주파수영역에서 선형시스템의 파라메트릭 식별

論　文
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A parametric Identification of Linear System in the Frequency Domain

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Abstract - This paper presents a proper rational transfer function synthesis in the continuous time system from noisy measurements. The proposed method identifies the coefficients vector of the transfer function from an overdetermined linear system that develops from rearranging the two dimensional system matrices and output vectors obtained from the observed frequency responses. By computer simulation, the performance improvement is verified.

Key Words : parametric identification, transfer function, TLS, SVD

1. Introduction

The system identification is the process of deriving a mathematical model from observed data in accordance with some predetermined criterion. The representation for SISO (Single-Input Single-Output) system is described by the ratio of Laplace transform for their input and output signals. Many studies regarding parametric identification methods, which also can be described as complex curve fitting, to synthesize the rational polynomial transfer function have been reported heretofore. Levy[1] introduced an approximation technique to synthesize transfer function using an experimentally obtained frequency response. Since that time, various methods to compensate for the bias introduced in Levy's scheme was given by many researchers. Sanathanan and Koerner[2] introduced an iterative approach that should eliminate any bias. Lawrence and Rogers[3] reformulated the solution of the linear least squares problem by applying recursive least squares method. Stahl[4] proposed the matrix adaptation method. Among these methods, any bias problem can be eliminated using the iterative approach, however the solution have been shown to sometime converge to a local minimum. A review of these methods and a mathematical representation were given by Whitfield[5]. However, Whitfield's constraint may give numerical problems if the constrained parameter should be zero or close to zero. A survey of SISO methods based on parametric identification was introduced in [6], which led to the TLS(Total Least Squares) solution. The presented formulation in [6] is also suffered from the deficiencies as Levy's estimate.

In this paper, we present a theoretical description for the proper rational transfer function and a parametric identification method via TLS. The proposed method provides better low frequency fit and an computer aided identification that estimates the vector of coefficients for the numerator and denominator polynomial on the rational transfer function from an overdetermined linear system constructed by the observed noisy frequency responses.

2. Parametric Identification

For a given set of experimental frequency response data, the transient behavior of linear dynamic time-invariant continuous time system shown in Fig. 1. is described by

$$G(s,\Theta)=\frac{b(s,\Theta)}{a(s,\Theta)}=\frac{\sum_{g=0}^{m} b_g s^g}{\sum_{h=0}^{n} a_h s^h} \tag{1}$$

where $\Theta=[a_0\ a_1\cdots a_{n-1}\ \vdots\ b_0\ b_1\cdots b_m]^T$, $n\geq m$, and $a_n=1$.

The frequency response function of (1) is represented

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through magnitude $|G(j\omega)|$ and phase $\phi(\omega)$ in complex plane.

$$G(j\omega,\Theta) = |G(j\omega)|\, e^{j\phi(\omega)} \tag{2}$$

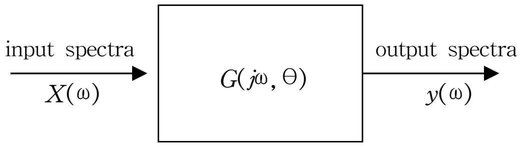


Fig. 1. Linear dynamic time-invariant continuous time system

If the error function is weighted by the numerator polynomial, we have accomplished better fits than Levy′s scheme from a following parametric optimization

$$\hat{\Theta} = \underset{\Theta}{\arg\min} \sum_{i=1}^{N} \left| \frac{1}{G(j\omega,\Theta)} b(j\omega,\Theta) - a(j\omega,\Theta) \right|^2 \tag{3}$$

where $\underset{\Theta}{\arg\min}\, f(\Theta)$ is the minimizing argument of $f(\Theta)$ and N is the number of measured frequency data. Then, for any frequency ω_i, (1) can be rewritten by

$$\sum_{h=0}^{n} a_h (j\omega_i)^h = \sum_{g=0}^{m} b_g (j\omega_i)^g \left(M_i + jN_i\right) \tag{4}$$

where $M_i = \frac{\cos\phi(\omega_i)}{|G(j\omega_i)|}$ and $N_i = -\frac{\sin\phi(\omega_i)}{|G(j\omega_i)|}$.

We define the indices $p, q, u, v \geq 0$ for m and n either even or odd as follows

$$m = 2k: \; p = \frac{m}{2}, \; q = \frac{m}{2} - 1$$
$$m = 2k+1: \; p = q = \frac{m-1}{2}$$
$$n = 2l: \; u = v = \frac{n}{2} - 1$$
$$n = 2l+1: \; u = \frac{n-1}{2}, \; v = \frac{n-1}{2} - 1$$

where $k, l = 0, 1, 2, \cdots$.

Hence (4) can be separated into the real and imaginary parts as

$$y_{Ri} = \sum_{\gamma=0}^{u}(-1)^{\gamma} a_{2\gamma}\omega_i^{2\gamma} - M_i \sum_{\alpha=0}^{p}(-1)^{\alpha} b_{2\alpha}\omega_i^{2\alpha} + N_i \sum_{\beta=0}^{q}(-1)^{\beta} b_{2\beta+1}\omega_i^{2\beta+1} \tag{5}$$

$$y_{Ii} = \sum_{\delta=0}^{v}(-1)^{\delta} a_{2\delta+1}\omega_i^{2\delta+1} - N_i \sum_{\alpha=0}^{p}(-1)^{\alpha} b_{2\alpha}\omega_i^{2\alpha} - M_i \sum_{\beta=0}^{q}(-1)^{\beta} b_{2\beta+1}\omega_i^{2\beta+1} \tag{6}$$

where y_{Ri} and y_{Ii} are scalar values, and then it can be classified by

$$n = 4l: \; y_{Ri} = -\omega_i^n, \; y_{Ii} = 0$$
$$n = 4l+1: \; y_{Ri} = 0, \; y_{Ii} = -\omega_i^n$$
$$n = 4l+2: \; y_{Ri} = \omega_i^n, \; y_{Ii} = 0$$
$$n = 4l+3: \; y_{Ri} = 0, \; y_{Ii} = \omega_i^n$$

By augmenting (5) and (6), define the equations in the form of vectors

$$x_{Ri} = [\, p_{Ri}^{even} \; \vdots \; 0 \; \vdots \; z_{Ri}^{even} \; \vdots \; z_{Ri}^{odd} \,] \tag{7}$$
$$x_{Ii} = [\, 0 \; \vdots \; p_{Ii}^{odd} \; \vdots \; z_{Ii}^{even} \; \vdots \; z_{Ii}^{odd} \,] \tag{8}$$
$$\Theta = [\, a_0\, a_2 \cdots a_u \vdots a_1\, a_3 \cdots a_v \vdots b_0\, b_2 \cdots b_p \vdots b_1\, b_3 \cdots b_q \,]^T \tag{9}$$

where $p_{Ri}^{even} \in R^u$, $z_{Ri}^{even} \in R^p$, and $z_{Ri}^{odd} \in R^q$ are obtained from (5), $p_{Ii}^{odd} \in R^v$, $z_{Ii}^{even} \in R^p$, and $z_{Ii}^{odd} \in R^q$ are also obtained from (6).

And then an overdetermined linear system $X\Theta = y$ is constructed, where $X \in R^{2N \times (n+m+1)}$ and $y \in R^{2N}$ are referred to

$$X = [\, x_{R1}^T \cdots x_{Ri}^T \cdots x_{RN}^T \vdots x_{I1}^T \cdots x_{Ii}^T \cdots x_{IN}^T \,]^T \tag{10}$$
$$y = [\, y_{R1} \cdots y_{Ri} \cdots y_{RN} \vdots y_{I1} \cdots y_{Ii} \cdots y_{IN} \,]^T \tag{11}$$

A solution to this case is known as a LS(Least Squares) problem. However, we could only use this method, if we knew that the measurements are noise-free and the assumed model is completely accurate. The TLS is used to find the best fit to the overdetermined linear system, when noise is on both sides of the equation. In the case, E and e are perturbations of X and y, then $(X+E)\Theta = (y+e)$ is satisfied. The TLS problem can be formulated by the following optimization problem.

$$\min_{E,e} \left\| [\, E \vdots e \,] \right\|_F, \text{ subject to } y + e \in Range(X + e) \tag{12}$$

where $\|\cdot\|_F$ denotes the Frobenius norm.

And the SVD(Singular Value Decomposition) of augmented matrix $[X \vdots y] \in R^{2N \times (n+m+2)}$ can be written as

$$D = U^T [X \vdots y]\, V = \begin{bmatrix} diag(\sigma_1, \sigma_2, \cdots, \sigma_{n+m+2}) & 0 \\ 0 & 0 \end{bmatrix} \tag{13}$$

where $U = [\, u_1 \; u_2 \cdots u_{2N} \,] \in R^{2N \times 2N}$ and

$V = [\, v_1 \; v_2 \cdots u_{n+m+2} \,] \in R^{(n+m+2)\times(n+m+2)}$.

If the smallest singular value is repeated,

$$\sigma_1 \geq \sigma_2 \geq \cdots \geq \sigma_k > \sigma_{k+1} = \cdots = \sigma_{n+m+2} > 0 \tag{14}$$

we can find a Householder matrix Q such that

$$[v_{k+1}\ v_{k+2} \cdots v_{n+m+2}]Q = \begin{bmatrix} W & z \\ 0 & \eta \end{bmatrix} \tag{15}$$

By the properties of the Householder matrix, $w = [z\ \ \eta]^T$ is the vector in $S_C = span\{v_{k+1}, v_{k+2}, \cdots, v_{n+m+2}\}$ such that the last component of $[v_{k+1}\ v_{k+2} \cdots v_{n+m+2}]w$ is maximized. And the TLS solution[7,8] is obtained as the following

$$\begin{bmatrix} \hat{\Theta} \\ -1 \end{bmatrix} = -\frac{w}{\eta} \tag{16}$$

3. Example

In order to evaluate the proposed method, we consider the perturbed data for the transfer function of [6] in the frequency range $10^{-3} \leq \omega \leq 10^{0}$.

$$G_M(s) = \frac{1}{s^2 + s + 1} \tag{17}$$

The transfer function obtained by Levy's method[1,9] is

$$G_{Levy}(s) = \frac{7.4843 \times 10^{-1}}{s^2 + 7.7051 \times 10^{-1} s + 7.6329 \times 10^{-1}} \tag{18}$$

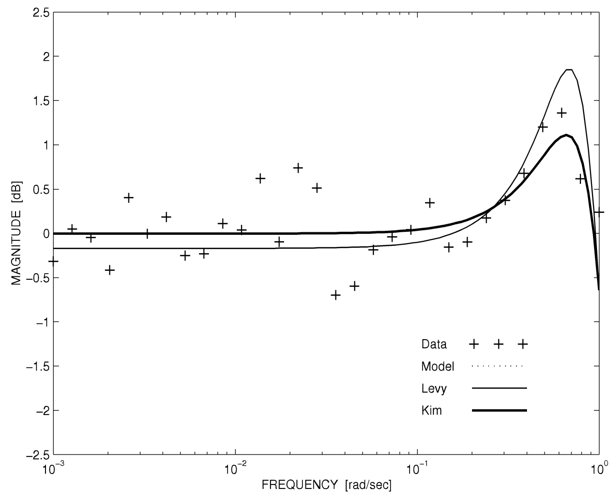
Whereas the transfer function identified by proposed method is

$$G_{Kim}(s) = \frac{9.1160 \times 10^{-1}}{s^2 + 9.7767 \times 10^{-1} s + 9.1202 \times 10^{-1}} \tag{19}$$

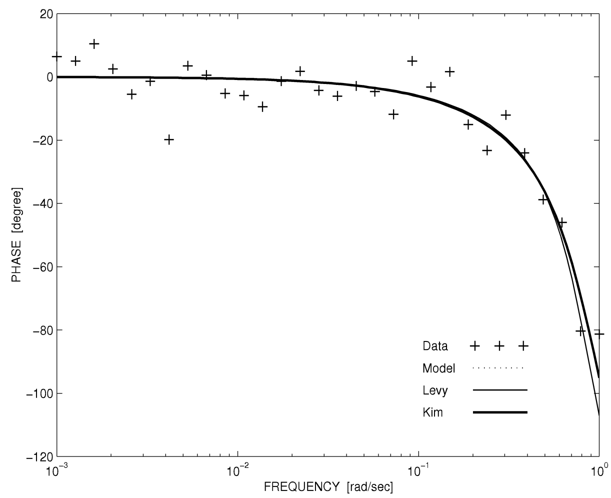
The frequency responses of model (17) are represented in Fig. 2 by the dotted lines, and the plus symbols (+) represent the perturbed data. Furthermore the magnitude and phase responses of Levy and proposed method are illustrated in Fig. 2. From these figures, it can be noticed that the suggested approach gives an improvement in the low frequency range and accuracy.

4. Conclusions

In this paper, we presented the theoretical description of the rational transfer function and the frequency domain identification methods via the TLS algorithms to execute the best solutions against noise. The proposed parametric optimization give an improved low frequency fit, because the error function is biased by the magnitude of the numerator polynomial. And consequently produce models are simple, easy to implement and can be used to automate the identification of the linear dynamic time-invariant continuous time system. Comparison with the previous results are carried out, and it also checked that the better frequency responses are achieved. The suggested algorithms may be used for the control system identification on the basis of frequency responses and various tuning techniques.



(a) log magnitude



(b) phase

Fig. 2 Frequency responses

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