# Radiation from a Current Filament Located inside a Cylindrical Frequency Selective Surface

Ali Uzer and Tuncay Ege

**We consider electromagnetic field radiation properties of a current filament placed at the origin of a cylindrical frequency selective surface (CFSS). The CFSS consists of free standing metal strips with two-dimensional periodicity. The analysis is based on a cylindrical Floquet mode wave expansion technique. We observed that near the half wavelength resonance frequencies, there exist some specific frequencies at which the surface becomes totally transparent.**

**Keywords: Cylindrical frequency selective surfaces, cylindrical floquet modes.**

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## I. Introduction

Non-planar frequency selective surfaces such as radomes or subreflectors in reflector antenna systems are used in practice. These surfaces may be studied by employing a Floquet theorem in a manner analogous to planar periodic surfaces [1]. By this technique, G. Loukos and J.C. Vardaxoglou [2] analyzed electromagnetic wave propagation inside strip grating frequency selective surface waveguides with a cylindrical cross-section. In this letter, we consider the two-dimensional problem of a cylindrical transverse magnetic field to z direction ($TM^z$) wave incidence on a cylindrical frequency selective surface (CFSS) made up of finite length strips. An axial current filament located at the origin generates the incident wave.

The cylindrical structure of radius $a$ consists of periodically arranged metal strips with periodicities $d$ and $b$ in $z$ and $\phi$ directions, respectively. As seen from the unit cell of the

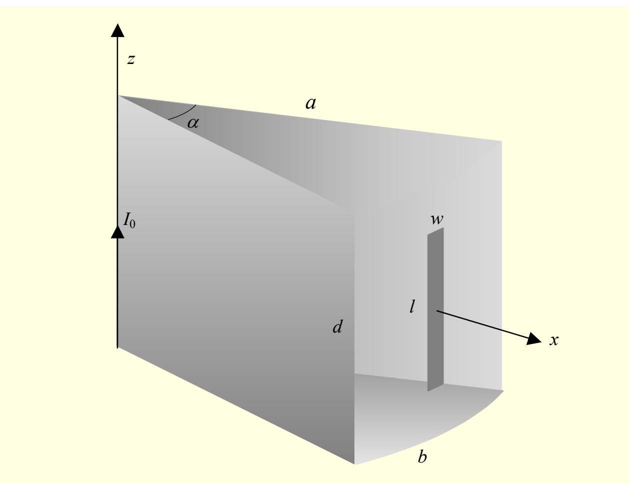


Fig. 1. Unit cell of the problem.

problem in Fig. 1, length $b$ subtends an angle $\alpha=2\pi/N$, where $N$ is the number of strips in $\phi$ direction. The strip dimensions are denoted by $l$ and $w$. We assume the strip width $w$ to be very small compared to the wavelength. In that case, a strictly axial electric current, $K_z$, should yield a good approximation for the current induced on the surface of the strip. Hence, all scattered field components may be derived from a single axial component of a vector potential, $A_z$. Floquet mode expressions for $A_z$ are given in [1] as

$$\begin{bmatrix} A_z^I \\ A_z^{II} \end{bmatrix} = \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \begin{bmatrix} a_{mn}^I J_{mN}(k_n\rho) \\ a_{mn}^{II} H_{mN}^{(2)}(k_n\rho) \end{bmatrix} \psi_{mn}, \begin{bmatrix} \rho < a \\ \rho > a \end{bmatrix}, \tag{1}$$

where

$$\psi_{mn} = \frac{e^{-jmN\phi} e^{-jk_{z_n}z}}{\sqrt{\alpha d}}, \tag{2}$$

$$k_n = \begin{bmatrix} \sqrt{k_0^2 - k_{z_n}^2} \\ -j\sqrt{k_{z_n}^2 - k_0^2} \end{bmatrix}, \begin{bmatrix} k_0^2 > k_{z_n}^2 \\ k_{z_n}^2 > k_0^2 \end{bmatrix}, \tag{3a}$$

$$k_0 = \omega\sqrt{\mu_0\varepsilon_0}\text{ , and} \tag{3b}$$

$$k_{z_n} = 2\pi n / d\ . \tag{3c}$$

Here, $a_{mn}^I$ and $a_{mn}^{II}$ are unknown weighting coefficients to be determined, while $J_{mN}$ and $H_{mN}$ denote the Bessel and Hankel functions of order $mN$, respectively, $\psi_{mn}$ is the Floquet modes, and $k_n$ denotes the propagation constants in the radial direction. In addition, $\mu_0$ and $\varepsilon_0$ denote the free space permeability and permittivity, respectively.

The magnetic and electric fields are found from $\mathbf{H}^{I,II} = \frac{1}{\mu_0}\nabla\times\mathbf{a}_z A_z^{I,II}$ and $\mathbf{E}^{I,II} = \frac{1}{j\omega\varepsilon_0}\nabla\times\mathbf{H}^{I,II}$. Weighting coefficients $a_{mn}^I$ and $a_{mn}^{II}$ of the two infinite summations are related to unknown current $K_z$ through the use of appropriate boundary conditions,

$$\begin{bmatrix} a_{mn}^I \\ a_{mn}^{II} \end{bmatrix} = \frac{-j\pi a\mu_0}{2} \begin{bmatrix} H_{mN}^{(2)}(k_n a) \\ J_{mN}(k_n a) \end{bmatrix} \langle K_z, \psi_{mn} \rangle . \tag{4}$$

The bracket in (4) denotes the inner product defined by

$$\langle f, g \rangle = \int_{-d/2}^{d/2} \int_{-\alpha/2}^{\alpha/2} f g^* \, d\phi dz\ , \tag{5}$$

while the asterisk denotes the complex conjugate.

In absence of the cylindrical structure, the magnetic vector potential of a current filament carrying a total current of $I_0$ and located at the origin is given in [3] as

$$A_z^{inc} = \frac{\mu_0 I_0}{4j} H_0^{(2)}(k_0\rho)\ . \tag{6}$$

An electric field integral equation is obtained by requiring the total tangential electric field to vanish on a strip; that is,

$$E_z^{inc} + E_z^{II} = 0 \quad \text{on } S_{strip}\ , \tag{7}$$

where $E_z^{inc}$ represents the electric field radiated by the current filament at the origin, and $S_{strip}$ is defined by,

$$S_{strip} = \{\rho = a; -{}^{w}\!/_{2a} < \phi < {}^{w}\!/_{2a}; -{}^{l}\!/_{2} < z < {}^{l}\!/_{2}\}\ . \tag{8}$$

For a moment method solution and Galerkin's method, unknown current $K_z$ is written as a sum of entire domain basis function $f_q$ with unknown coefficient $i_q$ ,

$$K_z = \sum_{q=1}^{Q} i_q f_q = \sum_{q=1}^{Q} i_q \sin\left(\tfrac{q\pi}{l}(z + \tfrac{l}{2})\right). \tag{9}$$

Then, we obtain a matrix equation in the form,

$$(Z_{pq})_{Q\times Q}(i_q)_{Q\times 1} = (V_p)_{Q\times 1} \qquad p, q = 1,2,\ldots,Q\ , \tag{10}$$

where

$$Z_{pq} = \frac{-\pi a}{2\omega\varepsilon_0} \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} k_n^2 J_{mN}(k_n a) H_{mN}^{(2)}(k_n a) \langle f_p, \psi_{mn} \rangle \langle f_p, \psi_{mn}^* \rangle, \tag{11}$$

$$V_p = \frac{I_0\omega\mu_0}{4} \sqrt{\alpha d}\, H_0^{(2)}(k_0 a) \langle f_p, \psi_{00} \rangle. \tag{12}$$

Finally, the average power radiated from length $d$ of the current filament in the presence of the CFSS is determined from

$$P_{rad} = -\mathrm{Re}\left\{ \lim_{\rho\to\infty} \int_0^d \int_0^{2\pi} E_z^{total} (H_\phi^{total})^* \rho \, d\phi dz \right\}, \tag{13}$$

and the normalized power is

$$\frac{P_{rad}}{P_{inc}} = 1 + \frac{8\,\mathrm{Re}\{ja_{00}^{II}\}}{\mu_0\sqrt{\alpha d}} + \frac{16}{\mu_0^2 \alpha d} \sum_{m=-\infty}^{\infty} |\, a_{m0}^{II} \,|^2\ , \tag{14}$$

where $P_{inc}$ is the power radiated in the absence of the CFSS for a unit current amplitude ($I_0$=1). Using the expressions for the basis functions and the inner products, (14) can be written as

$$\frac{P_{rad}}{P_{inc}} = 1 - 2\,\mathrm{Re}\{C_{K_z}\} C_0 J_0(k_0 a) + \left| C_0 C_{K_z} \right|^2 C_J , \qquad (15a)$$

where

$$C_{K_z} = \sum_{q=1}^{Q} i_q \frac{1-(-1)^q}{q} , \qquad (15b)$$

$$C_0 = \frac{2\pi^2 wl}{\alpha d} , \text{ and} \qquad (15c)$$

$$C_J = \sum_{m=-\infty}^{\infty} \left| \frac{\sin(m\pi w/b)}{m\pi w/b} J_{mN}(k_0 a) \right|^2 . \qquad (16)$$

Plots of $C_J$ versus frequency for values of $N \geq 6$ reveal that the sum tends to zero at several frequencies, which correspond to the zeros of $J_0(k_0 a)$. Close to these frequencies, the second and third terms in (15a) vanish and consequently normalized power equals unity. That is, at these frequencies radiated power becomes identical to the incident power. Since the cut-off frequencies of $\mathrm{TM}^z_{0n}$ modes in a conducting circular waveguide having radius $a$ are determined by the zeros of $J_0(k_0 a)$, these frequencies for which the power is totally transmitted will be referred from here onwards as "$\mathrm{TM}^z$ cut-off frequencies." On the other hand, the moment method solution of (15a) yields additional frequencies at which the power is totally transmitted, and these frequencies are different than the "$\mathrm{TM}^z$ cut-off frequencies" discussed above.

## II. Numerical Results

In the calculations of infinite summations, we use different asymptotic expressions [4] for the large and small argument regions of Bessel functions. But for imaginary values of the propagation constants $k_n$ in (3a), modified Bessel functions are used.

The moment method solution requires the accurate computation of matrix elements $Z_{pq}$ in (11), which involves products of Bessel functions in the form

$$J_{mN}(k_n a) H^{(2)}_{mN}(k_n a) \approx j / |\pi m N| \quad \text{as } |mN| \to \infty, \qquad (17)$$

thereby resulting in a slowly convergent infinite series. The convergence of the series is accelerated using the technique discussed in [5] to obtain values of $Z_{pq}$ accurately and efficiently.

The moment method solution of the induced current is obtained by using seven sinusoidal basis functions. For the frequency ranges given in all plots, strip width $w$ remains less than 1/20 of the wavelength.

For the plots in Figs. 2 and 3, the unit cell dimensions are $b$=50 mm, $d$=100 mm, $w$=4 mm, and $l$=90 mm, but the radius $a=bN/2\pi$ varies with the number of elements in $\phi$ direction, $N$=4, 6, 8, 16, 32, and 64. As shown in Fig. 2, the normalized powers for all $N$ values become zero at the resonance frequency occurring near $f$=1.84 GHz (corresponding to $l$=0.55$\lambda_0$). Numerical results show that if

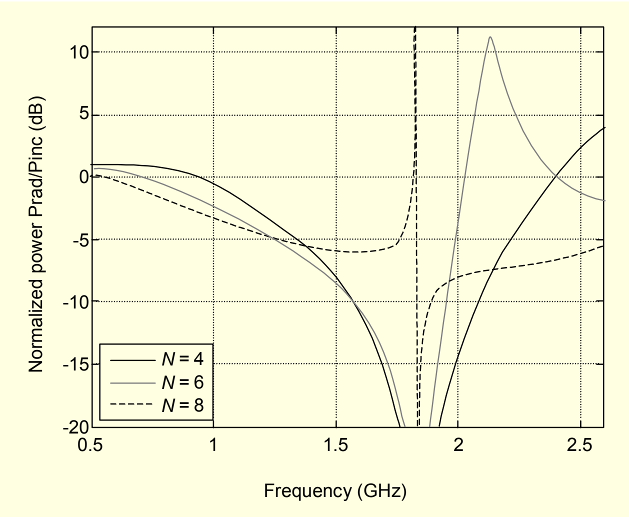


Fig. 2. Variation of normalized power with frequencies for different numbers of elements: N=4, 6, 8.

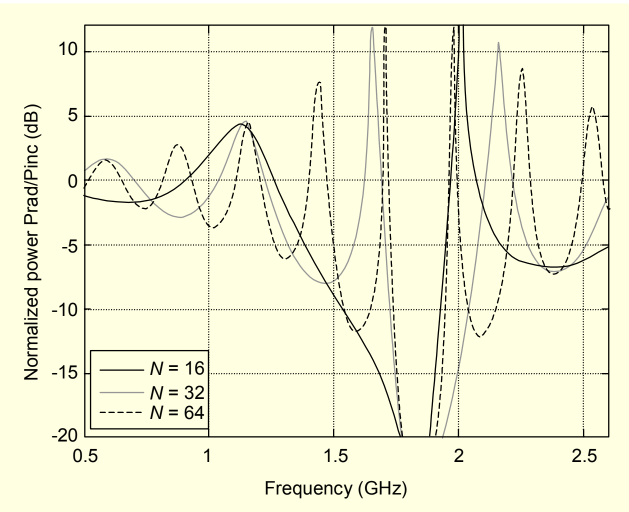


Fig. 3. Variation of normalized power with frequencies for different numbers of elements: N=16, 32, 64.

$N<6$ no "TM$^z$ cut-off frequency" exists, but for $N=6$ and $N=8$ the "TM$^z$ cut-off frequencies" are at $f=2.405$ and 1.805 GHz, respectively. Note that for $N=8$, the "TM$^z$ cut-off frequency" is very close to the resonance frequency at 1.84 GHz, so a spike occurs at 1.805 GHz.

As the number of elements increases to $N=64$, the curves near the resonance frequency become sharper. For example, if $N=64$, the "TM$^z$ cut-off frequencies" are 0.513, 0.811, 1.11, 1.40, 1.70, 1.99, 2.28, and 2.58 GHz. Hence, the normalized power equals unity near these frequencies.

The plots given in Fig. 4 for $N=64$ strip elements show that the resonance frequency changes with the strip lengths. For $l=85$, 90, and 95 mm, the resonances occur at frequencies 1.94, 1.84, and 1.73 GHz, respectively.

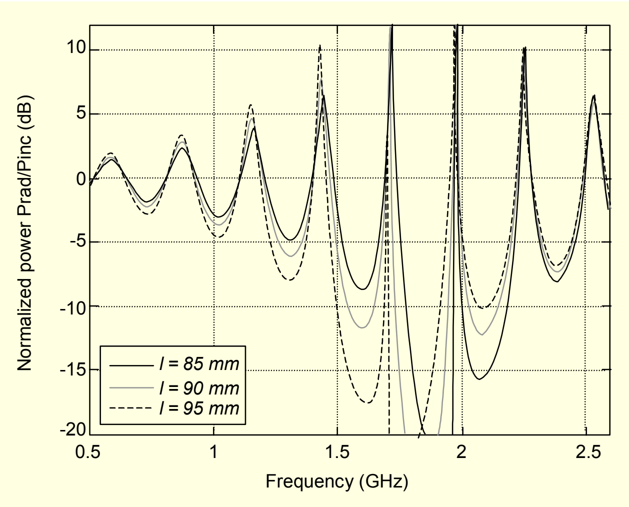


Fig. 4. Variation of normalized power with frequencies for different strip lengths: $l=85$, 90, 95 mm.

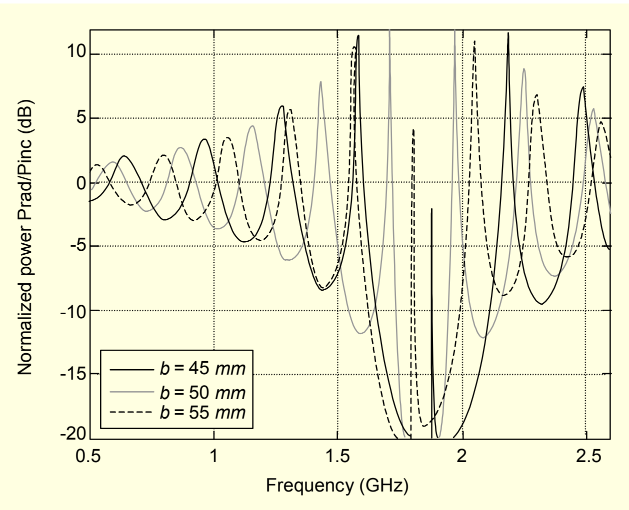


Fig. 5. Variation of normalized power with frequencies for different unit cell widths: $b=45$, 50, 55 mm

We observe that the resonance frequency also varies if the width of unit cell $b$ is varied, as shown in Fig. 5. The plots for $N=64$ strip elements and $b=45$, 50, and 55 mm show that the surface resonates at frequencies 1.88, 1.84, and 1.80 GHz, respectively. The sharp ripples for the $b=45$ and 55 mm curves are due to the "TM$^z$ cut-off frequencies" that are very close to the resonance frequencies.

As seen from the figures, there are regions where normalized power exceeds unity. This implies that the real part of the input impedance seen by the current filament becomes greater than the input impedance that would be seen if the metal strips were not present (that is, the filament radiates into the free space). Therefore, given curves may also be visualized as the normalized input resistances seen by the current filament located at the origin.

## III. Conclusion

For a TM$^z$ cylindrical wave excitation, the spectral response of the cylindrical structure consisting of metal strips is found to exhibit resonances that depend on the surface periodicity. At the resonance frequency, which occurs when the strip length is nearly the half wavelength, no real power is radiated by the current filament, so the surface is totally reflective. In addition to the resonance frequency, there exist some frequencies at which the surface becomes totally transparent, whereby all the power is transmitted outside the cylindrical structure. Some of these frequencies at which the surface is transparent correspond to cut-off frequencies of $TM_{0n}^z$ modes of a conducting circular waveguide, which we referred to as "TM$^z$ cut-off frequencies."

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