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Empirical Rock Strength Logging in Boreholes Penetrating Sedimentary Formations

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Abstract : The knowledge of rock strength is important in assessing wellbore stability problems, effective sanding, and the estimation of in situ stress field. Numerous empirical equations that relate unconfined compressive strength of sedimentary rocks (sandstone, shale, and limestone, and dolomite) to physical properties (such as velocity, elastic modulus, and porosity) are collected and reviewed. These equations can be used to estimate rock strength from parameters measurable with geophysical well logs. Their ability to fit laboratory-measured strength and physical property data that were compiled from the literature is reviewed. While some equations work reasonably well (for example, some strength-porosity relationships for sandstone and shale), rock strength variations with individual physical property measurements scatter considerably, indicating that most of the empirical equations are not sufficiently generic to fit all the data published on rock strength and physical properties. This emphasizes the importance of local calibration before one utilizes any of the empirical relationships presented. Nonetheless, some reasonable correlations can be found between geophysical properties and rock strength that can be useful for applications related to wellbore stability where having a lower bound estimate of in situ rock strength is especially useful.

Keywords : unconfined compressive strength, sedimentary rocks, wellbore stability, empirical strength log, velocity, Young's modulus, porosity

요 약 : 퇴적암반에서의 암석 강도는 시추공 굴착 과정에서 동반될 수 있는 시추공 불안정이나 효율적인 모래 생산 평가, 또는 시추공 압축 파쇄대를 이용한 지반 응력장 추정 문제 등 다양한 범위의 지질 역학적 문제들에 있어서 선결적으로 규명되어야 할 요소 중 하나이다. 본 연구에서는 여러 종류의 퇴적암(사암, 셰일, 석회암, 방해석 등)에서의 일축 압축 강도와 기타 물성(속도, 탄성계수, 공극률)을 관계 짓는 경험식들을 수집하여 검토하였다. 이들 경험식들은 시추공 물리검층을 통해 측정 가능한 파라미터들로 암석 강도를 추정하는데 이용될 수 있다. 일부 관계식들(예를 들어 사암이나 셰일의 강도-공극률 관계식)은 일정정도 만족스러운 강도 추정 도구로 이용될 수도 있지만, 각각의 물성 측정치에 대한 암석 강도가 상당히 분산되어 있어 모든 자료를 만족스럽게 예측하기에는 충분히 일반화 시킬 수 없다는 문제점이 있다. 따라서 이들 경험식들을 이용하기 전에 해당 지역에 대한 강도-물성 관계 캘리브레이션의 중요성이 강조된다. 그럼에도 불구하고 현장 암석 강도의 최저 한계 정보를 제공할 수 있는 몇몇 경험식들은 시추공 불안정 분석과 관련하여 유용하게 이용될 수 있다.

주요어 : 일축 압축 강도, 퇴적암, 시추공 불안정, 경험적 강도 검층, 속도, 영율, 공극률

Introduction

The unconfined compressive strength (UCS) of sedimentary rocks are key parameters needed to address a range of geomechanical problems ranging from limiting wellbore instabilities during drilling (e.g. Moos *et al.*, 2003), to assessing sanding potential (e.g. Santarelli *et al.*, 1989) and quantitatively constraining stress magnitudes using observations of wellbore failure (e.g. Zoback *et al.*, 2003). Laboratory-based UCS is typically determined through triaxial tests on cylindrical samples that are obtained from depths of interest. In practice, however, many geomechanical problems in reservoirs must be addressed when core samples are unavailable for laboratory testing. In fact, core samples of

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overburden formations (where many wellbore instability problems are encountered) are almost never available for testing.

As practical approach to these problems, a number of empirical relations have been proposed that relate rock strength to parameters measurable with geophysical well logs. The use of such relations is often the only way to estimate strength in many situations due to the absence of core for laboratory tests. The basis for these relations is the fact that many of the same factors that affect rock strength also affect other physical properties such as velocity, elastic moduli and porosity. In many cases, such relationships have been suggested for sedimentary rocks mainly because the strength information is greatly demanded in reservoirs for the purpose of drilling, maintenance and depletion of wellbores.

In general, a strength-physical property relationship for a specific rock formation is developed based on calibration through laboratory tests on rock cores from the given field. If there is no core sample available for calibration, the next best thing would be to use empirical strength relations based on measurable physical properties. However, because there are multiple choices of strength models for various rock types in different geological settings, it is necessary to understand the characteristics of the strength models and their range of applicability prior to utilizing them. For the purpose of review of the suitability of existing strength relations for various sedimentary rocks (sandstone, shale, and carbonate rocks), 30 different empirical relations (both published and proprietary) are summarized here and they were compared with an extensive dataset of published laboratory-determined rock physical/mechanical properties. Through this review the goal of the present study is to synthesize and compare the many relations proposed over the years and provide insight into the appropriateness of the various proposed criteria for rock strength when no core sample is available for testing.

Wellbore Instability

Wellbore instability is an important factor that should be considered in the process of drilling, maintenance and depletion of boreholes. There are several types of phenomena leading to wellbore instability; borehole breakouts (Bell and Gough, 1979; Zoback *et al.*, 1985), hydraulic fracturing due to excessive drilling mud pressure (Stock *et al.*, 1985), and drilling-induced tensile fractures (Lund and Zoback, 1999). Borehole breakout is typical type of wellbore instability induced by compressive rock failure around wellbore wall due to stress concentration (Bell and Gough, 1979; Zoback *et al.*, 1985; Zheng *et al.*, 1989; Santarelli *et al.*, 1989). Although the excessive breakout should be avoided for the successful completion of borehole, the occurrence of breakouts is beneficial for effective sand production in sandstone formations and importantly for the determination of in-situ state of underground stress.

When a vertical wellbore is drilled in a formation subjected to the maximum horizontal principal stress (S_{Hmax}) and the minimum horizontal principal stress (S_{hmin}), the local state of stress around the wellbore can be expressed as follows (Fairhurst, 1968):

$$\begin{aligned} \sigma_r &= \Delta P \\ \sigma_\theta &= S_{Hmax}+S_{hmin}-2(S_{Hmax}-S_{hmin})\cos 2\theta - 2P_p-\Delta P \\ \sigma_z &= S_v-2\nu\,(S_{Hmax}-S_{hmin})\cos 2\theta - P_p \end{aligned} \quad (1)$$

where σ_θ, σ_z and σ_r are tangential, vertical, and radial principal stresses, respectively, acting at the borehole wall; θ is the azimuth angle with respect to the S_{hmin} direction; ν is the Poisson's ratio of the rock; ΔP is the difference between borehole fluid pressure and pore pressure. When the state of local stresses acting on the rocks surrounding the borehole exceeds rock strength, rock failure occurs. It is typical that breakouts form at two diametrically opposed zones around the borehole wall along the S_{hmin} spring line where the tangential stress concentration is highest (Fig. 1). Because of this, breakouts can be used to estimate the direction of in situ stress (Bell and Gough, 1979; Plumb and Hickman, 1985; Zoback *et al.*, 1985; Haimson and Herrick, 1986; Moos and Zoback, 1990).

Stress magnitudes evaluation has evolved from models suggested by Zoback *et al.* (1985), Vernik and Zoback (1992), Vernik *et al.* (1992), Haimson and Song (1995), Brudy *et al.* (1997), and others. The hypothesis in these models is that the broken-out zone corresponds to the region where the state of stress exceeds rock strength; consequently, the boundary of the breakout is the locus of points where the criterion of rock failure is just met by the local stress condition (for example points B and B' or their mirror image in Fig. 1). In other words, at the breakout margin, the state of local stress field is in equilibrium with rock strength there. Thus, if we have a rock strength criterion for the rocks together with borehole breakout widths typically determined from wellbore image logging, we can estimate the magnitude of in situ stress field using equation (1).

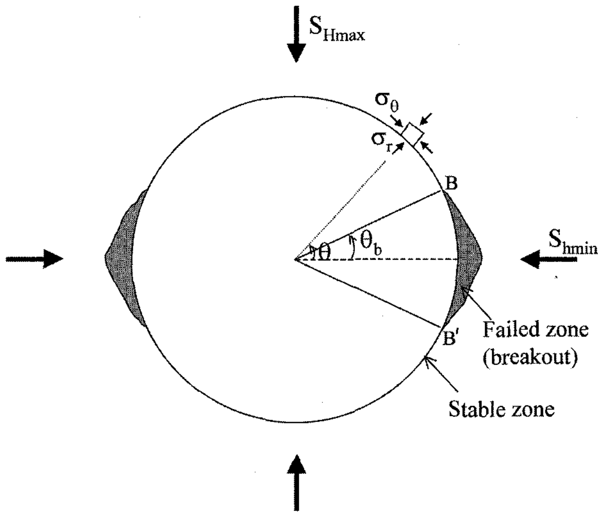

Fig. 1. Schematic cross-section of a borehole showing a compressive failure zone known as 'breakout' (between points B and B' along the borehole wall). Also shown are the principal in situ (far-field) horizontal stresses S_{Hmax} and S_{hmin}, and the radial and tangential stresses σ_r and σ_θ at the borehole wall at an angle θ from Shmin direction. Not shown are σ_v and σ_z, which act parallel to the borehole axis.

Rock Strength Criteria

Many different rock failure criteria have been proposed to describe rock strength under different stress conditions based on the different types of laboratory tests (i.e. uniaxial, triaxial, and polyaxial tests). The commonly used Mohr-Coulomb criterion has the form of

$$\sigma_1 = \text{UCS} + \sigma_3 \tan^2(45^\circ + \Phi/2) \tag{2}$$

where UCS is unconfined compressive strength (i.e. uniaxial compressive strength) and Φ is angle of internal friction, both of which are material constants. Other well-known strength criteria are the Drucker and Prager (1952) criterion and the modified Lade criterion (Ewy, 1998), which can also be expressed in terms of UCS and Φ (Colmenares and Zoback, 2002). Thus, if UCS and Φ can be estimated, it is possible to construct any of the commonly used failure criteria above, which can fully define rock strength at depth.

It is noteworthy that the impact of Φ on wellbore stability analysis is much less significant than that of UCS. Typical rocks have a relatively narrow range of internal friction angle (roughly between 15° and 40°), and the uncertainty in rock strength due to uncertain value is only an order of the least principal stress magnitude (if the Mohr-Coulomb failure criterion is utilized for example). Because the magnitude of the least principal stress dealt with in practice is considerably lower than UCS, we can simply deduce that the impact of uncertainty in Φ value on estimating in situ rock strength is minor compared to that of UCS. Thus, the esti-

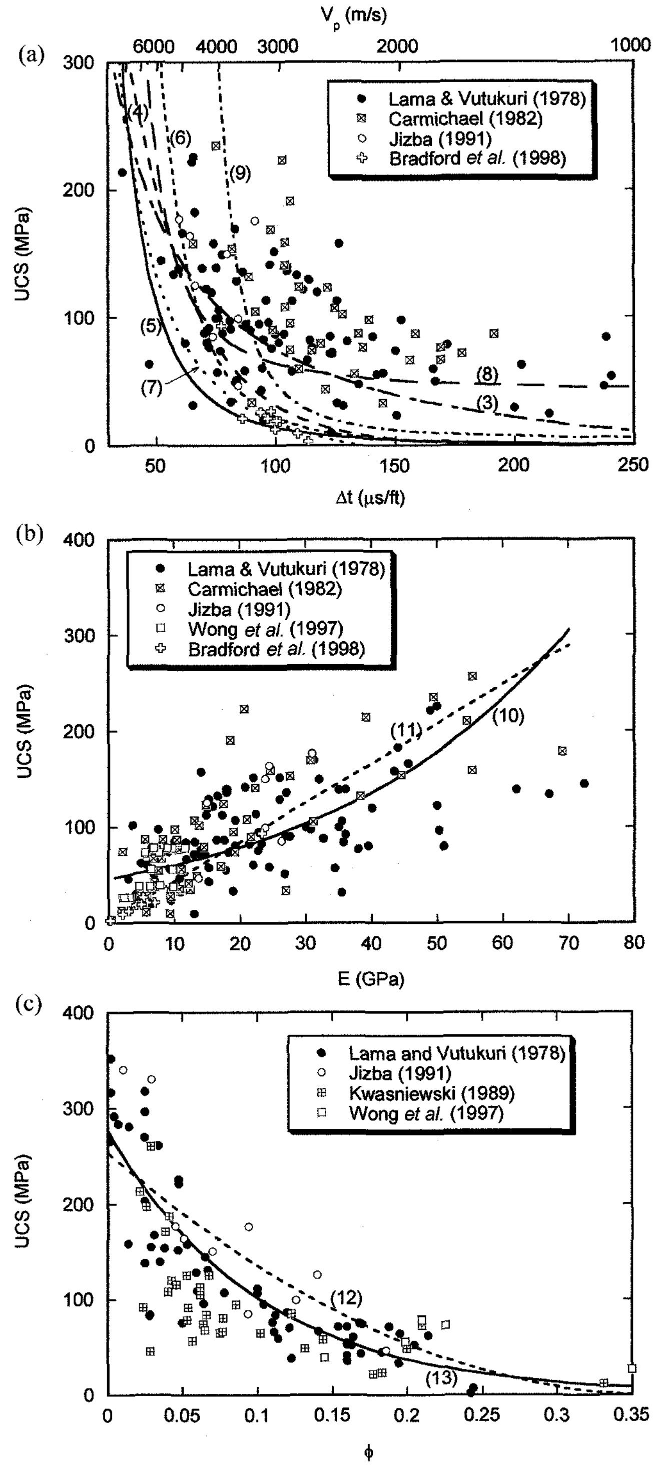

Fig. 2. Laboratory determined UCS data for sandstones plotted as a function of (a) slowness, Δt, (b) Young's modulus, E, and (c) porosity, ϕ, which are overlapped with numerous empirical strength relationships listed in Table 1.

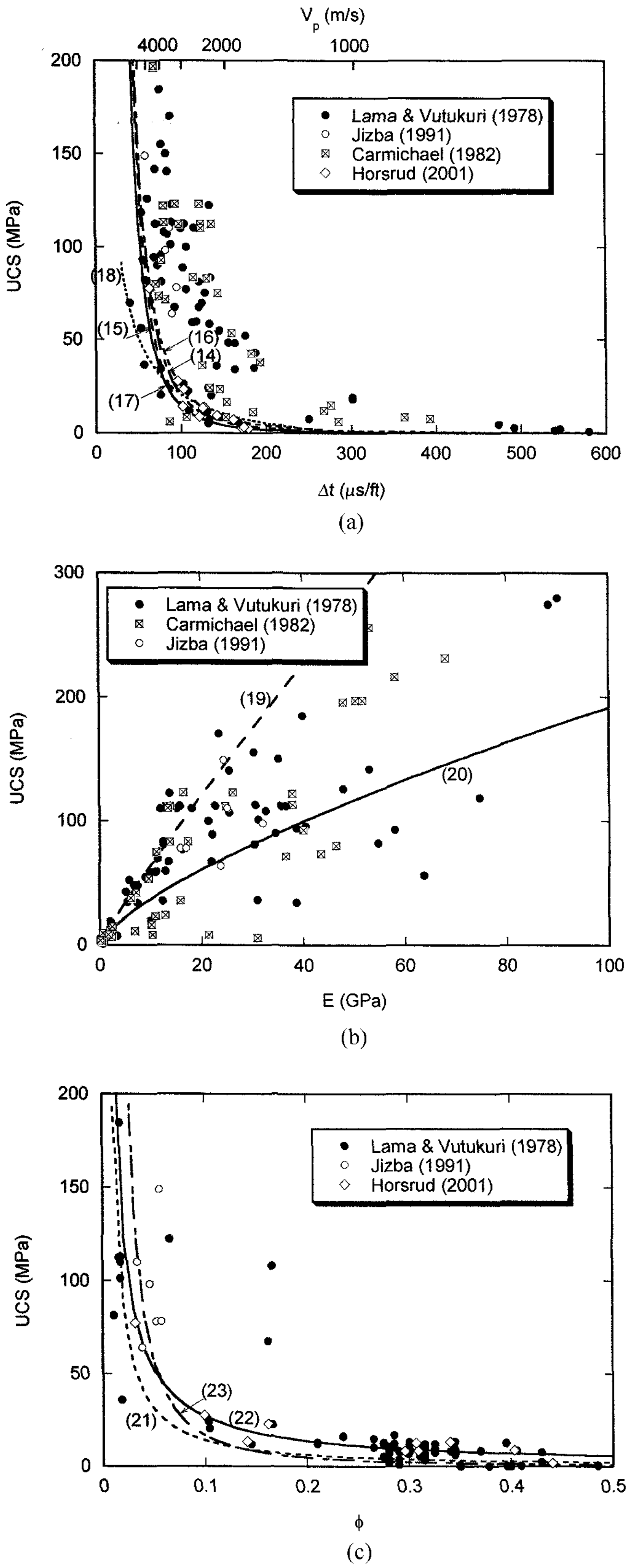

Fig. 3. Laboratory determined UCS data for shales plotted as a function of (a) slowness, Δt, (b) Young's modulus, *E*, and (c) porosity, ϕ, which are overlapped with numerous empirical strength relationships listed in Table 2.

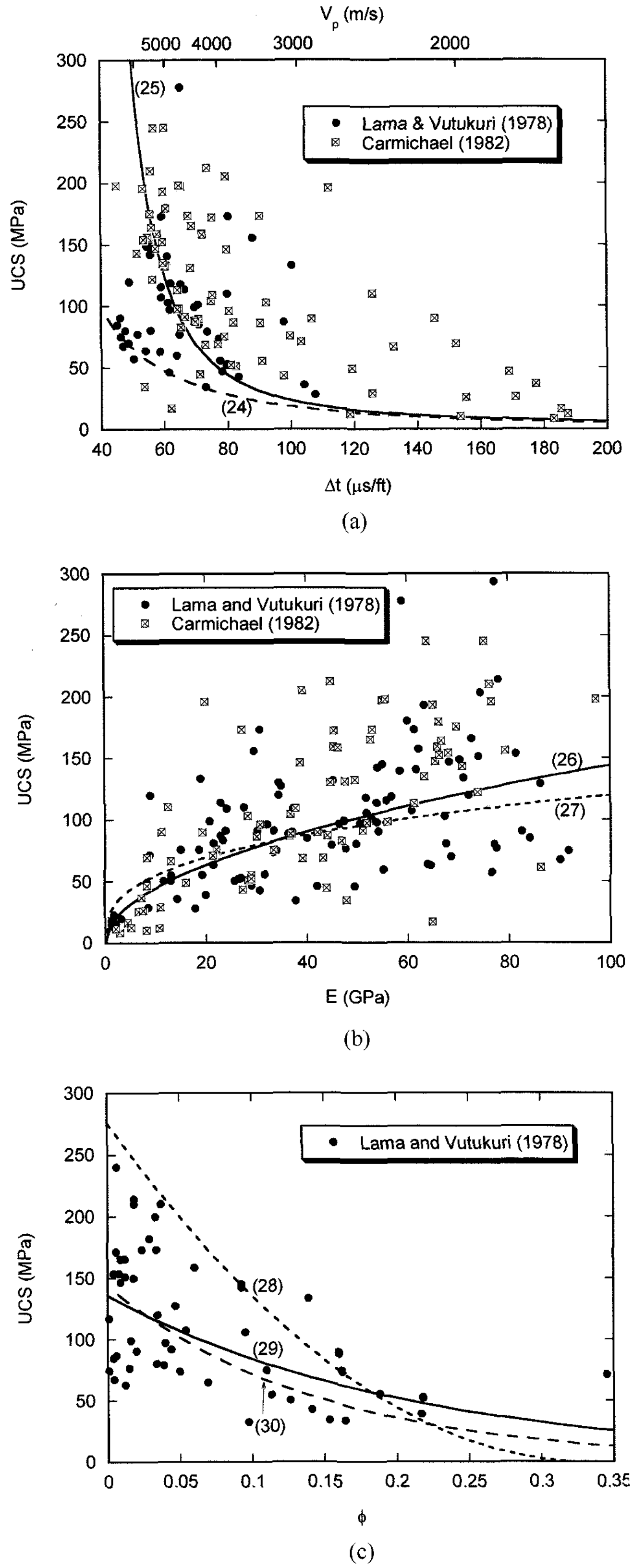

Fig. 4. Laboratory determined UCS data for carbonate rocks (limestone and dolomite) plotted as a function of (a) slowness, Δt, (b) Young's modulus, *E*, and (c) porosity, ϕ, which are overlapped with numerous empirical strength relationships listed in Table 3.

mation of UCS is of the practical importance for characterizing rock strength at depth.

Physical/mechanical property data for sedimentary rocks

Nearly all proposed formulae for determination of rock strength from geophysical logs utilize one (or more) of the following parameters: P-wave velocity (V_p), or equivalently, slowness ($\Delta t = V_p^{-1}$), Young's modulus (E), and porosity (ϕ). V_p or Δt are directly measurable, E can be derived from velocities and density measurements, and ϕ can be usually derived from density measurements assuming rock matrix and fluid densities.

Conceptually, the justification for the empirical relations discussed below is the general correlation between these parameters and unconfined compressive strength. These general correlations are seen in the laboratory data presented in Figs. 2, 3 and 4 for sandstone, shale, and limestone and dolomite, respectively. Despite the considerable scatter in the data, for each rock type, there is a marked decrease in strength with Δt and ϕ, and an increase in strength with E. The rock strength and physical property data presented in these figures were compiled from the literature (Lama and Vutukuri, 1978; Carmichael, 1982; Kwasniewski, 1989; Jizba, 1991; Wong *et al.*, 1997; Bradford *et al.*, 1998; Horsrud, 2001, see symbols). Lama and Vutukuri (1978) and Carmichael (1982) tabulated extensive lists of various mechanical properties of sedimentary rocks from different locations around the world. Kwasniewski (1989) listed UCS and porosity data of various sandstones. Jizba (1991) presented mechanical properties of sandstones and shales with a wide range of porosity recovered from different depths in a borehole in Texas, USA. Wong *et al.* (1997) presented a table of strength and other physical properties of several representative porous sandstones. Bradford *et al.* (1998) and Horsrud (2001) reported laboratory test results on the North Sea sandstone and shale, respectively. The compiled data constitute a database of about 260 sandstones, 100 shales, and 140 limestones and dolomites.

Empirical Strength Equations for Sedimentary Rocks

Sandstone

Equations (3)~(13) in Table 1 present a number of relationships in common practice (both published and proprietary) for estimating the unconfined compressive strength of sandstone from geophysical logging data. These relations were derived for case studies carried out for markedly different rocks in markedly different geological settings, around the world. To the degree possible, the regions and/or the general rock properties appropriate for each equation are indicated in Table 1. If no reference is shown, the given empirical relation is unpublished. Equations (3)~(5) utilize V_p (or expressed equivalently as Δt) measurements from well

Table 1. Empirical relationships between UCS and other physical properties in sandstone. Units used are: Δt: s/ft, E: GPa, ϕ: fraction, and V_{clay}: fraction.

Eq. No.	UCS (MPa)	Regions where developed	General remarks	Reference
(3)	$0.035V_p-31.5$	Thuringia, Germany	-	Freyburg (1972)
(4)	$1200 \exp(-0.036\Delta t)$	Bowen Basin, Australia	Fine grained, both consolidated and unconsolidated sandstones with all porosity range	McNally (1987)
(5)	$1.4138\times10^{7}\ \Delta t^{-3}$	Gulf Coast, USA	Weak and unconsolidated sandstones	-
(6)	$3.3\times10^{-20}\ \rho^2 V_p^4 \left(\frac{1+\nu}{1-\nu}\right)^2 \cdot (1-2\nu)(1+0.78V_{clay})$	Gulf Coast, USA	Applicable to sandstones with UCS > 30 MP	Fjaer *et al.* (1992)
(7)	$1.745\times10^{-9}\ \rho V_p^2-21$	Cook Inlet, Alaska, USA	Coarse grained sandstones and conglomerate	Moos *et al.* (1999)
(8)	$42.1 \exp(1.9\times10^{-11}\ \rho V_p^2)$	Australia	Consolidated sandstones with $0.05 < \phi < 0.12$ and UCS > 80MP	-
(9)	$3.87 \exp(1.14\times10^{-10}\ \rho V_p^2)$	Gulf of Mexico, USA	-	-
(10)	$46.2 \exp(0.027E)$	-	-	-
(11)	$2.28+4.1089E$	Worldwide	-	Bradford *et al.* (1998)
(12)	$254\ (1-2.7\phi)^2$	Sedimentary basins worldwide	Very clean, well-consolidated sandstones with $\phi < 0.3$	Vernik *et al.* (1993)
(13)	$277 \exp(-10\phi)$	-	Sandstones with $2 <$ UCS < 360 MPa and $0.002 < \phi < 0.33$	-

logs. Equations (7)~(9) utilize both density and V_p data, and equation (6) utilizes V_p, density ρ, Poisson's ratio v (which requires V_s measurements) and clay volume V_{clay} (from gamma ray logs). Equations (10) and (11) utilize Young's modulus, E, derived from V_p, V_s, ρ, and equations (12) and (13) utilize log-derived porosity measurements to estimate UCS.

The first impression one gets from seeing the fit between the measured strength and velocity data in the lab with the seven empirical relations appropriate for the UCS-Δt domain in Fig. 2(a) is that the scatter is remarkably large (a roughly ~100 MPa variation of strength) at any given Δt. It is noteworthy that except for equations (3) and (8) (derived for relatively strong rocks), all of the relations appear to badly underpredict the strength data for high slowness ($\Delta t > 100$ s/ft), or very low velocities ($V_p < 3000$ m/s). Such velocities are characteristic of very weak sandstone such as found in the Gulf of Mexico (GOM), but one needs to keep in mind that there is essentially no very weak sand represented in most of the strength data available except those provided by Bradford *et al.* (1998). Similarly, for fast, high strength rocks, equation (5) (derived for low strength rocks) does a particularly poor job of fitting the data.

It is worth noting that the estimated strengths from equations (4)~(7) and (9) are very similar to one another for high slowness (Δt higher than about 120 s/ft) sandstone (Fig. 2(a)) as most of these equations are derived for the GOM or Gulf Coast sandstone (Table 1). The variation of rock strength estimated using these relations is within 10 MPa. Fig. 2(a) also shows that the data of the very weak North Sea sandstone provided by Bradford *et al.* (1998) are fairly well fitted by equations (5)~(7) and very close to equations (4) and (9). These suggest that the rock strength of very weak sandstone from the GOM, the North Sea, and probably other sedimentary basins are characterized by a similar strength-velocity trend.

The use of Young's modulus for estimating UCS is less straightforward than that of velocity, because it generally requires the static-dynamic conversion or frequency correction. Equations (10) and (11) derived using Young's modulus fit the available data shown in Fig. 2(b) reasonably well in the lower Young's modulus range. Equation (11) tends to underestimate strength at low Young's modulus and overestimate strength at high Young's modulus. However, there is considerable scatter at any given value of Young's modulus (Fig. 2(b)).

With respect to porosity, both of the porosity relations listed in Table 1 seem to generally overestimate strength, except for the very lowest porosities. Equation (12) lies along the upper bound of UCS data, overestimating sandstone strength. Equation (12) was derived for very clean and well-consolidated sandstone, and should not be used to estimate UCS of sands with porosity higher than 0.37, where the predicted UCS starts to increase as porosity increases. Thus, care should be taken when equation (12) is used to estimate UCS of high porosity unconsolidated sandstone, typical of sea floor soft sediments. An extremely wide range of UCS (a range of ~300 MPa) is observed in the data at $\phi < 0.05$ (Fig. 2(c)). This suggests that porosity alone is not a good indicator for strength of low porosity sandstone. Such a wide scatter in rock strength can be attributed to different diagenetic processes (e.g. quartz vs. calcite cement, etc.) as sandstone are compacted.

Overall, it is probably fair to say that none of the equations in Table 1 do a very good job of fitting the data in Fig. 2. That said, it is important to keep in mind that the

Table 2. Empirical relationships between UCS and other physical properties in shale. Units used are: Δt: s/ft, E: GPa, and ϕ: fraction.

Eq. No.	UCS (MPa)	Regions where developed	General remarks	Reference
(14)	$0.77\ (304.8/\Delta t)^{2.93}$	North Sea	Mostly high porosity Tertiary shale	Horsrud (2001)
(15)	$0.43\ (304.8/\Delta t)^{3.2}$	Gulf of Mexico, USA	Pliocene and younger	-
(16)	$1.35\ (304.8/\Delta t)^{2.6}$	Worldwide	-	-
(17)	$0.5\ (304.8/\Delta t)^{3}$	Gulf of Mexico, USA	-	-
(18)	$10\ (304.8/\Delta t-1)$	North Sea	Mostly high porosity Tertiary shale	Lal (1999)
(19)	$7.97E^{0.91}$	North Sea	Mostly high porosity Tertiary shale	Horsrud (2001)
(20)	$7.22E^{0.712}$	-	Strong and compacted shale	-
(21)	$1.001\phi^{-1.143}$	-	Low porosity (ϕ < 0.1) high strength (~79 MPa) shale	Lashkaripour and Dusseault (1993)
(22)	$2.922\phi^{-0.96}$	North Sea	Mostly high porosity Tertiary shales	Horsrud (2001)
(23)	$0.286\phi^{-1.762}$	-	High porosity (ϕ > 0.27) shales	-

validity of any of these relations is best judged in terms of how well it would work for the rocks for which they were originally derived. Thus, calibration is extremely important before utilizing any of the relations shown. Equation (7), for example, seems to systematically underpredict most of the data in Fig. 2(a), yet worked very well for the relatively clean sands from the North Sea (Bradford *et al.*, 1998) since it was derived for an equivalently clean coarse-grained sandstone (Moos *et al.*, 1999).

Shale (or Mudstone)

The empirical relations for the strength of shale listed in Table 2 are based on model calibration for unconsolidated porous shales of Tertiary, or younger age except for equations (20) and (21) developed for rather strong shales. Note that equations (14)~(17), principally utilizing Δt for UCS estimation, are expressed in the same form of power law function of Δt with slightly different coefficients and exponents. These equations show nearly the same trends, providing a lower bound of the data (Fig. 3(a)). As mentioned above, it is prudent to underestimate strength to be conservative for applications to wellbore stability. However, the difference between these relations and the measured strengths is quite marked for fast, low Δt, rocks. In the low Δt range (< 100 s/ft), most of UCS data are higher by 50~100 MPa than the estimated values from equations (14)~(18). For slower rocks ($\Delta t > 100$ s/ft), these equations fit about 1/3 of data within 10 MPa. Still almost all data are located above the model predictions, implying that the UCS-Δt relationships provide only a lower bound of UCS of shale. It should be noted that equations (14)~(18) were calibrated for samples collected from the North Sea and Gulf of Mexico where high porosity, unconsolidated Tertiary or younger shales are dominant, while the majority of rock strength data presented in Fig. 3(a) came from shales that underwent a higher degree of diagenesis except for the North Sea shale (Horsrud, 2001). Thus, the use of the empirical equations leads to significant misfits in most cases, while estimating the North Sea shale data fairly well. Hence, such relations form a useful means for estimating shale strength in weak formations.

The two relations (Equations (19) and (20)) that utilize Young's modulus for estimating UCS show a remarkable difference in their general trends (Fig. 3(b)). This is because the two equations were developed based on markedly different rock types: equation (19) was developed for high porosity North Sea shale and equation (20) from relatively strong compacted shale. Perhaps the only conclusion that can be reached from this comparison is that equation (19) appears to predict shale strength in the lower E range (< 30 GPa) fairly well, while it considerably overestimates strength for rocks with higher E.

Equations (21)~(23) that utilize porosity are in a similar form of power law function and exhibit a similar decreasing trend of UCS as a function of ϕ (Fig. 3(c)). Unlike the case for sandstone, porosity appears to be a good parameter that can be used to estimate UCS of shale, especially for high porosities ($\phi > 0.1$). The three equations (21)~(23) all predict shale strength fairly well, fitting most of available data within 10 MPa. This is a very useful result since the weak shales are major constituents of most sedimentary basins and reservoir that often cause major wellbore stability problems. Their strength can be relatively well constrained with empirical relations that utilize porosity as a constitutive parameter.

Table 3. Empirical relationships between UCS and other physical properties in limestone and dolomite. Units used are: Δt: s/ft, E: GPa, and ϕ: fraction.

Eq. No.	UCS, MPa	Region where developed	General comments	Reference
(24)	$(7682/\Delta t)^{1.82}/145$	-	-	Militzer and Stoll (1973)
(25)	$10^{(2.44+109.14/\Delta t)}/145$	-	-	Golubev and Rabinovich (1976)
(26)	$13.8E^{0.51}$	-	Limestone with 10 < UCS < 300 MPa	
(27)	$25.1E^{0.34}$	-	Dolomite with 60 < UCS < 100 MPa	
(28)	$276(1-3\phi)^2$	Korobcheyev deposit, Russia	-	Rzhevsky and Novik (1971)
(29)	$143.8\exp(-6.95\phi)$	Middle East	Representing low to moderate porosity ($0.05 < \phi < 0.2$) and high UCS (30 < UCS < 150 MPa)	
(30)	$135.9\exp(-4.8\phi)$	-	Representing low to moderate porosity ($0 < \phi < 0.2$) and high UCS (10 < UCS < 300 MPa)	

While equations (21) and (23) estimate nearly the same UCS, equation (22) predicts slightly higher UCS (by 4 to 10 MPa) than the other two. In the lower porosity range (< 0.1), the fit is not as good but there are only a limited number of available data. Statistically, however, equation (21) appears to do a better job than the other two, which supports the fact that the former equation was developed based on low porosity and high strength shale.

Limestone and Dolomite

Table 3 lists seven empirical equations relating the strength of limestone and dolomite to measurable geophysical parameters. Both limestone and dolomite were analyzed as one carbonate rock group as there is insufficient information to separate the relation between strength and mechanical properties for the individual rock types. Unfortunately, this results in an extraordinarily wide variation of strength of limestone and/or dolomite with any given parameters (Fig. 4). For example, at low porosity, high velocity and high stiffness, strength varies by almost a factor of four, regardless of whether uses velocity, Young's modulus or porosity to estimate strength. Thus, empirical equations relating the strength of carbonate rocks to geophysical parameters do a fairly poor job whether considering velocity, modulus or porosity data, which emphasizes the importance of being able to calibrate strength in any given case. Nevertheless, there are some meaningful points we can extract from Fig. 4. While equation (24) gives statistically less satisfactory results than equation (25) to all of the data, the former equation defines a clear lower bound of measured strength data for any given Δt (Fig. 4(a)). As the importance of conservative strength estimation is emphasized for wellbore stability problems, equation (24) gives a good first approximation of the lower limit of carbonate rock strength when slowness (or velocity) is known. With respect to E, both equations (26) and (27) pass through the average of data set, predicting similar strength values (Fig. 4(b)). Equation (26) gives slightly better statistical results than equation (27), probably because the former equation utilized a wider range of UCS data than the latter when developed. In terms of porosity, equations (29) and (30) estimate average values of UCS for a given ϕ, while equation (28) defines an upper bound of the given data set. Thus, equation (28) is unfavorable at low porosities, and at porosities greater than 0.1, equations (29) and (30) seem to work well.

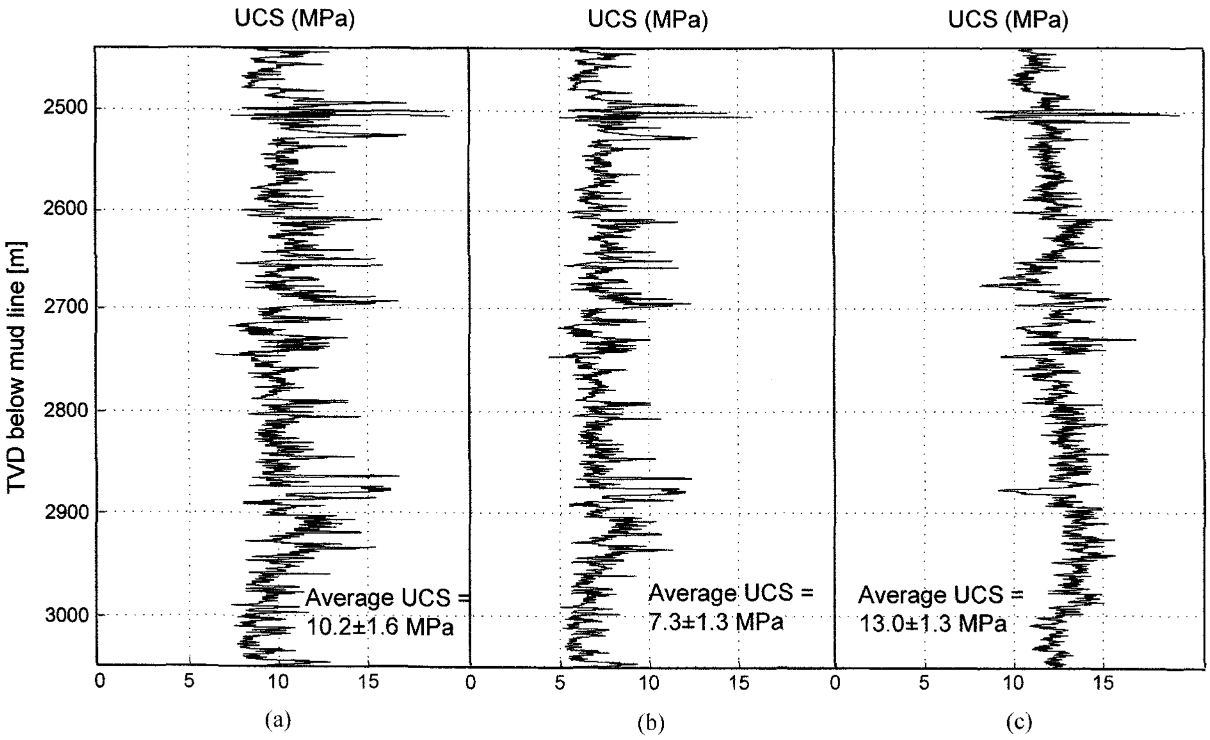

Fig. 5. Unconfined compressive strength (UCS) profile for a shale 2,440~3,050 m section in a vertical well in the Gulf of Mexico. The UCS profiles in (a), (b) and (c) were determined from geophysical logs using equations (14), (15) and (22), respectively.

Application to Empirical Strength Logs

An example of how rock strength is determined from geophysical logs using three of the empirical relations in Table 2 is illustrated in Fig. 5 for a shale section in a vertical well in the Gulf of Mexico. At the interval from 2,440 to 3,050 m, available logging data include compressional wave velocity and density. Using the velocity data, the UCS was determined using equations (14) and (15) (Fig. 5(a) and (b), respectively). While the overall shape of the two strength logs is approximately the same (as both are derived from the V_p data), the strength derived using equation (14) is 10.2 ± 1.6 MPa whereas that derived with equation (15) has an average strength of 7.3 ± 1.3 MPa. Porosity was derived from the density log assuming a matrix density of 2.65 g/cm^3 and a fluid density of 1.1 g/cm^3. The porosity-derived UCS is shown in Fig. 5(c) utilizing equation (22). Using this relation, the mean strength is 13.0 ± 1.3 MPa. It is noteworthy that there is an almost factor of two variation in mean strength. However, as equation (15) was derived for shales in the Gulf of Mexico region, it is probably more representative of actual strengths at depth. Again, while there are multiple options for determining strength from logs, it is best to use relations derived for formations characteristic of a particular region, and better yet, to calibrate the relation one proposes to use with laboratory measurements on representative core samples.

Conclusions

Estimation of unconfined compressive strength of sedimentary rocks through empirical relations were reviewed and discussed. It is clear that a few of the empirical relations discussed above appear to work fairly well for some subsets of the rocks tested in the laboratory. For example, if we focus on relatively weak rocks which are of most interest in cases of wellbore stability, use of Δt with equations (5) and (7) seem to provide a reasonable fit to the strength of weak sands. In addition, equation (13) allows one to utilize porosity measurements to estimate weak sand strength when porosity is relatively high ($\phi > 0.1$). With weak shales, equation (17) seems to work well when using Δt and equations (22) and (23) seem to work well at relatively high porosity ($\phi > 0.15$). It is more difficult to generalize about limestones and dolomites, but relation (24) appears to fit some of the weaker rocks with high velocity ($\Delta t < 80$ s/ft) and equation (29) appears to allow one to estimate strength from porosity data over a narrow range of porosities ($0.1 < \phi < 0.25$). While most of other relations do a poor job in fitting measured data for the reasons discussed above, it should not be forgotten that these relations were originally proposed because they fit some subset of data. Therefore, they do work, but not necessarily for the data represented by the published studies available to us. Moreover, a number of the strength-physical property correlations are especially useful in applications related to wellbore stability by providing a lower bound estimate of in situ rock strength. These relations may provide a good first approximation of the lower strength bound when no other information on rock strength is available. It is somewhat obvious, however, that calibration of empirical relations between strength and physical properties is generally required for any correlation to be used with some degree of confidence.

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