A Unified Framework for Transmitter Power Control in Cellular Radio Systems

Chin Choy Chai, Ying Lu, Yong Huat Chew, and Tjeng Thiang Tjhung

In earlier works on transmitter power control in cellular radio systems, the problem of whether a signal-to-interference ratio (SIR) threshold is achievable is determined by apparently different rules for homogeneous and heterogeneous SIR systems. In this paper, we present a unified and more universal framework for both cases. We also highlight the conditions under which a given SIR threshold vector for the heterogeneous SIR system is achievable, although so far there is no general solution to this problem.

Keywords: Power control, achievability of SIR, homogeneous and heterogeneous services, CDMA, SIR threshold.

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I. Introduction

In wireless cellular code division multiple access (CDMA) communication systems, interference can have a significantly negative impact on both system capacity and quality-of-service (QoS) if not properly dealt with. Power control is an effective technique to mitigate interference, maintain the required link QoS, and increase system capacity. Extensive research has been carried out on this subject. It is commonly assumed that this system is interference-limited; that is, the receiver noise level at the desired link is negligible compared to the interference from other unwanted links, such that the QoS depends only on the signal-to-interference ratio (SIR). Based on such a system model, we aim to find the minimum possible transmitter power that can achieve the required SIR threshold. Obviously, it is desirable to be able to predict whether the required SIR threshold is achievable using the adopted power control technique.

This problem has been investigated in earlier works. J. Zander [1], [2] and Sudheer A. Grandhi [3] studied cellular communication systems with an identical SIR threshold for each mobile in so-called homogeneous SIR systems. They found that the largest achievable SIR is equal to the reciprocal of the positive maximum modulus eigenvalue of the normalized link gain matrix. A homogeneous SIR system model is mainly suitable for networks that support only one class of service. Future cellular mobile radio systems have to support different classes of service with different SIR requirements. This prompted Wu to investigate cellular systems with different SIR thresholds, so-called heterogeneous SIR systems [4]. In order to make use of the results in [1]-[3], he used margin parameter δ to reconstruct the required SIR threshold vector.

Looking at these works on transmitter power control, the solution to whether an SIR threshold vector is achievable for a homogeneous SIR appears to be different from that for heterogeneous SIR cellular systems. However, from these previous works, we found that for both homogeneous SIR and heterogeneous SIR cases we can determine whether an achievable SIR exists for a particular power control scheme using a unified criterion. In this paper, our main contribution is in presenting a unified framework on the power control systems for both homogeneous SIR and heterogeneous SIR scenarios and in deriving a generalized theorem to determine whether an SIR threshold vector is achievable for both cases. Therefore, our result is a generalization of the results reported in papers [1]-[4].

We describe the system model and summarize some of the findings in the literature in section II. In section III, we present a unified criterion that determines whether an SIR threshold is achievable, regardless of whether it is a heterogeneous SIR or homogeneous SIR system. In section IV, we explain how our results can lead to the findings in [1]-[4]. In section V, we highlight the conditions under which a given SIR threshold vector is achievable for a heterogeneous SIR system. Finally, we conclude this paper in section VI.

II. System Model

1. Definitions

Consider an interference-limited cellular radio communication system with N users whose QoS requirements depend only on their respective SIR. We first need to state some definitions to facilitate our later analysis and explanation.

i) For a cellular radio system, a minimum uplink SIR threshold, γ_i, $i \in [1, N]$, is required for each mobile. We define vector $\boldsymbol{\gamma} = [\gamma_1, \cdots, \gamma_i, \cdots \gamma_N]^T$ as the uplink SIR threshold vector. Similarly, we can define the downlink SIR threshold vector as $\tilde{\boldsymbol{\gamma}} = [\tilde{\gamma}_1, \cdots, \tilde{\gamma}_i, \cdots, \tilde{\gamma}_N]^T$.

ii) For vectors $\mathbf{a} = \{a_i\}_{i=1}^N$ and $\mathbf{b} = \{b_i\}_{i=1}^N$ having the same dimension, $\mathbf{a} \le \mathbf{b}$ means $a_i \le b_i$ for every i and $\mathbf{a} > \mathbf{b}$ means $a_i > b_i$ for every i. For power vector $\mathbf{p}$, the notation $\mathbf{p} > \mathbf{0}$ denotes that all elements of $\mathbf{p} = \{p_i\}_{i=1}^N$, $p_i > 0, i = 1, \ldots, N$, where $\mathbf{0}$ is a vector with all zero elements.

iii) An SIR threshold vector of $\boldsymbol{\gamma} = [\gamma_1, \cdots, \gamma_i, \cdots \gamma_N]^T$ is achievable if a power vector, $\mathbf{p} > \mathbf{0}$, exists such that $\Gamma_i \ge \gamma_i$ for all mobiles, where Γ_i is the received SIR for mobile i.

2. System Model

We consider the uplink transmission first. Let p_i denote the transmitter power of user i. G_{ij} denotes the uplink channel gain (which models the effects of path loss, log-normal shadowing, and fading) from user j to the base station (BS) of user i. Hence, G_{ii} corresponds to the desired communication links and G_{ij} corresponds to the unwanted communication links. Using these notations, we can derive the uplink received SIR for mobile i as

$$\Gamma_i = \frac{p_i G_{ii}}{\sum_{j \ne i}^{N} p_j G_{ij}}. \tag{1}$$

In order to satisfy the QoS requirement of each mobile, we have

$$\Gamma_i = \frac{p_i G_{ii}}{\sum_{j \ne i}^{N} p_j G_{ij}} \ge \gamma_i, \quad i = 1, \ldots, N. \tag{2}$$

That is,

$$p_i \ge \sum_{j \ne i}^{N} \gamma_i \frac{G_{ij}}{G_{ii}} p_j, \quad i = 1, \cdots, N. \tag{3}$$

We can express the above inequality in the forms of a matrix and vector as

$$\mathbf{p} \ge \mathbf{W}_A \, \mathbf{p}, \tag{4}$$

where $\mathbf{p} = [p_1, \cdots, p_i, \cdots, p_N]^{\mathrm{T}}$ and the elements of matrix $\mathbf{W}_A$ are

$$\{W_A\}_{ij} = \begin{cases} \gamma_i \dfrac{G_{ij}}{G_{ii}}, & i \ne j; \\ 0 \quad , & i = j. \end{cases} \tag{5a}$$

In contrast to the normalized link gain matrix defined in [1]–[3], we shall call matrix $\mathbf{W}_A$, which is the SIR-threshold-embedded normalized uplink gain matrix, as the uplink system gain matrix. In the particular homogeneous case where all services have the same SIR threshold, i.e., $\gamma_i = \gamma, \forall i$, then we obtain

$$\{W_A\}_{ij} = \begin{cases} \gamma \dfrac{G_{ij}}{G_{ii}} \quad , & i \ne j; \\ 0 \quad , & i = j. \end{cases} \tag{5b}$$

As expected, we can see from (5a) and (5b) that the homogeneous SIR system is a special case of the heterogeneous SIR system where $\gamma_i \ne \gamma_j, i \ne j$.

In the downlink transmission, we can similarly define SIR-threshold-embedded normalized downlink gain matrix $\mathbf{Z}_A$ as the downlink system gain matrix, which consists of the following elements,

$$\{Z_A\}_{ij} = \begin{cases} \tilde{\gamma}_i \dfrac{\tilde{G}_{ji}}{\tilde{G}_{ii}} & , \quad i \neq j; \\ 0 & , \quad i = j, \end{cases} \tag{6}$$

where $\tilde{G}_{ji}$ is the downlink channel gain from the base station of user j to user i. To satisfy the QoS requirement of each mobile, we obtain

$$\tilde{\mathbf{p}} \geq \mathbf{Z}_A \, \tilde{\mathbf{p}}, \tag{7}$$

where $\tilde{\mathbf{p}} = [\tilde{p}_1, \cdots, \tilde{p}_i, \cdots, \tilde{p}_N]^T$ and $\tilde{p}_i$ is the transmitter power from the base station of user i. It is obvious that the power control problem can be expressed by the unified matrix and vector form for the uplink, (4), as well as the downlink transmission, (7). For simplicity, we will focus on the uplink case in this paper. All the results obtained also hold for the downlink case. Thus we shall avoid the words "uplink" or "downlink" in the sequel.

3. Perron-Frobenius Theorem

Before presenting our results, we will first summarize the Perron-Frobenius Theorem because it is the basis of the transmitter power control theory.

Theorem 1 [3],[5],[6]. Let $\mathbf{A}$ be an $N \times N$ irreducible nonnegative matrix with eigenvalues $\{\lambda_i\}_{i=1}^N$. Then,

1) $\mathbf{A}$ has a positive real eigenvalu λ^* with $\lambda^* = \max_{1 \leq i \leq N} \{|\lambda_i|\}$;
2) λ^* above has an associated eigenvector $\mathbf{p}^*$ with strictly positive entries;
3) λ^* has an algebraic multiplicity equal to 1;
4) all eigenvalues λ of $\mathbf{A}$ other than λ^* satisfy $|\lambda| < |\lambda^*|$ if and only if there is a positive integer, k, with all entries of $\mathbf{A}^k$ being strictly positive;
5) the minimum real λ, such that the inequality $\lambda \mathbf{p} \geq \mathbf{A}\mathbf{p}$ has solutions for $\mathbf{p} \geq \mathbf{0}$, is $\lambda = \lambda^*$;
6) the maximum real λ, such that the inequality $\lambda \mathbf{p} \leq \mathbf{A}\mathbf{p}$ has solutions for $\mathbf{p} \geq \mathbf{0}$, is $\lambda = \lambda^*$; and
7) the matrix $\mathbf{A}$ cannot have two linearly independent nonnegative eigenvectors.

Based on the above points, we can derive a lemma to supplement Theorem 1.

Lemma 1. λ^* is the unique eigenvalue of $\mathbf{A}$, which has corresponding nonnegative eigenvectors.

Proof. From 1-3 in Theorem 1, we know that λ^* is the simple root of the characteristic equation $|\lambda \mathbf{I} - \mathbf{A}| = 0$; hence the other eigenvalues of $\mathbf{A}$ are different from λ^*. Since the eigenvectors corresponding to different eigenvalues are linearly independent, together with 1-7 in Theorem 1 we can conclude that only λ^* has corresponding nonnegative eigenvectors among all the eigenvalues of $\mathbf{A}$. □

Lemma 2. $\mathbf{W}_A$ is an irreducible nonnegative matrix [3].

With Lemma 2, we can see that Theorem 1 and Lemma 1 can be applied to the system gain matrix, $\mathbf{W}_A$.

III. A Unified Framework

In section II.2 we define SIR-threshold-embedded normalized link gain matrix $\mathbf{W}_A$ as the system gain matrix. In $\mathbf{W}_A$, the original normalized link gain matrix is "modified" by the SIR thresholds, which can be identical for homogeneous SIR systems or dissimilar for heterogeneous SIR systems. Hence, this definition of a system gain matrix provides us with a unified framework on the formulation of the power control problem for both homogeneous SIR and heterogeneous SIR systems.

According to the unified framework presented above, the homogeneous SIR system is only a special case of a heterogeneous SIR system. The power control problem for these two cases can both be expressed using a unified form as

$$\mathbf{p} \geq \mathbf{W}_A \, \mathbf{p} \, . \tag{8}$$

The difference between these two cases is that all elements in the SIR threshold vector for a homogeneous system are identical, i.e., $\boldsymbol{\gamma} = \{\gamma\}_{N \times 1}$.

Because of Lemma 2 and Theorem 1, the maximum modulus eigenvalue λ_A^* of $\mathbf{W}_A$ is real, positive, and has an eigenvector $\mathbf{p}_A^*$ with strictly positive entries. Thus we can introduce Proposition 1 and Proposition 2.

Proposition 1. Let λ_A^* denote the maximum modulus eigenvalue of system gain matrix $\mathbf{W}_A$. If $\lambda_A^* \leq 1$, then the SIR threshold vector is achievable, and the power vector achieving this SIR threshold vector is the positive eigenvector $\mathbf{p}_A^*$ corresponding to λ_A^*.

Proof. Since λ_A^* is the maximum modulus eigenvalue of matrix

$\mathbf{W}_A$, and $\mathbf{p}_A^* > \mathbf{0}$ is its corresponding positive eigenvector, we have

$$\lambda_A^* \mathbf{p}_A^* = \mathbf{W}_A \mathbf{p}_A^* \quad . \tag{9}$$

If $\lambda_A^* \le 1$, we get

$$\mathbf{p}_A^* \ge \lambda_A^* \mathbf{p}_A^* = \mathbf{W}_A \mathbf{p}_A^* \quad . \tag{10}$$

From (8) we know (10) indicates that there exists a positive power vector $\mathbf{p}_A^*$ satisfying $\Gamma_i \ge \gamma_i$ for all the mobiles, i.e., the SIR threshold vector is achievable. □

Proposition 2. If an SIR threshold vector is achievable, we have $\lambda_A^* \le 1$.

Proof. First, assume that if the SIR threshold vector is achievable, $\lambda_A^* > 1$. Because the SIR threshold vector is achievable, we have $\mathbf{p}_A > \mathbf{0}$ such that $\mathbf{p}_A \ge \mathbf{W}_A \mathbf{p}_A$. That is, there exists a value of μ, for example $\mu = 1$, such that $\mu \mathbf{p}_A \ge \mathbf{W}_A \mathbf{p}_A$ has solutions for $\mathbf{p}_A > \mathbf{0}$. Since $\mu = 1 < \lambda_A^*$, which is obviously a contradiction against 1-5 in Theorem 1, the assumption is wrong and Proposition 2 is proved. □

With Propositions 1 and 2, we naturally get the following theorems.

Theorem 2. An SIR threshold vector is achievable if and only if $\lambda_A^* \le 1$.

Using the same proving method as the above, we can also conclude the following:

Theorem 3. An SIR threshold vector is not achievable if and only if $\lambda_A^* > 1$.

IV. Previous Results Seen from Unified Framework

We have seen that Proposition 1, Proposition 2, and Theorem 2 give the general rules to determine whether an SIR threshold vector is achievable for both homogeneous SIR and heterogeneous SIR systems. In this section, we shall demonstrate that all the results in papers [1]-[4] can be derived from the above general rules.

1. Homogeneous SIR Cellular Systems

For a homogeneous SIR system, we can extract the SIR threshold from system gain matrix $\mathbf{W}_A$, which can be rewritten as

$$\mathbf{W}_A = \gamma \mathbf{W} , \tag{11}$$

where the entry of matrix $\mathbf{W}$ is

$$W_{ij} = \begin{cases} \dfrac{G_{ij}}{G_{ii}} & , \quad i \ne j; \\ 0 & , \quad i = j, \end{cases} \tag{12}$$

and matrix $\mathbf{W}$ is defined as the normalized link gain matrix in [1]-[3]. According to (8), the problem of power control for this case becomes

$$\mathbf{p} \ge \gamma \mathbf{W} \mathbf{p} . \tag{13}$$

If we let λ^* be the real, positive, and maximum modulus eigenvalue of matrix $\mathbf{W}$, and $\mathbf{p}^*$ be the positive eigenvector corresponding to λ^*, we have

$$\lambda^* \mathbf{p}^* = \mathbf{W} \mathbf{p}^* . \tag{14}$$

Multiplying γ to both sides of (14), we get

$$\gamma \lambda^* \mathbf{p}^* = \gamma \mathbf{W} \mathbf{p}^* = \mathbf{W}_A \mathbf{p}^* . \tag{15}$$

Thus, $\gamma\lambda^*$ is a positive eigenvalue of $\mathbf{W}_A$ and has a corresponding positive eigenvector $\mathbf{p}^*$. Because of Lemma 1, we can conclude that

$$\lambda_A^* = \gamma \lambda^* \quad \text{and} \quad \mathbf{p}_A^* = \mathbf{p}^* . \tag{16}$$

From Proposition 1, if $\lambda_A^* \le 1$, SIR threshold γ is achievable. The power vector achieving γ is the positive eigenvector $\mathbf{p}_A^*$ of $\mathbf{W}_A$, corresponding to λ_A^*. That is, if the SIR threshold is $\gamma \le 1/\lambda^*$, then SIR threshold γ is achievable and the power vector achieving γ is the positive eigenvector $\mathbf{p}^*$ of $\mathbf{W}$, corresponding to λ^*. This is just the result of papers [1]-[3].

2. Heterogeneous SIR Cellular Systems

For a heterogeneous SIR system, given SIR threshold vector $\boldsymbol{\gamma}$, system gain matrix $\mathbf{W}_A$ can be expressed as

$$\mathbf{W}_A = \begin{pmatrix} \gamma_1 & 0 & \cdots & 0 \\ 0 & \gamma_2 & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & \gamma_N \end{pmatrix} \mathbf{W} , \tag{17}$$

where $\{\gamma_i\}_{i=1}^{N}$ are the elements of SIR threshold vector $\boldsymbol{\gamma}$.

If we extract common margin parameter δ from each element of $\boldsymbol{\gamma}$, we can rewrite $\boldsymbol{\gamma}$ as

$$\boldsymbol{\gamma} = \delta \boldsymbol{\gamma}^0, \tag{18}$$

where $\boldsymbol{\gamma}^0 = \{\gamma_1^0, \gamma_2^0, ..., \gamma_i^0, ..., \gamma_N^0\}$ is called the protection ratio vector, and $\gamma_i^0, i = 1, ..., N,$ is the protection ratio for each individual mobile user at a given instant. Thus $\mathbf{W}_A$ becomes

$$\mathbf{W}_A = \delta \begin{pmatrix} \gamma_1^0 & 0 & \cdots & 0 \\ 0 & \gamma_2^0 & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & \gamma_N^0 \end{pmatrix} \mathbf{W} = \delta \mathbf{W}^0, \tag{19}$$

where $\mathbf{W}^0 = \{\gamma_i^0 \cdot W_{ij}\}_{N \times N}$. Hence, the problem of power control becomes

$$\mathbf{p} \geq \delta \mathbf{W}^0 \mathbf{p}. \tag{20}$$

Following the same analysis as for the above homogeneous SIR systems,

$$\lambda_A^* = \delta \lambda_0^* \text{ and } \mathbf{p}_A^* = \mathbf{p}_0^*, \tag{21}$$

where λ_0^* is the real, positive, and maximum modulus eigenvalue of matrix $\mathbf{W}^0$, and $\mathbf{p}_0^*$ is the positive eigenvector of $\mathbf{W}^0$ corresponding to λ_0^*.

From Proposition 1, if $\lambda_A^* \leq 1$, i.e., $\delta \leq 1/\lambda_0^*$, SIR threshold vector $\boldsymbol{\gamma}$ is achievable. That is, the largest value of the margin parameter is $\delta^* = 1/\lambda_0^*$, and the largest achievable SIR threshold vector related to $\boldsymbol{\gamma}^0$ is $\boldsymbol{\gamma}^* = \delta^* \boldsymbol{\gamma}^0$. This is the result in [4].

To summarize, we need not extract SIR threshold γ used in (11) for the homogeneous SIR systems or the margin parameter δ in (19) for the heterogeneous SIR systems in order to determine whether an SIR threshold vector is achievable. Instead, the unified rule in section III, which is derived from system gain matrix $\mathbf{W}_A$, can be used to determine whether the required SIR threshold vector is achievable for cellular systems with either homogeneous SIR thresholds or heterogeneous SIR thresholds.

3. A Physical Interpretation on the Unified Framework

In Figs. 1 and 2, we use a system with two users as an example to give a physical interpretation of the unified framework. In system gain matrix $\mathbf{W}_A$, the normalized link gain is multiplied to the SIR threshold corresponding to each desired link, that is, $\{W_A\}_{ij} = \gamma_i \{W\}_{ij}$. We can view SIR threshold γ_i (of the desired link from user i to its own base station) as an additional gain factor to the link gain, G_{ij}, of the unwanted link from user $j (j \neq i)$ to the base station of user i. Mathematically, we can use an equivalent unwanted link gain, $G_{ij}^{'}$, to denote this effect, where $G_{ij}^{'} = \gamma_i G_{ij}$, and the corresponding equivalent desired link gain is $G_{ii}^{'} = G_{ii}$. Therefore, the larger the required SIR threshold, the larger the equivalent amount of "interference" imposed on the desired link gain $G_{ii}^{'}$. Note that since SIR threshold γ_i has been factored into the equivalent unwanted link gain $G_{ij}^{'}$, the equivalent SIR threshold $\gamma_i^{'}$ now becomes unity (or equivalently, the desired link gain can be modified to $G_{ii}^{''} = G_{ii} / \gamma_i$, and in this case the equivalent unwanted link gain will become $G_{ij}^{''} = G_{ij}$).

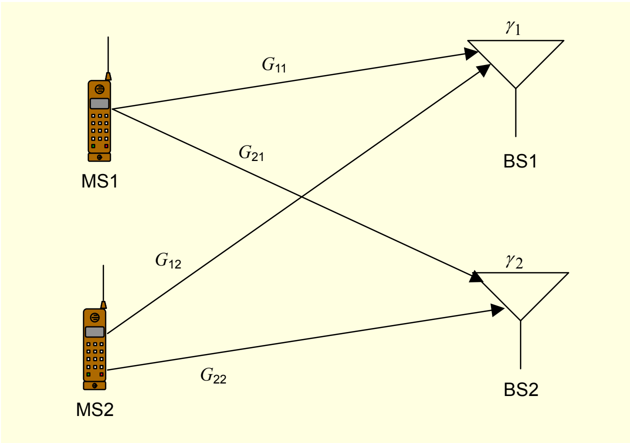

Fig. 1. Original model based on normalized link matrix $\mathbf{W}$.

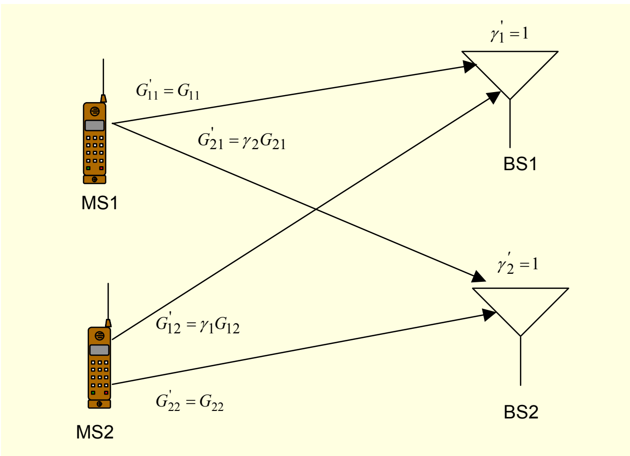

Fig. 2. Equivalent model based on system gain matrix $\mathbf{W}_A$.

In homogeneous SIR systems, because the required SIR thresholds for all the desired links are identical, which is γ, the gains of all the unwanted links are weighted equally

Table 1. A physical interpretation on the unified framework.

Original heterogeneous SIR system		Equivalent heterogeneous SIR systems	
Normalized link gain matrix	$\mathbf{W}=\{W_{ij}\}_{N\times N}$	System gain matrix	$\mathbf{W}_A=\{\gamma_i W_{ij}\}_{N\times N}$
Gain of the desired links	G_{ii}, i=1,…,N	Equivalent gain of the desired links	$G'_{ii}=G_{ii}$, $i=1,\ldots,N$
Gain of the unwanted links	G_{ij}, $j\neq i$	Equivalent gain of the unwanted links	$G'_{ij}=\gamma_i G_{ij}$, $j\neq i$
SIR threshold at the receiver	γ_i, $i=1,\ldots,N$	Equivalent SIR threshold at the receiver	$\gamma'_i=1$, $i=1,\ldots,N$

according to $G'_{ij}=\gamma G_{ij}$. Therefore, the largest achievable homogeneous SIR threshold depends only on the normalized link gains. In heterogeneous SIR systems, the gains of all the unwanted links are weighted unequally according to $G'_{ij}=\gamma_i G_{ij}$. So, whether an SIR threshold is achievable will depend not only on the link gains but also on the SIR thresholds of all system users.

In Table 1, we summarize how system gain matrix $\mathbf{W}_A$, which combines link gains and the required SIR thresholds of the system, provides us with a unified framework to the problem.

V. Notes on the Achievable SIR

In homogeneous SIR systems, once maximum modulus eigenvalue λ^* of normalized link gain matrix $\mathbf{W}$ is determined, the largest achievable SIR threshold of a homogeneous SIR system is given by $\gamma^*=1/\lambda^*$. However, so far there is no definite answer to the achievable SIR threshold vector for heterogeneous SIR systems. In this section, we highlight the conditions under which a given SIR threshold vector for the heterogeneous system is achievable.

Proposition 3. If the SIR threshold vector for a heterogeneous system satisfies $\max_{1\le i\le N}[\gamma_i]\le\gamma^*$, then $\lambda_A^*\le 1$, which means that the SIR threshold vector, $\boldsymbol{\gamma}$, for the heterogeneous SIR system is achievable.

Proof. For a heterogeneous SIR system with N users, let us assume that when $\max_{1\le i\le N}[\gamma_i]\le\gamma^*$, $\lambda_A^*>1$.

$$\because \lambda_A^*\mathbf{p}_A^*=\mathbf{W}_A\mathbf{p}_A^*=\begin{pmatrix}\gamma_1 & 0 & \cdots & \cdots & 0\\ 0 & \ddots & \ddots & \ddots & \vdots\\ \vdots & \ddots & \gamma_i & \ddots & \vdots\\ \vdots & \ddots & \ddots & \ddots & 0\\ 0 & \cdots & \cdots & 0 & \gamma_N\end{pmatrix}\mathbf{W}\mathbf{p}_A^*$$

$$\therefore\begin{cases}\dfrac{\lambda_A^*}{\gamma_1}p_{A1}^*=\sum_{j\neq 1}^{N}\dfrac{G_{1j}}{G_{11}}p_{Aj}^*\\ \vdots\\ \dfrac{\lambda_A^*}{\gamma_i}p_{Ai}^*=\sum_{j\neq i}^{N}\dfrac{G_{ij}}{G_{ii}}p_{Aj}^*\\ \vdots\\ \dfrac{\lambda_A^*}{\gamma_N}p_{AN}^*=\sum_{j\neq N}^{N}\dfrac{G_{Nj}}{G_{NN}}p_{Aj}^*\end{cases}$$

Since $\lambda_A^*>1$ and $\max_{1\le i\le N}[\gamma_i]\le\gamma^*$, we have $\lambda_A^*/\gamma_i>1/\gamma^*$ for all i. Let $\gamma_0=\max_{1\le i\le N}\{\gamma_i\}$. We then have $\lambda_A^*/\gamma_0\le\lambda_A^*/\gamma_i$ for all i and $\lambda_A^*/\gamma_0>1/\gamma^*$. Thus, we get

$$\begin{cases}\dfrac{\lambda_A^*}{\gamma_0}p_{A1}^*\le\dfrac{\lambda_A^*}{\gamma_1}p_{A1}^*=\sum_{j\neq 1}^{N}\dfrac{G_{1j}}{G_{11}}p_{Aj}^*\\ \vdots\\ \dfrac{\lambda_A^*}{\gamma_0}p_{Ai}^*\le\dfrac{\lambda_A^*}{\gamma_i}p_{Ai}^*=\sum_{j\neq i}^{N}\dfrac{G_{ij}}{G_{ii}}p_{Aj}^*\\ \vdots\\ \dfrac{\lambda_A^*}{\gamma_0}p_{AN}^*\le\dfrac{\lambda_A^*}{\gamma_N}p_{AN}^*=\sum_{j\neq N}^{N}\dfrac{G_{Nj}}{G_{NN}}p_{Aj}^*\end{cases}.$$

That is, $\lambda_A^*/\gamma_0\cdot\mathbf{p}_A^*\le\mathbf{W}\mathbf{p}_A^*$. However, since $\lambda_A^*/\gamma_0>1/\gamma^*=\lambda^*$, this result contradicts 1-6 in Theorem 1. So, the assumption that $\lambda_A^*>1$ is wrong. Hence, when $\max_{1\le i\le N}[\gamma_i]\le\gamma^*$, it implies $\lambda_A^*\le 1$. From Proposition 1, this means the SIR threshold vector $\boldsymbol{\gamma}$ is achievable. □

Proposition 4. If $\min_{1\le i\le N}[\gamma_i]>\gamma^*$, then $\lambda_A^*>1$, which means that this heterogeneous SIR threshold vector is not achievable.

Proof. Assume that when $\min_{1\le i\le N}[\gamma_i]>\gamma^*$, $\lambda_A^*\le 1$. Since $\lambda_A^*\le 1$ and $\min_{1\le i\le N}[\gamma_i]>\gamma^*$, we have $\lambda_A^*/\gamma_i<1/\gamma^*$. Similar to the proof to Proposition 3, we can get $\lambda_A^*/\gamma_0\cdot\mathbf{p}_A^*\ge\mathbf{W}\mathbf{p}_A^*$, where

$\gamma_0 = \min_{1\le i\le N}\{\gamma_i\}$ and $\lambda_A^*/\gamma_0 < 1/\gamma^* = \lambda^*$. This contradicts 1-5 in Theorem 1, so the assumption is not valid. Therefore, if $\min_{1\le i\le N}[\gamma_i] > \gamma^*$, then $\lambda_A^* > 1$. From Theorem 3, this means that SIR threshold vector $\boldsymbol{\gamma}$ is not achievable. □

With Propositions 3 and 4, the achievable SIR of a given heterogeneous SIR system is related to its largest achievable homogeneous SIR threshold, which is $\boldsymbol{\gamma}^* = \{\gamma^*\}_{i=1}^N$. When heterogeneous SIR requirement $\boldsymbol{\gamma}$ is imposed on an originally homogeneous SIR system, and if $\boldsymbol{\gamma} \le \boldsymbol{\gamma}^*$, heterogeneous SIR threshold $\boldsymbol{\gamma}$ is achievable. And if $\boldsymbol{\gamma} > \boldsymbol{\gamma}^*$, heterogeneous SIR threshold $\boldsymbol{\gamma}$ is not achievable.

However, if some elements of $\boldsymbol{\gamma}$ are bigger than γ^* and some elements of $\boldsymbol{\gamma}$ are smaller than γ^*, then there is no definite answer about whether such $\boldsymbol{\gamma}$ is achievable. Next, we use examples to illustrate the applications of the above results.

1. Example I

Let us consider a system consisting of three base stations, namely BS A, BS B, BS C, and four mobile users *a*, *b*, *c*, and *d*. The channel gains of their communication links are given as follows:

i) User *a* to BS A: $G_{Aa} = 1\times10^{-4}$,
user *a* to BS B: $G_{Ba} = 1\times10^{-5}$,
user *a* to BS C: $G_{Ca} = 8\times10^{-5}$,
ii) User *b* to BS A: $G_{Ab} = 3\times10^{-4}$,
user *b* to BS B: $G_{Bb} = 1\times10^{-4}$,
user *b* to BS C: $G_{Cb} = 3\times10^{-5}$,
iii) User *c* to BS A: $G_{Ac} = 8\times10^{-5}$,
user *c* to BS B: $G_{Bc} = 1.5\times10^{-4}$,
user *c* to BS C: $G_{Cc} = 7\times10^{-5}$,
iv) User *d* to BS A: $G_{Ad} = 3\times10^{-5}$,
user *d* to BS B: $G_{Bd} = 3\times10^{-5}$,
user *d* to BS C: $G_{Cd} = 5\times10^{-4}$.

The CDMA processing gain is 8, and binary phase shift keying is used. Supposing there are three classes of services, so-called class 1, 2 and 3, and each requires a target bit error rate (BER) performance of 10^{-4}, 10^{-3} and 10^{-2}, respectively. The respective target bit energy to noise power spectral density ratio (E_b/N_0) used to achieve such BER performances are given by 6.85 (= 8.4 dB), 4.80 (= 6.8 dB) and 2.65 (= 4.2 dB). Equivalently, we can divide the target E_b/N_0 by the processing gain to obtain the corresponding SIR thresholds, $\gamma_1 = 0.86$, $\gamma_2 = 0.60$ and $\gamma_3 = 0.33$ for class 1, class 2, and class 3 services.

We assume that the transmitter power of each user is controlled by the BS associated with the largest channel gain. Therefore, in this example, the transmitter power of mobile users *a* and *b* will be controlled by BS A, user *c* by BS B, and user *d* by BS C.

Therefore, the channel link gain matrix is given by

$$\mathbf{G} = \begin{bmatrix} 1\times10^{-4} & 3\times10^{-4} & 8\times10^{-5} & 3\times10^{-5} \\ 1\times10^{-4} & 3\times10^{-4} & 8\times10^{-5} & 3\times10^{-5} \\ 1\times10^{-5} & 1\times10^{-4} & 1.5\times10^{-5} & 3\times10^{-5} \\ 8\times10^{-5} & 3\times10^{-5} & 7\times10^{-5} & 5\times10^{-4} \end{bmatrix}.$$

Normalized link gain matrix **W**, shown in (12), and system gain matrix $\mathbf{W}_A$, shown in (5a), can be written as

$$\mathbf{W} = \begin{bmatrix} 0 & 3.0 & 0.8 & 0.3 \\ 0.33 & 0 & 0.27 & 0.1 \\ 0.067 & 0.67 & 0 & 0.2 \\ 0.16 & 0.06 & 0.14 & 0 \end{bmatrix}$$

$$\mathbf{W}_A = \begin{bmatrix} 0 & 3.0\gamma_a & 0.8\gamma_a & 0.3\gamma_a \\ 0.33\gamma_b & 0 & 0.27\gamma_b & 0.1\gamma_b \\ 0.067\gamma_c & 0.67\gamma_c & 0 & 0.2\gamma_c \\ 0.16\gamma_a & 0.06\gamma_d & 0.14\gamma_d & 0 \end{bmatrix},$$

where $\gamma_i \in \{\gamma_1, \gamma_2, \gamma_3\}$ depends on the service class of user *i*, where *i* is *a*, *b*, *c*, or *d*.

Next, we shall illustrate the following three cases separately.

Case i) Homogeneous SIR systems

Using the results in section IV, since the real, positive, and maximum modulus eigenvalue of **W** is given by $\lambda^* = 1.26$, the maximum SIR that mobiles can achieve is $\gamma^* = 1/\lambda^* = 0.795$. Therefore, all four users can select either class 2 or class 3 services, whilst the SIR requirement for class 1 service is not achievable.

To look at the later, let us consider the case where all four users select class 1 service. Then, $\gamma_a = \gamma_b = \gamma_c = \gamma_d = \gamma_1 = 0.86$, and the real, positive, and maximum modulus eigenvalue of $\mathbf{W}_A$ is $\lambda_A^* = 1.26 > 1$. Based on our unified framework (see Theorem 2), we can see that such a system cannot be supported.

Case ii) Heterogeneous SIR systems

Based on our unified framework, if user *a* and user *c* are using class 1, user *b* is using class 2, and user *d* is using class 3 service, we have $\lambda_A^* = 0.8961 < 1$. Therefore, such system can be supported and the optimal solution for transmitter power is given by the associated eigenvector $[0.935 \quad 0.256 \quad 0.236 \quad 0.073]^T$.

On the other hand, if users *a* and *b* are using class 1 service, while users *c* and *d* are using class 2 service, since

$\lambda_A^* = 1.014 > 1$, we predict that this combination of users cannot be supported.

Case iii)

As long as all four users select either class 2 or class 3 services, their SIR requirements can always be met, regardless of the combination of their services. This conclusion can be predicted using $\max[\gamma_2, \gamma_3] < \gamma^*$ by applying Proposition 3. It is also not difficult to verify that by increasing the processing gain to $G = 9$, we have $\gamma_1^{'} = 6.85/G = 0.76$, $\gamma_2^{'} = 4.80/G = 0.53$, and $\gamma_3^{'} = 2.65/G = 0.29$. Therefore, since $\max[\gamma_1^{'}, \gamma_2^{'}, \gamma_3^{'}] = 0.76 < \gamma^* = 0.795$, all service classes can be supported regardless of their combinations.

2. Example II

We now illustrate the use of the generalized theorem in determining all the feasible combinations of user services that a cellular system can support. Let us suppose that there are three co-channel mobile users, *a*, *b* and *c*, with their respective channel gains to three associated co-channel cells, A, B and C, as follows:

$G_{Aa} = 3\times10^{-4}$, $G_{Ba} = 1\times10^{-5}$, $G_{Ca} = 3\times10^{-5}$
$G_{Ab} = 1\times10^{-5}$, $G_{Bb} = 1\times10^{-4}$, $G_{Cb} = 3\times10^{-5}$
$G_{Ac} = 3\times10^{-5}$, $G_{Bc} = 3\times10^{-5}$, $G_{Cc} = 5\times10^{-4}$.

Assume that the system is providing three classes of mobile services, so-called class 1, class 2 and class 3 services, with the respective required BER performances of 10^{-2}, 10^{-3} and 10^{-4}. If we assume binary phase shift keying modulation without signal spreading, the values of E_b/N_0 to achieve such BER performance are given by $\gamma_1 = 2.65$ (= 4.2 dB), $\gamma_2 = 4.80$ (= 6.8 dB) and $\gamma_3 = 6.85$ (= 8.4 dB), respectively. The system gain matrix is given by

$$\mathrm{G} = \begin{bmatrix} 3\times10^{-4} & 1\times10^{-5} & 3\times10^{-5} \\ 1\times10^{-5} & 1\times10^{-4} & 3\times10^{-5} \\ 3\times10^{-5} & 3\times10^{-5} & 5\times10^{-4} \end{bmatrix}$$

$$\mathrm{W}_A = \begin{bmatrix} 0 & 0.0333\gamma_a & 0.1000\gamma_a \\ 0.1000\gamma_b & 0 & 0.3000\gamma_b \\ 0.0600\gamma_c & 0.0600\gamma_c & 0 \end{bmatrix},$$

where $\gamma_j \in \{\gamma_1, \gamma_2, \gamma_3\}$, with j = *a, b, c*, indicates the service class that co-channel users *a, b,* and *c* are requesting. We can then use the algorithm to determine all possible combinations or sets of services that can be supported for these three co-channel users. The value of λ_A is first calculated for each set of services, and if $\lambda_A < 1$, this set is feasible. Otherwise, if $\lambda_A > 1$, such a set of services is unfeasible.

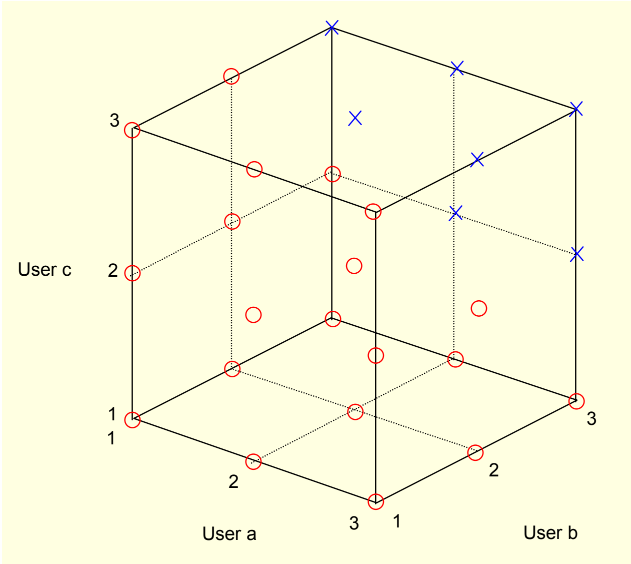

Fig. 3. Feasible region for the cellular system in Example II, where "o" denotes feasible sets and "x" denotes unfeasible sets. The index on all axes represent class 1, class 2 and class 3 services.

The feasible region can be shown as the 3-dimensional plot in Fig. 3, where each coordinate {*x*, *y*, *z*} represents the set of services requested by {user *a*, user *b*, user *c*}. We use "o" to denote a feasible set or combination and "x" to denote an unfeasible combination. As examples, for coordinate {3, 3, 3}, where all users *a, b,* and *c* are requesting class 3 service, we have $\lambda_A = 1.26 > 1$, and therefore this set is unfeasible; whereas for {1, 1, 1}, we have $\lambda_A = 0.49 < 1$, and therefore this set is feasible. We observe that the feasible region appears at the lower range of the coordinates, which correspond to the sets of services (as denoted by "o") with a higher BER (or less stringent BER performance).

Similarly, by using an *n*-tuple vector, we can extend this application to an arbitrary *n*-user system with *m* service classes. However, it is then difficult to plot the results in an *n*-dimensional graph for $n > 3$.

VI. Conclusions

In this paper, we study the SIR-based transmitter power control problem for cellular radio systems. In contrast to the different rules for determining whether an SIR threshold is achievable for homogeneous SIR and heterogeneous SIR systems in earlier works, we present a unified framework and general theorem for both cases using a definition of the system gain matrix, $\mathbf{W}_A$, in which the target SIR is embedded. We show that an SIR threshold is achievable for both cases when $\lambda_A^* \le 1$, where λ_A^* is the largest modulus eigenvalue of $\mathbf{W}_A$. We have also highlighted the conditions under which a given

SIR threshold vector is achievable for the heterogeneous SIR system. These results show how the achievable heterogeneous SIR threshold of a cellular radio system is closely related to the maximum modulus eigenvalue λ^* of the normalized link gain matrix, **W**.

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