

A BAYESIAN VIEW ON FARADAY ROTATION MAPS – SEEING THE MAGNETIC POWER SPECTRUM IN CLUSTERS OF GALAXIES

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ABSTRACT

Magnetic fields are an important ingredient of galaxy clusters and are indirectly observed on cluster scales as radio haloes and radio relics. One promising method to shed light on the properties of cluster wide magnetic fields is the analysis of Faraday rotation maps of extended extragalactic radio sources. We developed a Fourier analysis for such Faraday rotation maps in order to determine the magnetic power spectra of cluster fields. In an advanced step, here we apply a Bayesian maximum likelihood method to the RM map of the north lobe of Hydra A on the basis of our Fourier analysis and derive the power spectrum of the cluster magnetic field. For Hydra A, we measure a spectral index of $-5/3$ over at least one order of magnitude implying Kolmogorov type turbulence. We find a dominant scale of about 3 kpc on which the magnetic power is concentrated, since the magnetic autocorrelation length is $\lambda_B = 3 \pm 0.5$ kpc. Furthermore, we investigate the influences of the assumption about the sampling volume (described by a window function) on the magnetic power spectrum. The central magnetic field strength was determined to be $\sim 7 \pm 2 \mu\text{G}$ for the most likely geometries.

Key words : clusters of galaxies – magnetic fields – rotational measure

I. INTRODUCTION

One method to investigate cluster magnetic field structure and strength is the detection of the Faraday rotation effect (for recent reviews see Carilli & Taylor 2002; Widrow 2002; Govoni & Feretti 2004). This effect is observed whenever linearly polarised radio emission passes through a magnetised medium. A linearly polarised wave can be described by two circularly polarised waves. Their motion along magnetic field lines in a plasma introduces a phase difference between the two waves resulting in a rotation of the plane of polarisation. If the Faraday active medium is external to the source of the polarised emission, one expects the change in polarisation angle to be proportional to the squared wavelength. The proportionality factor is called rotation measure (RM). This quantity can be evaluated in terms of the line of sight integral over the product of the electron density and the magnetic field component along the line of sight.

Enßlin & Vogt (2003) proposed a method to determine the magnetic power spectra by Fourier transforming RM maps. Based on these considerations, Vogt & Enßlin (2003) applied this method and determined the magnetic power spectrum of three clusters (Abell 400, Abell 2634 and Hydra A) from RM maps of radio sources located in these clusters. They determined field strengths of $\sim 12 \mu\text{G}$ for the cooling flow cluster Hydra A, $3 \mu\text{G}$ and $6 \mu\text{G}$ for the non-cooling flow clusters Abell 2634 and Abell 400, respectively. Their analysis revealed spectral slopes of the power spectra with

spectral indices $-2.0 \dots -1.6$. However, it was realised that using the proposed analysis, it is difficult to reliably determine differential quantities such as spectral indices due to the complicated shapes of the used emission regions which lead to a redistribution of magnetic power within the spectra.

In order to determine a power spectrum from observational data, maximum likelihood estimators are widely used in astronomy. These methods and algorithms have been greatly improved especially by the Cosmic Microwave Background (CMB) analysis which is facing the problem of determining the power spectrum from large CMB maps. Kolatt (1998) proposed to use such an estimator to determine the power spectrum of a primordial magnetic field from the distribution of RM measurements of distant radio galaxies.

Based on the initial idea of Kolatt (1998), the methods developed by the CMB community (especially Bond *et al.* 1998) and our understanding of the magnetic power spectrum of cluster gas (Enßlin & Vogt 2003), we present here a Bayesian maximum likelihood approach to calculate the magnetic power spectrum of cluster gas given observed Faraday rotation maps of extended extragalactic radio sources (Sect. II). The power spectrum enables us to determine also characteristic field length scales and strength. After testing our method on artificial generated RM maps with known power spectra (Sect. III), we apply our analysis to a Faraday rotation map of Hydra A (Sect. IV). The data were kindly provided by Greg Taylor. In addition, this method allows to determine the uncertainties of our measurement and, thus, we are able to give errors on the calculated quantities. Based on these calcula-

tions, we derive statements for the nature of turbulence for the magnetised gas in the cooling core galaxy cluster Hydra A (Sect. V).

Throughout the rest of the paper we assume a Hubble constant of $H_0 = 70 \text{ km s}^{-1} \text{ Mpc}^{-1}$, $\Omega_m = 0.3$ and $\Omega_\Lambda = 0.7$ in a flat universe. All equations follow the notation of Enßlin & Vogt (2003).

II. MAXIMUM LIKELIHOOD ANALYSIS

One of the most common used methods of Bayesian statistic is the maximum likelihood method. The likelihood function for a model characterised by p parameters a_p is equivalent to the probability of the data Δ given a particular set of a_p and can be expressed in the case of (near) Gaussian statistic of Δ as

$$\mathcal{L}_{\Delta}(\dagger) = \frac{\infty}{(\epsilon\pi)^{\sqrt{|\mathcal{C}|}}|\mathcal{C}|^{\infty/\epsilon}} \cdot \exp\left(-\frac{\infty}{\epsilon}\Delta^T \mathcal{C}^{-\infty} \Delta\right), \quad (1)$$

where $|\mathcal{C}|$ indicates the determinant of a matrix, $\Delta_i = RM_i$ are the actual observed data, n indicates the number of observationally independent points and $\mathcal{C} = C(a_p)$ is the covariance matrix. This covariance matrix can be defined as

$$C_{ij}(a_p) = \langle \Delta_i^{obs} \Delta_j^{obs} \rangle = \langle RM_i^{obs} RM_j^{obs} \rangle, \quad (2)$$

where the brackets $\langle \rangle$ denote the expectation value and, thus, $C_{ij}(a_p)$ describes our expectation based on the proposed model characterised by a particular set of a_p 's. Now, the likelihood function $\mathcal{L}_{\Delta}(\dagger)$ has to be maximised for the parameters a_p . Note, that although the magnetic fields might be non-Gaussian, the RM should be close to Gaussian due to the central limit theorem. Observationally, RM distributions are known to be close to Gaussian (*e.g.*, Taylor & Perley 1993; Feretti *et al.* 1999a,b; Taylor *et al.* 2001).

Since we are interested in the magnetic power spectrum, we have to find an expression for the covariance matrix $C_{ij}(a_p) = C_{RM}(x_{\perp i}, x_{\perp j})$, where $x_{\perp i}$ is the displacement of point i from the z -axis, which can be identified as the RM autocorrelation $\langle RM(x_{\perp i}) RM(x_{\perp j}) \rangle$. This has then to be related to the magnetic power spectra. For a line of sight parallel to the z -axis and displaced by x_{\perp} from it, the RM arising from polarised emission passing from the source $z_s(x_{\perp})$ through a magnetised medium to the observer located at infinity is expressed by

$$RM(x_{\perp}) = a_0 \int_{z_s(x_{\perp})}^{\infty} dx n_e(x) B_z(x), \quad (3)$$

where $a_0 = e^3/(2\pi m_e^2 c^4)$, $x = (x_{\perp}, z)$, $n_e(x)$ is the electron density and $B_z(x)$ is the magnetic field component along the line of sight.

In the following, we will assume that the magnetic fields in galaxy clusters are isotropically distributed throughout the Faraday screen. If one samples such

a field distribution over a large enough volume they can be treated as statistically homogeneous and statistically isotropic. Therefore, any statistical average over a field quantity will not be influenced by the geometry or the exact location of the volume sampled. Following Enßlin & Vogt (2003), we can define the elements of the RM covariance matrix using the RM autocorrelation function $C_{RM}(x_{\perp i}, x_{\perp j}) = \langle RM(x_{\perp i}) RM(x_{\perp j}) \rangle$ and introduce a window function $f(x)$ which describes the properties of the sampling volume

$$C_{RM}(x_{\perp}, x'_{\perp}) = \tilde{a}_0^2 \int_{z_s}^{\infty} dz \int_{z'_s}^{\infty} dz' f(x) f(x') \times \langle B_z(x_{\perp}, z) B_z(x'_{\perp}, z') \rangle, \quad (4)$$

where $\tilde{a}_0 = a_0 n_{e0}$, the central electron density is n_{e0} and the window function is defined by

$$f(x) = \mathbf{1}_{\{x_{\perp} \in \Omega\}} \mathbf{1}_{\{z \geq z_s(x_{\perp})\}} \mathbf{g}(x) \mathbf{n}_e(x) / \mathbf{n}_{e0}, \quad (5)$$

where $\mathbf{1}_{\{\text{condition}\}}$ is equal to unity if the condition is true and zero if not and Ω defines the region for which RM 's were actually measured. The electron density distribution $n_e(x)$ is chosen with respect to a reference point x_{ref} (usually the cluster centre) such that $n_{e0} = n_e(x_{ref})$, *e.g.*, the central density, and $B_0 = \langle B^2(x_{ref}) \rangle^{1/2}$. The dimensionless average magnetic field profile $g(x) = \langle B^2(x) \rangle^{1/2} / B_0$ is assumed to scale with the density profile such that $g(x) = (n_e(x) / n_{e0})^{\alpha_B}$.

With the understanding of the magnetic power spectrum of cluster gas $\varepsilon_B(k)$ as described by Enßlin & Vogt (2003), one can then derive the expression for the covariance matrix $C_{RM}(x_{\perp}, x'_{\perp} = x_{\perp} + r_{\perp})$.

$$C_{RM}(x_{\perp}, x_{\perp} + r_{\perp}) = 4\pi^2 \tilde{a}_0^2 \int_{z_s}^{\infty} dz f(x) f(x+r) \times \int_{-\infty}^{\infty} dk \varepsilon_B(k) \frac{J_0(kr_{\perp})}{k}, \quad (6)$$

where $J_0(kr_{\perp})$ is the 0th Bessel function.

Since the magnetic power spectrum is the interesting function, we parametrise $\varepsilon_B(k) = \sum_p \varepsilon_{B_p} \mathbf{1}_{\{k \in [k_p, k_{p+1}]\}}$, where ε_{B_p} is constant in the interval $[k_p, k_{p+1}]$, leading to

$$C_{RM}(\varepsilon_{B_p}) = 4\pi^2 \tilde{a}_0^2 \int_{z_s}^{\infty} dz f(x) f(x+r) \times \sum_p \varepsilon_{B_p} \int_{k_p}^{k_{p+1}} dk \frac{J_0(kr_{\perp})}{k}, \quad (7)$$

where the ε_{B_p} are to be understood as the model parameter a_p for which the likelihood function $\mathcal{L}_{\Delta}(\dagger)$ has to be maximised given the Faraday data Δ .

In order to maximise the likelihood function, we employed the algorithm developed by Bond *et al.* (1998). For more detailed description of its implementation, we refer to Vogt & Enßlin (submitted).

III. TESTING THE ALGORITHM

In order to test our algorithm, we applied our maximum likelihood estimator to generated RM maps with a known magnetic power spectrum $\varepsilon_B(k)$. Enßlin & Vogt (2003) give a prescription (their Eq. (37)) for the relation between the amplitude of RM , $|R\hat{M}(k_\perp)|^2$, and the magnetic power spectrum in Fourier space. Using this relation, we assumed for the magnetic field power spectra a Kolmogorov type power spectrum $\varepsilon_B^{\text{obs}}(k) \propto k^{-5/3}$ (for $k \geq k_c$, where k_c is the energy injection scale).

As Faraday screen, we assumed a box with sides being 150 kpc long and a depth of $L = 300$ kpc. For the sake of simplicity, we assumed a uniform electron density profile with a density of $n_{e0} = 0.001 \text{ cm}^{-3}$. We applied the Fourier analysis as described in Enßlin & Vogt (2003) to a part of this map in order to mimick a limited source size. The resulting power spectrum is shown in Fig. 1 as dashed line in comparison with the input power spectrum as dotted line.

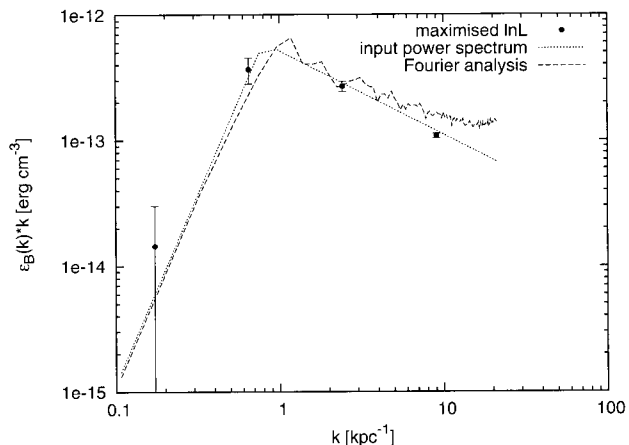


Fig. 1.— Power spectra for a simulated RM map with a known power spectrum. The input power spectrum is shown in comparison to the one found by the Fourier analysis as described in Vogt & Enßlin (2003) and the one which was derived by our maximum likelihood estimator. One can see the good agreement within one σ between input power spectrum and the power spectrum derived by the maximum likelihood method.

The maximum likelihood method is numerically limited by computational power since it involves matrix multiplication and inversion, where the latter is a N^3 process. Therefore, we chose to randomly average neighbouring points with a scheme which led to a map with spatially inhomogeneously resolved cells. The resulting map is highly resolved on top and lowest on the bottom with some random deviations which makes it similarly to the error weighting of the observed data. We used $N = 1500$ independent points for the analysis.

The resulting power spectrum is shown as filled circles with $1\text{-}\sigma$ error bars in Fig. 1. As can be seen

from this figure, the input power spectrum and the power spectrum derived by the maximum likelihood estimator agree well within the one σ level. Integration over this power spectrum results in a field strength of $(4.7 \pm 0.3)\mu\text{G}$ in agreement with the input magnetic field strength of $5\mu\text{G}$.

IV. APPLICATION TO HYDRA A

We applied this maximum likelihood estimator introduced and tested in the last sections to the Faraday rotation map of the north lobe of the radio source Hydra A (Taylor & Perley 1993). The data were kindly provided by Greg Taylor. For a description of the window function $f(x)$, we refer to Vogt & Enßlin (2003). However, the scaling parameter α_B between electron density profile and global magnetic field distribution has to be considered as a free parameter.

For this purpose, we used a high fidelity RM map presented in Vogt *et al.* (2004) which was generated by the newly developed algorithm *Pacman* (Dolag *et al.* 2004) using the original polarisation data. The *Pacman* map which was used is shown in the right panel of Fig. 2.

For the same reasons as mentioned in Sect. III, we averaged the data. The analysed RM map was determined by a gridding procedure. The original RM map was divided into four equally sized cells. In each of these the original data were averaged employing an error weighting scheme. Then the cell with the smallest error was chosen and again divided into four equally sized cells and the original data contained in the so determined cell were averaged. The last step was repeated until the number of cells reached a defined value $N = 1500$. This is partly due to the limitation by computational power but also partly because of the desired suppression of small scale noise by a strong averaging of the noisy regions.

The final RM map which was analysed is shown in Fig. 2. The most noisy regions in Hydra A are located in the coarsely resolved northernmost part of the lobe. We chose not to resolve this region any further but to keep the large-scale information which is carried by this region.

V. RESULTS AND DISCUSSION

Based on the described treatment of the data and a properly defined window function, we calculated power spectra for various scaling exponents α_B . The resulting power spectra are plotted in Fig. 3.

In Fig. 3, one can see that the power spectrum derived for $\alpha_B = 1.0$ has a completely different shape whereas the other power spectra show only slight deviation from each other and are vertically displaced implying different normalisation factors, *i.e.*, central magnetic field strengths which increase with increasing α_B . The straight dashed line which is also plotted in Fig. 3 indicates a Kolmogorov like power spectrum

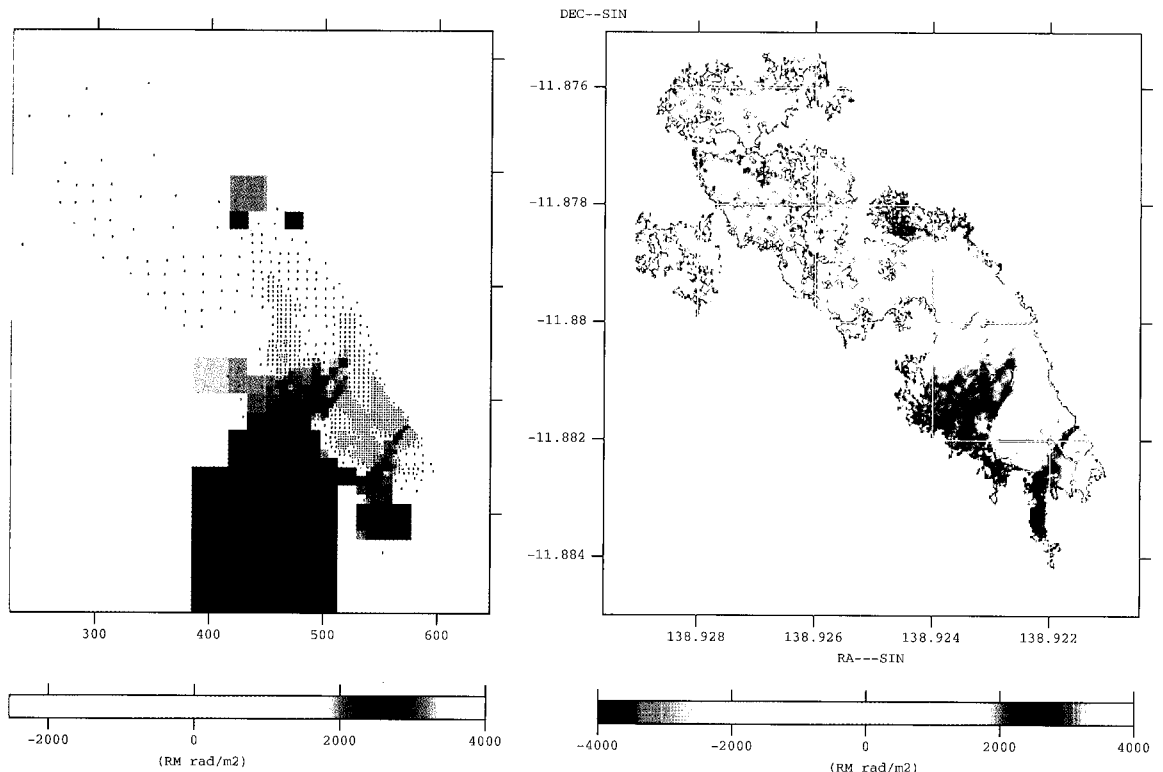


Fig. 2.— The final RM map from the north lobe of Hydra A which was analysed with the maximum likelihood estimator; left: error weighted map. The dots indicate the coordinates which correspond to the appropriate error weighted RM value, which resulted from averaging over the indicated area. Right: original *Pacman* map. Note that the small scale noise for the diffuse part of the lobe is averaged out and only the large scale information carried by this region is maintained.

being equal to $5/3$ in our prescription. One can see, that the power spectra follow this slope over at least on order of magnitude.

The likelihood function offers the possibility to calculate the actual probability of a set of parameters given the data (see Eq. (1)). Thus, we calculated the log likelihood $\ln \mathcal{L}_\Delta(\varepsilon_{B,p})$ value for various power spectra derived for the different window functions varying in the scaling exponent α_B . In Fig. 4, the log likelihood is shown in dependence of the used scaling parameter α_B .

As can be seen from Fig. 4, there is a plateau of most likely scaling exponents α_B ranging from 0.1 to 0.8. An $\alpha_B = 1$ seems to be very unlikely for our model. The sudden decrease for $\alpha_B < 0.1$ might be due to non-Gaussian effects. The magnetic field strength derived for this plateau region ranges from $9 \mu\text{G}$ to $5 \mu\text{G}$. The correlation length of the magnetic field λ_B was determined to range between 2.5 kpc and 3.0 kpc whereas the RM correlation length was determined to be in the range of 4.5...5.0 kpc. These ranges have to be considered as a systematic uncertainty since we are not yet able to distinguish between these scenarios observationally. Another systematic effect might be given by uncertainties in the electron density as incorporated

in the window function (see Eq. (5)) itself. Varying the electron density normalisation parameters leads to a vertical displacement of the power spectrum while keeping the same shape.

VI. CONCLUSIONS

We presented a maximum likelihood estimator for the determination of cluster magnetic field power spectra from RM maps of extended polarised radio sources. We introduced the covariance matrix for RM under the assumption of statistically homogeneously distributed magnetic fields throughout the Faraday screen. We successfully tested our approach on simulated RM maps with known power spectra.

We applied our approach to the RM map of the north lobe of Hydra A. We calculated different power spectra for various window functions being especially influenced by the scaling parameter between electron density profile and global magnetic field distribution. The scaling parameter α_B was determined to be most likely in the range of 0.1...0.8.

We realised that there is a systematic uncertainty in the values calculated due to the uncertainty in the window parameters. Taking this into account, we deduced

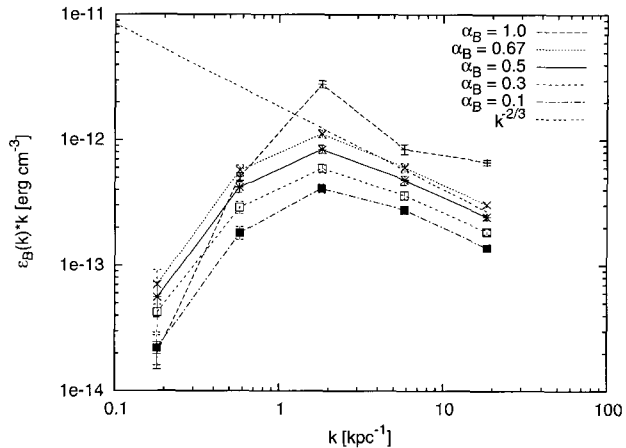


Fig. 3.— Power spectra for various scaling exponents α_B in the relation $B(r) \sim n_e(r)^{\alpha_B}$ of the window function were used.

for the central magnetic field strength in the Hydra A cluster $B = (7 \pm 2) \mu\text{G}$ and for the magnetic field correlation length $\lambda_B = (3.0 \pm 0.5) \text{ kpc}$. If the geometry uncertainties could be removed the remaining statistical errors are an order of one magnitude smaller. The difference of these values from the ones found in an earlier analysis of the same dataset of Hydra A which yielded $B = 12 \mu\text{G}$ and $\lambda_B = 1 \text{ kpc}$ Vogt & Enßlin (2003) is a result of the improved *RM* map using the *Pacman* algorithm (Dolag *et al.* 2004; Vogt *et al.* 2004) and a better knowledge of the magnetic cluster profile, *i.e.*, here $\alpha_B \approx 0.5$ (instead of $\alpha_B = 1.0$ as in Vogt & Enßlin 2003).

The cluster magnetic field power spectrum of Hydra A follows a Kolmogorov like power spectrum over at least one order of magnitude. However, from our analysis it seems that there is a dominant scale $\sim 3 \text{ kpc}$ on which the magnetic power is concentrated.

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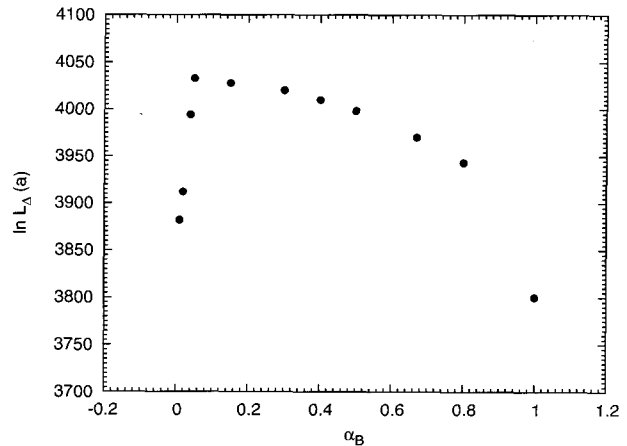


Fig. 4.— The log likelihood $\ln \mathcal{L}_\Delta(-)$ of various power spectra assuming different α_B . One can see that $\alpha_B = 0.1 \dots 0.8$ are in the plateau of maximum likelihood. The sudden decrease for $\alpha_B < 0.1$ in the likelihood might be due to non-Gaussian effects becoming too strong.

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