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Hybrid Representations for Enveloping Modeling in Gearing

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Abstract – Hybrid method of representing geometric entities as combination of boundary (B-rep) and functional (F-rep) representations is presented which can be used as a basis of solid modeling kernel. It contains whole functionality of classic B-rep kernel, and also supports enveloping (sweep of solid body). Principles and keysolutions are considered. Example of a real task that comes from gearing is provided.

Keywords: **CAD, gearing, B-rep, F-rep, enveloping**

1. Introduction

Functionality of modern 3D solid modeling systems depends on geometric kernel. Popular geometric kernels, like ACIS or ParaSolid, are based on boundary representation (B-rep) of model. A solid body is considered as set of faces that are oriented surfaces, limited by boundaries, or simply speaking, by edges. Every time the model is changed by appling boolean operations or local operations (fillet, chamfer, draft, etc.) the faces set is recalculated. This approach shows good performance in meshing and visualization, and also in drawings, since edges set is part of solid body, and therefore can be obtained directly. There and many other factors make B-rep the main solution for geometric kernels of CAD systems. At the same time it has some limitations. Most dramatic of them, it's virtually impossible to sweep a solid body along a path. This task is also called eveloping and has many applications, such as gearing or the design of cabinets. There are papers dedicated to generating of swept solid [3, 6, 7]. But they still have some drawbacks from gearing viewpoint. For example, in approach, presented in [3], the original body must be polyhedron, and it still gives us an approximate solution, this makes it useless in gearing. The second problem is that currently implemented algorithms for boolean operations may produce funny results or perhaps not even handle some cases, e.g., if the bodies for operation have coincident faces. As an alternative of B-rep, there is functional representation (F-rep). In F-rep, it's assumed that the solid body is described by a function that is always positive inside the body, negative outside the body and zero on the border [11]. F-rep solves a number of problems of gearing [10]. There is also [12], that basically repeats the solutions from [10] in English.

Pure F-rep is useless in design because there are many difficulties in visualization and especially in drawings. Existing algorithms [9] don't satisfy customer's requirements since these algorithms demand excessive resources. Another hybrid system based on voxel data and F-rep is described in [1].

This paper describes solutions for a geometric kernel that uses B-rep and F-rep representations and is able to use them both in the vital calculations. Although there is no simple way to covert B-rep to F-rep, normally a 3D solid model is presented in CSG (constructive solid geometry) form, e.g. as sequence of topology and boolean operations, normally solid body is presented in B-rep, but F-rep can be used when neccessary.

2. Functional Representation of Standard Operations

In CAD systems, now there is main way to receive 3D model when user has to desribe a set of 2,5D models, that basically 2D sketch with the law that says how to receive 3D model from a 2D one, it may be by vectors of extrusion (extrude operation), axis and angle of rotation (revovle operation), another 2D sketch (sweep operation), etc. Then the user has to apply boolean operations (unite, subtract, intersect) over 2,5D models. Also there are local operations that are applyed over existing 3D models (fillets, chamfers, face draft, etc).

In this section we shall show an example of an F-rep of these operations. Let's consider some properties of 2,5D model first:

1. Every 2,5D model contains at least 1 basic closed sketch:
   $C^k = \{ C_i^k \}$,where C_i^k – figure of skecth
   Figure of sketch:
   $C_i^k = < t_i^k, l_i^k, a_i^k, p_i^k >$, where:

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t_i^k – type of figure (line, arc of circle, etc),
l_i^k –linear parameters of figure,
a_i^k –angular parameters of figure,
p_i^k –control point of figure.
Depending of type of figure and method of representation, the parameters set may vary. The line can be presented by two control points or one control point, one linear and one angular parameter. Same for a circular arc.

2. Every point (x, y, z) of R^3 space has a corresponing point (x_c, y_c) on sketch plane

So, the task of buliding a 3D defining function can be changed by the defining function $F_c(x_c, y_c)$ in 2D space, that is a well-known task of recognision, if a given point is inside a closed sketch or outside of it. We are not going to concentrate on this task,since it has been considered by many authors, for instance in [2]. Instead we will consider examples how to get to point (x_c, y_c) from (x, y, z), which makes sense in the gearing area. First example is the revolve operation. Imagine that we rotate sketch around an axis, belonging to a sketch plane. In order to simplify our understanding, assume that this axis is an X-axis (otherwise, depending on implementation we have to provide a more complex formula with the same principle or apply a 2D transformation first).

$$\begin{cases} x_c = x \\ y_c = \sqrt{y^2+z^2} \end{cases} \quad (1)$$

The next example is the extrude operation. We assume that the sketch plane is XY and extrude direction is a vector $d=(x_d, y_d, z_d)$, where the length of this vector defines the depth of the extrusion.

$$\begin{cases} z & z \leq 0 \\ z_d - z & z \geq z_d \\ F_c\left(x - x_d\frac{z}{z_d},\ y - y_d\frac{z}{z_d}\right) & 0<z<z_d \end{cases} \quad (2)$$

If the extrusion is defined as an extrusion to a pre-defined geometry, we can apply the "intersect" operation over (2) and the halfspace of a given geometry.

A third example is the sweep of planar sketch along a helix, that generates a screw surface with h as a pitch and a rotation around axis X.

$$\begin{cases} x_c = x \bmod h - P_Y\alpha \\ y_c = (y+z)/(\sin\alpha + \cos\alpha) \end{cases} \quad (3)$$

Booolean operations over solid bodies are changed by R-functions [11] over defining functions of bodies. Union is changed to R-disjunction V_R, and intersection is changed to R-conjunction Λ_R. The following functions can be assumed as R-functions:

$$x \wedge_R y = \min(x, y) = \frac{x+y-|x-y|}{2}$$
$$x \vee_R y = \max(x, y) = \frac{x+y+|x-y|}{2} \quad (4)$$

3. Model of Enveloping Process

According to classic differential geometry, enveloping of family $F(x, y, z, t)$ of surfaces with equation $F(x, y, z)=0$ in each value of parameter t can be presented by the following system of equations [7, 8]:

$$\begin{cases} F(x, y, z, t) = 0 \\ \partial F/\partial t = 0 \end{cases} \quad (5)$$

Such an approach is called a differential or classic.

It's a traditional method in gearing. Initially, it was supposed to be solved manually, by excluding t parameter from (5). Despite its evidence and various forms of presentation the mentioned classic method has a number of drawbacks:

- method doesn't distinguish between the points of the actual touching and the so called "from-inside" touching which is impossible for points of sweep surface;
- method allows us to define only the points of local touching, i.e. differencial vicinity of the assumed points of contact;
- function $F(x, y, z, t)$ has to be differentiated, it cann't be applied for points, belonging to non-tangent edges;
- (5) is a criteria of belonging or not beloning to swept surface only. If point doesn't satisfy (5) there is no way to find out, whether the point is inside of outside the swept body.

A method that is called the non-differential [10] and is more adequate to actual enveloping process doesn't have these drawbacks. According to the non-differential method, defining function F_s of swept body:

$$F_s(x, y, z) = -\max_{t \in T} F(x, y, z, t) \quad (6)$$

where $F(x, y, z)$ is the defining function of body to sweep. Non-differential equalent of (5) is $F_s=0$. Using **max**, rather then **min**, guarantees that points of real touching will be found. Since the (6) doesn't contain derivatives it can be applied as for points, belonging to surface, as for points, belonging to non-tangent edges. A sign of F_s in (6) is important because even if the point doesn't belong to a swept surface, the value of (6) tells us whether a point is inside or outside a swept body. In (5) and (6) we assumed that the sweep law depends on only one independent parameter t. This assumption is right for gearing and for most manufacturing processes. But there are cases, when this assumption is wrong. For example in automative design, when a designer wants to figure out a form of wheel space. The wheel's position depends on steering and the ocassional defects on the road. These two factors are independent. For such cases we have to use a multi-parametric case (7) of (6).

$$F_s(x, y, z) = -\max_{t_1, t_2, \dots, t_n \in T} F(x, y, z, t_1, t_2, \dots, t_n) \qquad (7)$$

Geometric models will be useless in CAD systems, if there is no way to build a triangular mesh, which is needed for displaying the 3D model and for stereolithography (ability to generate STL files). Visualization is also an important thing for generating high-quality snapshots of 3D models. Meshing of non-enveloping surfaces is implemented within existing geometric kernel based on B-rep. For enveloping surfaces we can use the method described in [3] in the case of a polyhedron, otherwise we have to employ F-rep for meshing. A traditional algorithm is the marching cubes algorithm [9], that builds a 3D grid and calculates the interesection of surface with each cell. In order to get results, how the original model looks like, the grid has to bebig enough and require multi-processors calculations. It's not acceptable in real CAD systems, since the engineers' workspace is equipped with a PC. The good news is, since we have a B-rep of model to sweep, we can limit the set of cells as belonging to faces of the original body only. For visualization of the enveloping surface we suggest using an algorithm based on the enveloping surface's section, it can be either a scan-line or a Z-buffer algorithm. An algorithm for generating sections of an enveloping surface is described in [5]. Another possibility for visualization is highlighting points that are formed by non-tangent edges of the source body. From a mechanical viewpoint, these parts of an enveloping surface are wrong, and an engineer should minimize it.

4. Example

Let's consider an example of a real design of an enveloping surface, that comes from the design of gears [4]. This example shows the steps that should be done for implementation of the presented approach and can be a good test case for debugging purposes. Imagine that we want to design a worm of a spiroid gear. The screw surface of this worm is the result of grinding by a grinding wheel along a helix (see Fig. 1), where the pitch is $2\pi P_\gamma$. The first step is a model of a grinding wheel in a $X_0Y_0Z_0$ coordinate system. Assume that $\boldsymbol{K}$ is the set of points belonging to a wheel.

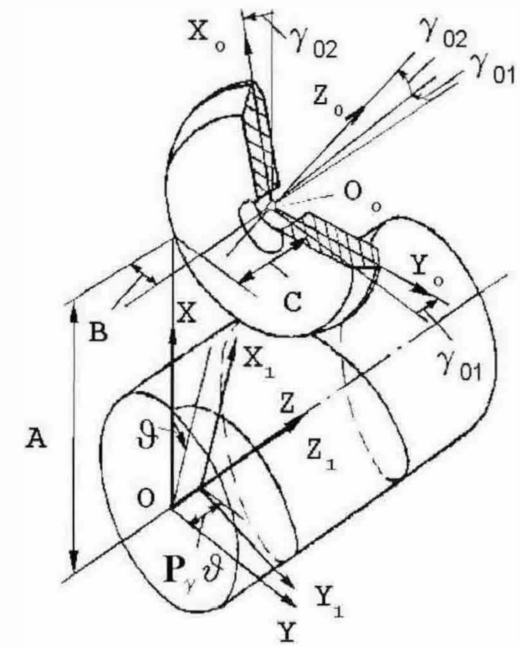

Fig. 1. Grinding by grinding wheel along helix.

$\boldsymbol{K}$ consists of 5 sets:

- sets $\boldsymbol{A_L}$ and $\boldsymbol{A_R}$ of the points are situated between the axis z_0 and left and right conic surfaces;
- set $\boldsymbol{C}$ of points is situated inside the outer wheel cylinder;
- sets $\boldsymbol{T_L}$ and $\boldsymbol{T_R}$ of the points situtateed to the left and to the right from the face planes of the wheel.

So the B-rep of the wheel are these surfaces with limiting circular edges. If a B-rep of the grinding wheel already exists, an F-rep of the grinding wheel can be recieved directly from the B-rep:

$$K = A_R \cap A_L \cap C \cap T_R \cap T_L \qquad (8)$$

Changing symbols of sets to equations of surfaces, and applying formula (4) we get the defining function of grinding wheel:

$$F_K = F_{A_R} \wedge_R F_{A_L} \wedge_R F_C \wedge_R F_{T_R} \wedge_R F_{T_L} \qquad (9)$$

If B-rep doesn't exist yet, user has to build the 2.5D model of the grinding wheel. Since the grinding wheel is revolution and following (1) user should build sketch $\boldsymbol{C^k}$ to revolve. $\boldsymbol{C^k}$ is limited by 5 figures:

- lines $\boldsymbol{A_L^k}$ and $\boldsymbol{A_R^k}$ that correspond to cones A_L and A_R;
- line $\boldsymbol{H^k}$ that corresponds to cylinder C;
- lines $\boldsymbol{T_L^k}$ and $\boldsymbol{T_R^k}$ that correspond planes T_L and T_R.

Assuming d as diameter of wheel, α_L, α_R - cones' angles, x_{0L}, x_{0R} position of face planes in sketch's coordinate system, L - height of cylinder C, defining function of sketch is:

$$F_c = (d-y) \wedge_R (x - x_{0L}) \wedge_R (x_{0R} - x) \wedge_R$$
$$(x\tan(\alpha_L) + d) \wedge_R ((L-x)\tan(\alpha_R) + d) \qquad (10)$$

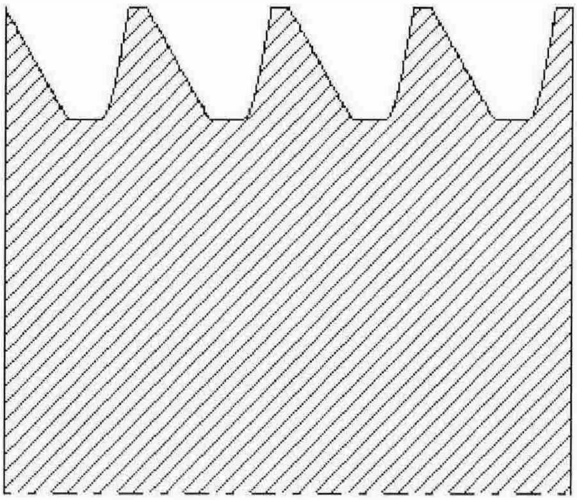

Fig. 2. Section view of spiroid worm.

Next step is law $A(t)=M(t)\cdot M_0$, representing movings of wheel, and generating family of functions F_1:

$$F_1(x, y, z, t)=F\left(A^{-1}(t)\begin{pmatrix} x \\ y \\ z \end{pmatrix}\right) \tag{11}$$

where M_0 is transformation from coordinate system XYZ to coordinate system $X_0Y_0Z_0$

$$M_0=\begin{pmatrix} \cos\gamma_{02} & 0 & -\sin\gamma_{02} & A \\ \sin\gamma_{01}\sin\gamma_{02} & \cos\gamma_{01} & \sin\gamma_{01}\cos\gamma_{02} & B \\ \cos\gamma_{01}\sin\gamma_{02} & -\sin\gamma_{01} & \cos\gamma_{01}\cos\gamma_{02} & C \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{12}$$

Assuming $t=\theta$, $M(\theta)$ desribes motion along helix, it's tranformation from coordinate system XYZ to coordinate system $X_1Y_1Z_1$:

$$M(\theta)=\begin{pmatrix} \cos\theta & \sin\theta & 0 & 0 \\ -\sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & P_\gamma\theta \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{13}$$

Matching $F_1(x, y, z, t)$ in (6) and applying intersection with defining function of blank (cylinder around axis Z), we get section view by ZX plane of enveloping surface grinded by wheel with α_L=30 and α_R=8 (Fig. 2).

5. Conclusion

Current situation in computer aided gear design is not free of problems. Companies who design gears have to spend money two times: one time for one of universal CAD system for design of cutters and other mechanical part, second time for system that does gearing calculations. The worse thing is that they have to waste time of engineers, since most of existing gearing systems are designed for concrete kind of gears and have special requirements for model of cutter (usually, it looks like set of values of parameters) and cannot easy import existing 3D models. Hybrid method presented in this paper is an attempt to solve these problems. Comparing with another hybrid of pure F-rep systems, it doesn't make more limitations that classic B-rep system has, engineer is able to use anything that he/she uses in existing 3D modeling systems. F-rep extension is not related to gearing only, it's unversal tool to solve any tasks, related to enveloping. So this kernel has good chances to grow over just scientific prototype and become part of real tool for engineers.

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