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H-FUZZY SUPRATOPOLOGICAL SPACES

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Abstract. We introduce the concepts of a gradation of supraopenness, $S(S^*)$-gradation preserving maps, and weakly $S(S^*)$-gradation preserving maps. And we investigate several properties of such concepts.

1. Introduction

Fuzzy topological spaces were first introduced in the literature by Chang [2] who studied a number of the basic concepts including fuzzy continuous maps and compactness. And fuzzy toplogical spaces are a very natural generalization of topological spaces. In [3], R. N. Hazra et.al introduced a new fuzzy topology and fuzzy topological space in terms of lattices L and L', both of which were taken to be $I = [0, 1]$. In this paper, we will call the new fuzzy topology an H-fuzzy topology. 0_X and 1_X will denote the characteristic functions of the crisp sets $\emptyset$ and X, respectively. An H-fuzzy topological space [3] is a pair (X, τ), where X is a non-empty set and $\tau : I^X \to I$ is a mapping satisfying the following properties:

(O1) $\tau(0_X) = \tau(1_X) = 1$.

(O2) If $\tau(A) > 0, \tau(B) > 0$, then $\tau(A \cap B) > 0$, for $A, B \in I^X$.

(O3) For every subfamily $\{A_i : i \in J\} \subset I^X$, if $\tau(A_i) > 0$, then $\tau(\cup_{i \in J} A_i) > 0$.

Then the mapping $\tau : I^X \to I$ is called an H-fuzzy topology or a gradation of openness on X.

If the H-fuzzy topoloy τ on X has the following property:

(O4) $\tau(I^X) \subset \{0, 1\}$, then τ corresponds in a one to one way to a fuzzy topology in Chang's sense [2].

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A mapping $\tau^* : I^X \to I$ is called an H-fuzzy cotopology or a gradation closedness [3] iff the following three conditions are satisfied:

(C1) $\tau^*(0_X) = \tau^*(1_X) = 1$.

(C2) If $\tau^*(A), \tau^*(B) > 0$, then $\tau^*(A \cup B) > 0$, for $A, B \in I^X$.

(C3) For every subfamily $\{A_i : i \in J\} \subset I^X$, if $\tau^*(A_i) > 0$, then $\tau^*(\cap_{i\in J} A_i) > 0$.

If τ is an H-fuzzy topology on X, then a mapping $\tau^* : I^X \to I$, defined by $\tau^*(A) = \tau(A^c)$ where A^c denotes the complement of A, is an H-fuzzy cotopology. Conversely, if τ^* is an H-fuzzy cotopology on X, then a mapping $\tau : I^X \to I$, defined by $\tau(A) = \tau^*(A^c)$, is an H-fuzzy topology on X [3].

Let (X, τ) be an H-fuzzy topological space and $A \in I^X$. Then the H-fuzzy closure of A, denoted by A^-, is defined by

$$A^- = \cap\{K \in I^X : \tau^*(K) > 0, A \subset K\},$$

where $\tau^*(K) = \tau(K^c)$ [3].

Let $f\colon (X, \tau) \to (Y, \sigma)$ be a map between two H-fuzzy topological spaces. Then the mapping f is called a gradation preserving (gp-) map if $\tau(f^{-1}(A)) \geq \sigma(A)$ for each $A \in I^Y$. And the mapping f is called a weakly gradation preserving (wgp-) if $\sigma(U) > 0 \Rightarrow \tau(f^{-1}(U)) > 0$ for each $U \in I^Y$ [3].

In 1987, M. E. Abd El-Monsef et al.[1] introduced a fuzzy supratopology as the following way: A subclass τ of I^X is called a fuzzy supratopology for the set X if

(1) $0_X, 1_X \in \tau$.

(2) For every subfamily $\{A_i : i \in J\} \subset I^X, \cup_{i\in J} A_i \in \tau$.

And the pair (X, τ) is called a fuzzy supratopological space.

2. Gradations of supraopenness

Definition 2.1. A gradation of supraopenness τ on X is a map $\tau : L^X \to L'$ satisfying the following properties, where $L = L' = [0, 1]$:

(S1) $\tau(0_X) = \tau(1_X) = 1$.

(S2) For every subfamily $\{A_i : i \in J\} \subset I^X$, if $\tau(A_i) > 0$, then $\tau(\cup_{i\in J} A_i) > 0$.

Then the pair (X, τ) is called an H-fuzzy supratopological space. If τ is a crisp (i.e. $L' = \{0, 1\}$), then the τ is a classical fuzzy supratopology on X [1].

DEFINITION 2.2. A mapping $\tau^* : I^X \to I$ is called a gradation of supraclosedness if the following two conditions are satisfied:
(C1) $\tau^*(0_X) = \tau^*(1_X) = 1$.
(C2) For every subfamily $\{A_i : i \in J\} \subset I^X$, if $\tau^*(A_i) > 0$, then $\tau^*(\cap_{i \in J} A_i) > 0$.

Obviously we get the following theorem from Definition 2.2.

THEOREM 2.3. *(1) Let τ be a gradation of supraopenness on X and $\tau^* : I^X \to I$ be a mapping defined by $\tau^*(A) = \tau(A^c)$, where A^c is the complement of A. Then τ^* is a gradiation of supraclosedness on X.*
(2) Let τ^* be a gradation of supraclosedness on X and $\tau : I^X \to I$ be a mapping defined by $\tau(A) = \tau^*(A^c)$. Then τ is a gradation of supraopenness on X.

Proof. (1). (C1) Obvious. (C2) For every subfamily $\{A_i : i \in J\} \subset I^X$, if $\tau^*(A_i) > 0$, then $\tau(A_i^c) > 0$. Since τ is a gradation of supraopenness, we get $\tau^*(\cap_{i \in J} A_i) = \tau(\cup_{i \in J} {A_i}^c) > 0$
(2). Similar to (1). □

DEFINITION 2.4. Let τ and σ be the gradation of supraopenness on X. We say that τ is finer than σ or σ is coarser than τ (denoted by $\tau > \sigma$) is $\tau(A) \geq \sigma(A)$ for every $A \in I^X$.

DEFINITION 2.5. Let (X, τ) be an H-fuzzy supratopological space and $A \in I^X$. Then
(1) The H-fuzzy supraclosure of A, denoted by $sl(A)$, is defined by

$$sl(A) = \cap \{K \in I^X : \tau^*(K) > 0, A \subset K\},$$

where $\tau^*(K) = \tau(K^c)$.
(2) The H-fuzzy suprainterior of A, denoted by $si(A)$, is defined by

$$si(A) = \cup \{K \in I^X : \tau(K) > 0, K \subset A\}.$$

Theorem 2.6. *Let (X, τ) be an H-fuzzy supratopoogical space and $A, B \in I^X$. Then*
(1) $si(1_X) = 1_X$,
(2) $si(A) \subset A$,
(3) $A \subset B \Rightarrow si(A) \subset si(B)$,
(4) $si(si(A)) = si(A)$,
(5) $si(A \cap B) \subset si(A) \cap si(B)$.

Proof. (1),(2) and (3) can be obtained from Definition 2.5.
(4) For each $A \in I^X$, we get $\tau(si(A)) = \tau(\cup\{K \in I^X : \tau(K) > 0, K \subset A\})$ by Definition 2.5. Since τ is a gradation of supraopenness on X, we can say $\tau(si(A)) > 0$. Consequently we have $si(si(A)) = si(A)$ from the concept of H-fuzzy suprainterior.
(5) From (2) we obtain $si(A \cap B) \subset si(A)$ and $si(A \cap B) \subset si(B)$, and so easily (5) is obtained. □

The following example shows that the equality of Theorem 2.6(5) is not true in general.

Example 2.7. Let $X = I$ and $\tau : I^X \to I$ be defined by

$$\tau(A) = \begin{cases} 0, & \text{if } A(x) \leq 1/2 \text{ for all } x \in X, \\ 1, & \text{otherwise,} \end{cases}$$

for each $A \in I^X - \{0_X\}$. Now we consider two fuzzy sets A, B defined as the following:

$$A(x) = x, \text{ for all } x \in X,$$
$$B(x) = 1 - x, \text{ for all } x \in X.$$

Then $\tau(A) = \tau(B) = 1$, but $\tau(A \cap B) = 0$. Thus τ is a gradation of supraopennes. And we have $si(A) \cap si(B) = A \cap B$ and $si(A \cap B) = 0_X$, from the gradation τ of supraopenness. Thus $si(A) \cap si(B)$ is not equal to $si(A \cap B)$.

Theorem 2.8. *Let (X, τ) be an H-fuzzy supratopological space and $A, B \in I^X$. Then*
(1) $sl(1_X) = 1_X$,
(2) $A \subset sl(A)$,

(3) $A \subset B \Rightarrow sl(A) \subset sl(B)$,
(4) $sl(A) = sl(sl(A))$,
(5) $sl(A) \cup sl(B) \subset sl(A \cup B)$.

Proof. Similar to Theorem 2.6. □

The following example shows the equality of Theorem 2.8 (5) is not true in general.

EXAMPLE 2.9. Let $X = I$ and $\tau : I^X \to I$ be a gradation of suraopenness defined as Example 2.7. Let two fuzzy sets A, B be defined as the following:

$$A(x) = x, \qquad \text{for all } x \in X,$$
$$B(x) = 1 - x, \qquad \text{for all } x \in X.$$

Since $\tau^*(A) = \tau(A^c) = \tau(B) > 0$ and $\tau^*(B) = \tau(B^c) = \tau(A) > 0$, we have $A = sl(A)$ and $B = sl(B)$. Let C be a fuzzy set such that $C \neq 1_X$ and $A \cup B \subset C$. Then $\tau^*(C) = \tau(C^c) = 0$, and so $sl(A \cup B) = 1_X$. Thus we have $sl(A) \cup sl(B) \neq sl(A \cup B)$.

THEOREM 2.10. *Let (X, τ) be an H-fuzzy supratopological space and $A \in I^X$. Then*
(1) $(si(A))^c = sl(A^c)$.
(2) $(sl(A))^c = si(A^c)$.

Proof. The proof is obtained from Definition 2.5. □

THEOREM 2.11. *Let (X, τ) be an H-fuzzy supratopological space and $A \in I^X$. Then*
(1) $\tau(A) > 0$ *iff* $A = si(A)$.
(2) $\tau^*(A) > 0$ *iff* $A = sl(A)$.

Proof. (1) Let $\tau(A) > 0$. Then $A \subset \cup\{K \in I^X : \tau(K) > 0, K \subset A\} = si(A)$. Hence we get $A = si(A)$ from Theorem 2.6(2).

For the converse, let $A = si(A)$. Then

$$\tau(A) = \tau(si(A)) = \tau(\cup\{K \in I^X : \tau(K) > 0, K \subset A\}).$$

Since τ is a gradation of supraopenness, we have $\tau(A) > 0$

(2) Similar to (1). □

DEFINITION 2.12. Let (X, τ) be an H-fuzzy topological space and $A \in I^X$. Then the H-fuzzy interior of A, denoted by A^o, is defined by

$$A^o = \cup\{K \in I^X : \tau(K) > 0, K \subset A\}.$$

THEOREM 2.13. *Let (X, τ) be an H-fuzzy topological space and $A \in I^X$. Then*
(1) $(A^o)^c = (A^c)^-$,
(2) $(A^-)^c = (A^c)^o$,
(3) $\tau(A) > 0$ iff $A = A^o$.

Proof. Similar to Theorem 2.10 and Theorem 2.11. □

THEOREM 2.14. *Let (X, τ) be an H-fuzzy topological space and $A \in I^X$. Then*
(1) ${1_X}^o = 1_X$,
(2) $A^o \subset A$,
(3) $A \subset B \Rightarrow A^o \subset B^o$,
(4) $(A^o)^o = A^o$,
(5) $(A \cap B)^o = A^o \cap B^o$.

Proof. (1), (2) and (3) are obvious.
(4) For each $A \in I^X$, it is obvious $\tau(A^o) > 0$. So from Theorem 2.13(3), we get $(A^o)^o = A^o$.
(5) From (2) and (3), it is obvious $(A \cap B)^o \subset A^o \cap B^o$. Since $\tau(A^o) > 0, \tau(B^o) > 0$ and τ is a gradation of openness, we can say $\tau(A^o \cap B^o) > 0$. Therefore

$$A^o \cap B^o = (A^o \cap B^o)^o \subset (A \cap B)^o.$$

Thus we have $A^o \cap B^o = (A \cap B)^o$. □

DEFINITION 2.15. Let (X, τ) be an H-fuzzy topological space and τ_s be a gradation of supraopenness on X. We call τ_s an associated gradation of supraopenness with τ on X if for every $A \in I^X, \tau(A) \leq \tau_s(A)$.

Given a gradation of openness on X, we can find an associated gradation of supraopenness with τ as the following example shows.

Example 2.16. Let (X,τ) be an H-fuzzy topological space. Define $\tau_s : I^X \to I$ as

$$\tau_s(A) = \begin{cases} \tau(A^o), & \text{if } A \subset A^{o-}, \\ 0, & \text{otherwise.} \end{cases}$$

Then clearly $\tau_s(0_X) = \tau_s(1_X) = 1$. For any index set J, let $\tau_s(A_i) > 0$ for all $i \in J$. Easily we can show $\cup A_i \subset (\cup A_i)^{o-}$, and from Theorem 2.13 and Theorem 2.14, we get $\tau_s(\cup A_i) = \tau((\cup A_i)^o) > 0$. Thus τ_s is a gradation of supraopenness on X. Let $\tau(A) > 0$ for $A \in I^X$. Since $A \subset A^{o-}$ and $\tau(A) = \tau(A^o)$, we have $\tau(A) \leq \tau_s(A)$ for each $A \in I^X$. Thus τ_s is an associated gradation of supraopenness with τ.

3. S-gradation preserving maps.

Definition 3.1. Let (X,τ) and (Y,σ) be H-fuzzy topological spaces and τ_s be an associated gradation of supraopenness with τ. The map $f : X \to Y$ is called

(1) an associated S-gradation preserving(S-gp-)map relative to τ_s if for each $A \in I^Y$,

$$\sigma(A) \leq \tau_s(f^{-1}(A)).$$

(2) an associated weakly S-gradation preserving(wS-gp-)map relative to τ_s if for each $A \in I^Y$,

$$\sigma(A) > 0 \Rightarrow \tau_s(f^{-1}(A)) > 0.$$

The concepts of associated S-gradation preserving(S-gp-)maps and associated weakly S-gradation preserving(wS-gp-)maps depend on a given associated gradation of supraopenness. Thus if no confusion will arise, we simply call them S-gradation preserving(S-gp-)maps and weakly S-gradation preserving(wS-gp-)maps, respectively.

From the notions of S-gp-map and wS-gp-map, we get that S-gp-maps implie wS-gp-maps. However, the converse is not necessarily true as is evident from the following example.

EXAMPLE 3.2. Let A_1, A_2 be fuzzy subsets of $X = I$, defined as

$$A_1(x) = \begin{cases} 0, & \text{if } 0 \leq x \leq 1/2, \\ 2x-1, & \text{if } 1/2 \leq x \leq 1; \end{cases}$$

$$A_2(x) = \begin{cases} 1, & \text{if } 0 \leq x \leq 1/4, \\ -4x+2, & \text{if } 1/4 \leq x \leq 1/2, \\ 0, & \text{if } 1/2 \leq x \leq 1. \end{cases}$$

Let $\tau\colon I^X \to I$ be defined by

$$\begin{aligned} \tau(0_X) &= \tau(1_X) = 1 \\ \tau(A_1) &= 1/2 \\ \tau(A_2) &= 1/3 \\ \tau(A_3) &= 1/4, \;\; A_3 = A_1 \cup A_2 \\ \tau(B) &= 0, \;\; \text{if } B \neq A_i, i = 1, 2, 3. \end{aligned}$$

Let $\sigma\colon I^X \to I$ be defined by

$$\begin{aligned} \sigma(0_X) &= \sigma(1_X) = 1 \\ \sigma(A_1) &= 1/4 \\ \sigma(A_2) &= 1/3 \\ \sigma(A_3) &= 1/2, \;\; A_3 = A_1 \cup A_2 \\ \sigma(B) &= 0, \;\; \text{if } B \neq A_i, i = 1, 2, 3. \end{aligned}$$

We define $\tau_s : I^X \to I$ as

$$\tau_s(A) = \begin{cases} \tau(A^o), & \text{if } A \subset A^{o-}, \\ 0, & \text{otherwise.} \end{cases}$$

Then the identity mapping $f\colon (X, \tau) \to (X, \sigma)$ is a wS-gp-map but not an S-gp-map.

THEOREM 3.3. *Let $(X, \tau), (Y, \sigma)$ be H-fuzzy topological spaces and τ_s be an associated gradation of supraopenness with τ. If $f : X \to Y$ is a map, then the following are equivalent:*

(1) f is a wS-gp-map,

(2) $\sigma^*(B) > 0 \Rightarrow {\tau_s}^*(f^{-1}(B)) > 0$, for $B \in I^Y$,

(3) $f(sl(A)) \subset f(A)^-$, for $A \in I^X$,

(4) $sl(f^{-1}(B)) \subset f^{-1}(B^-)$, for $B \in I^Y$,

(5) $f^{-1}(B^o) \subset si(f^{-1}(B))$, for $B \in I^Y$.

Proof. (1) $\Rightarrow$ (2). Let $\sigma^*(B) > 0$ for $B \in I^Y$. Then we get $\sigma(B^c) > 0$, and so $\tau_s(f^{-1}(B^c)) > 0$ follows from (1). Finally we get ${\tau_s}^*(f^{-1}(B)) > 0$ by Theorem 2.3.

(2) $\Rightarrow$ (3). For $A \in I^X$, it is obvious $\sigma^*(f(A)^-) > 0$ from Theorem 2.8 and Theorem 2.11. Thus we have ${\tau_s}^*(f^{-1}(f(A)^-)) > 0$ by the condition (2).

And $sl(f^{-1}(f(A)^-)) = f^{-1}(f(A)^-)$ follows from Theorem 2.11, so consequently we get $f(sl(A)) \subset f(A)^-$.

(3) $\Rightarrow$ (4). It is obvious.

(4) $\Rightarrow$ (5). Let $B \in I^Y$. Then $\sigma(B^o) > 0$ implies $\sigma^*((B^o)^c) > 0$. From the condition (4) and Theorem 2.14, we get

$$sl(f^{-1}(B^o)^c) \subset f^{-1}(((B^o)^c)^-) = f^{-1}((B^o)^c).$$

It is obtained $si(f^{-1}(B)) \subset f^{-1}(B^o)$ from Theorem 2.10.

(5) $\Rightarrow$ (1). Suppose $\sigma(B) > 0$ for $B \in I^Y$, then $B = B^o$ and from the condition (5) we get $si(f^{-1}(B)) \supset f^{-1}(B^o) = f^{-1}(B)$.

It follows that $si(f^{-1}(B)) = f^{-1}(B)$. Thus from Theorem 2.11 and Definition 2.12, we get $\tau_s(f^{-1}(B)) > 0$. □

THEOREM 3.4. *Let $(X,\tau), (Y,\sigma)$ be H-fuzzy topological spaces and τ_s be an associated gradation of supraopenness with τ . A map $f : X \to Y$ is S-gradation preserving if and only if $\sigma^*(B) \le {\tau_s}^*(f^{-1}(B))$, for $B \in I^Y$,*

Proof. The proof is obvious from Theorem 2.3. □

COROLLARY 3.5. *Let $(X,\tau), (Y,\sigma)$ be H-fuzzy topological spaces and τ_s be an associated gradation of supraopenness with τ. If $f : X \to Y$ is an S-gp-map, then we have*

(1) $f(sl(A)) \subset (f(A))^-$, for $A \in I^X$,

(2) $sl(f^{-1}(B)) \subset f^{-1}(B^-)$, for $B \in I^Y$,

(3) $f^{-1}(B^0) \subset si(f^{-1}(B))$, for $B \in I^Y$.

DEFINITION 3.6. Let (X,τ) and (Y,σ) be H-fuzzy supratopological spaces. The map $f : X \to Y$ is called

(1) an S^*-gradation preserving(S^*-gp-)map, if for each $A \in I^Y$,

$$\sigma(A) \le \tau(f^{-1}(A)).$$

(2) a weakly S^*-gradation preverving(wS^*-gp-)map, if for each $A \in I^Y$,

$$\sigma(A) > 0 \Rightarrow \tau(f^{-1}(A)) > 0.$$

Clearly S^*-gp-map $\Rightarrow$ wS^*-gp-map but the converse may not be true.

EXAMPLE 3.7. Let τ, σ be two gradations of openness and let τ_s be an associated gradation of supraopenness defined as it in Example 3.2. And we define $\sigma_s : I^X \to I$ as $\sigma_s(A) = \sigma(A^o)$, if $A \subset A^{o-}$. Otherwise, $\sigma_s(A) = 0$.

Then the identity mapping $f\colon (X, \tau_s) \to (X, \sigma_s)$ is a wS^*-gp-map but not an S^*-gp-map.

THEOREM 3.8. *Let $(X, \tau), (Y, \sigma)$ be H-fuzzy supratopological spaces. If $f : X \to Y$ is a map, then the following are equivalent:*
(1) f is a wS^*-gp-map,
(2) $\sigma^*(B) > 0 \Rightarrow \tau^*(f^{-1}(B)) > 0$, for $B \in I^Y$,
(3) $f(sl(A)) \subset sl(f(A))$, for $A \in I^X$,
(4) $sl(f^{-1}(B)) \subset f^{-1}(sl(B))$, for $B \in I^Y$,
(5) $f^{-1}(si(B)) \subset si(f^{-1}(B))$, for $B \in I^Y$.

Proof. (1) $\Rightarrow$ (2). Let $\sigma^*(B) > 0$ for $B \in I^Y$. Then $\sigma(B^c) > 0$, and from the condition (1) $\tau(f^{-1}(B^c)) > 0$. Thus we get $\tau^*(f^{-1}(B)) > 0$ by Theorem 2.3.

(2) $\Rightarrow$ (3). For $A \in I^X$, we get $\tau^*(f^{-1}(sl(f(A)))) > 0$, since $\sigma^*(sl(f(A))) > 0$. It follows that $sl(f^{-1}(sl(f(A)))) = f^{-1}(sl(f(A)))$ from Theorem 2.11, and so we get $f(sl(A)) \subset f(A)^-$.

(3) $\Rightarrow$ (4). It is obvious from the condition (3).

(4) $\Rightarrow$ (5). Let $B \in I^Y$. Then $\sigma(si(B)) > 0$ implies $\sigma^*((si(B))^c) > 0$. From the condition (4) and Theorem 2.10, we get

$$sl(f^{-1}(si(B))^c) \subset f^{-1}(sl((si(B))^c)) = f^{-1}((si(B))^c).$$

Thus we have $si(f^{-1}(B)) \subset f^{-1}(si(B))$.

(5) $\Rightarrow$ (1). Suppose $\sigma(B) > 0$ for $B \in I^Y$. Then $B = si(B)$ and from the condition (5) we get

$$si(f^{-1}(B)) \supset f^{-1}(si(B)) = f^{-1}(B).$$

Thus we get $\tau(f^{-1}(B)) > 0$. $\square$

Theorem 3.9. *Let (X, τ) and (Y, σ) be H-fuzzy supratopological spaces. A map $f : X \to Y$ is S^*-gradation preserving map if and only if $\sigma^*(B) \leq \tau^*(f^{-1}(B))$, for $B \in I^Y$.*

Proof. Similar to Theorem 3.4. □

The following corollary is obtained from Definition 3.6 and Theorem 3.8.

Corollary 3.10. *Let $(X, \tau), (Y, \sigma)$ be H-fuzzy supratopological spaces. If $f : X \to Y$ is an S^*-gp-map, then we have*
(1) $f(sl(A)) \subset sl(f(A))$, for $A \in I^X$,
(2) $sl(f^{-1}(B)) \subset f^{-1}(sl(B)))$, for $B \in I^Y$,
(3) $f^{-1}(si(B)) \subset si(f^{-1}(B))$, for $B \in I^Y$.

Definition 3.11. Let (X, τ) and (Y, σ) be H-fuzzy topological spaces and let τ_s and σ_s be associated gradations of supraopenness with τ and σ, respectively. The map $f : X \to Y$ is called

(1) an associated S^*-gradation preserving(S^*-gp-)map relative to τ_s and σ_s if for each $A \in I^Y$,

$$\sigma_s(A) \leq \tau_s(f^{-1}(A)).$$

(2) an associated weakly S^*-gradation preserving(wS^*-gp-)map relative to τ_s and σ_s if for each $A \in I^Y$,

$$\sigma_s(A) > 0 \Rightarrow \tau_s(f^{-1}(A)) > 0.$$

The concepts of associated S^*-gradation preserving(S^*-gp-)maps and associated weakly S^*-gradation preserving(wS^*-gp-)maps depend on given associated gradations of supraopenness. Thus if no confusion will arise, we simply call them S^*-gradation preserving(S^*-gp-)maps and weakly S^*-gradation preserving(wS^*-gp-)maps, respectively.

Remark. Let $(X, \tau), (Y, \sigma)$ be H-fuzzy topological spaces and let τ_s, σ_s be associated gradations of supraopenness with τ, σ, respectively. If $f : X \to Y$ is a map, we get the diagrams:

gp-map $\Rightarrow$ S-gp-map $\Leftarrow$ S^*-gp-map.

wgp-map $\Rightarrow$ wS-gp-map $\Leftarrow$ wS^*-gp-map.

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