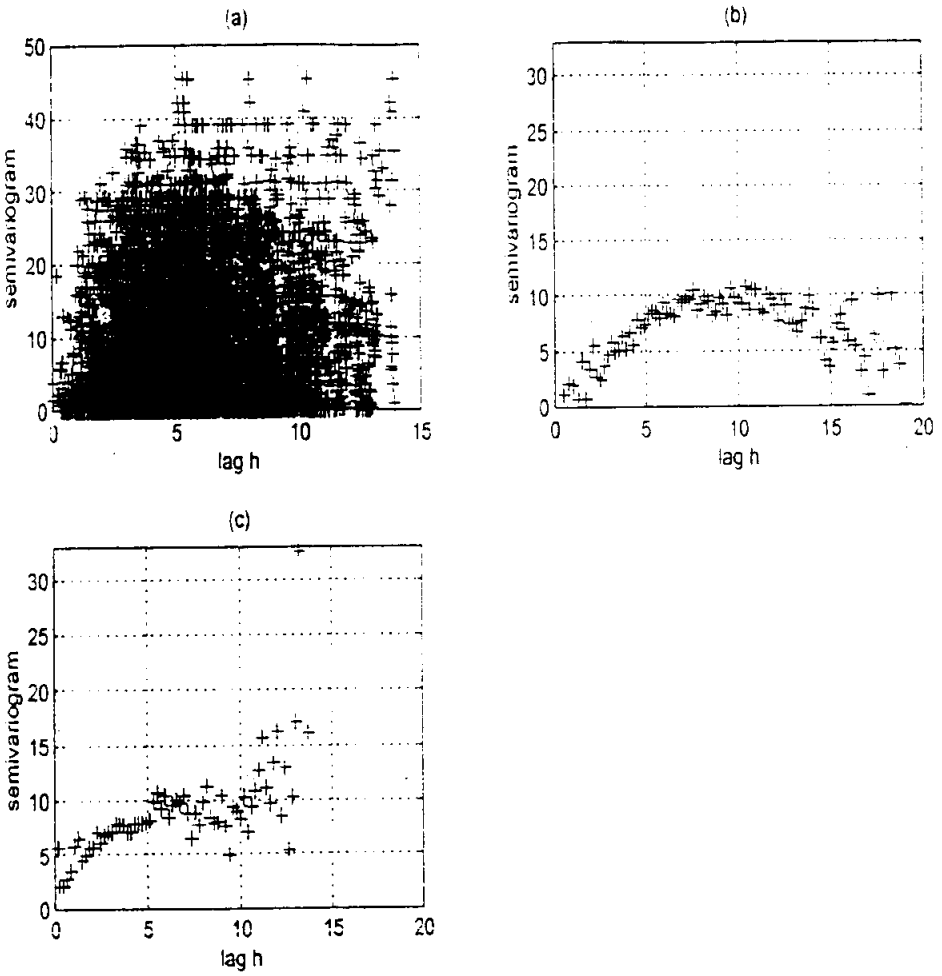

Figure 1. Semivariograms

(a) Without averaging process.

(b) With averaging distance $d = 0.2$ m and without symmetry.

(c) With averaging distance $d = 0.2$m and symmetry.

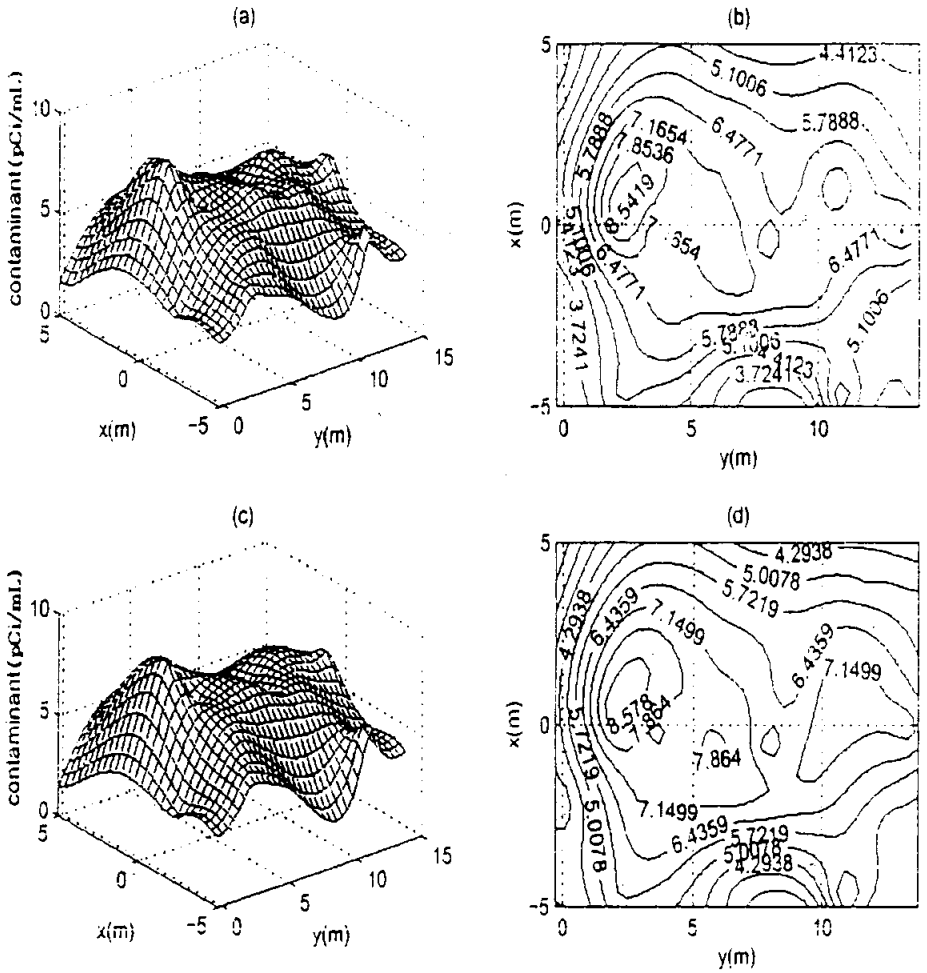

Figure 2. Surface and contour plots of the GCC

(a) Mesh of xy-cut plane $z = 58$m without symmetry.

(b) Contour of xy-cut plane $z = 58$m without symmetry.

(c) Mesh of xy-cut plane $z = 58$m with symmetry.

(d) Contour of xy-cut plane $z = 58$m with symmetry.

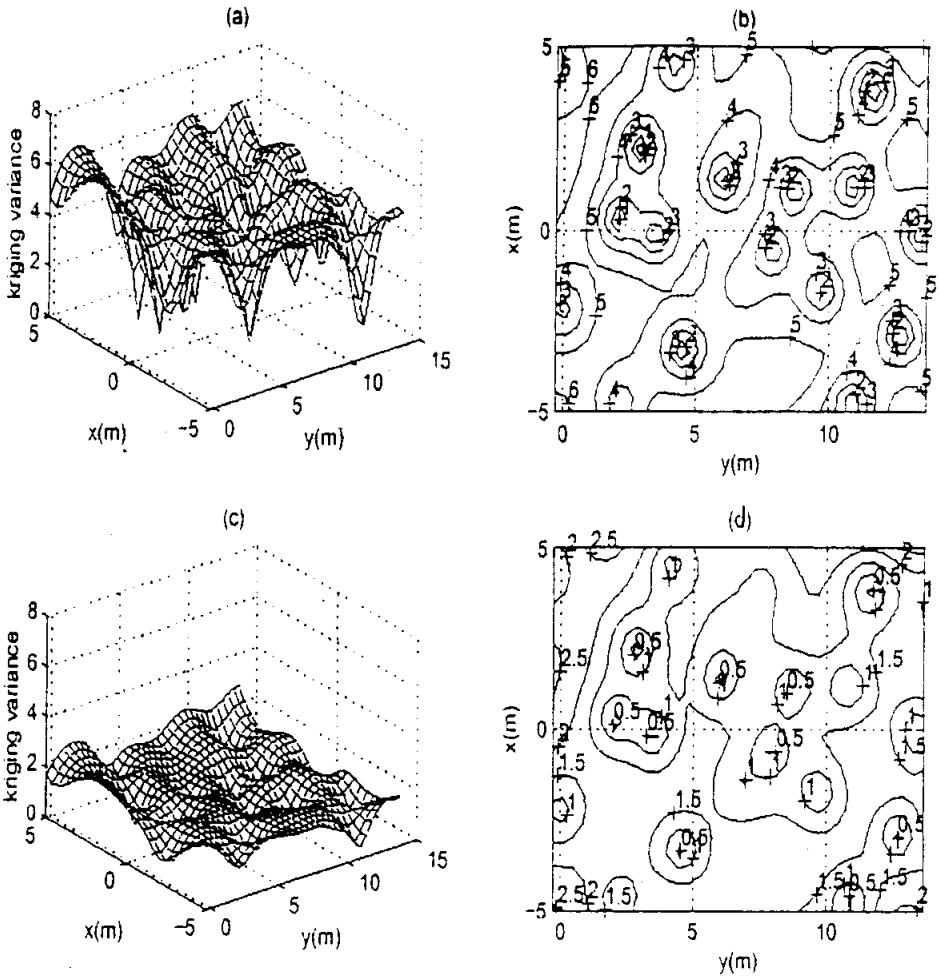

Figure 3. Surface and contour plots of the kriging variance
(a) Mesh of xy-cut plane $z = 58$m without symmetry.
(b) Contour of xy-cut plane $z = 58$m without symmetry.
(c) Mesh of xy-cut plane $z = 58$m with symmetry.
(d) Contour of xy-cut plane $z = 58$m with symmetry.

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