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A Case Study of Bus-Gearboxes Maintenance using Arithmetic Processes

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Abstract. In this study, we employed an arithmetic process (AP) approach to resolve gearbox maintenance problems. The approach is realistic and direct in modelling the characteristics of a deteriorating system such as a gearbox since a decreasing AP can model a gearbox's successive operating times and an increasing AP can model the corresponding consecutive repair times. First of all, two test statistics were used to check whether the process is arithmetic or not. Next, model parameters of the AP were estimated using the simple linear regression method. Finally, the optimal replacement policy based on minimising the long-run average cost per day was determined for each type of gearbox.

Keywords: deteriorating systems, maintenance, repair, replacement, arithmetic processes, renewal processes

1. INTRODUCTION

Most of the research on the maintenance of a repairable system has made either the perfect or minimal repair assumption. Perfect repair means that, after repair, a failed system is as good as new, i.e. a system's successive operating times constitute a renewal process, see Barlow and Proschan (1965). For a perfect repair model, if the time needed to repair a system is considered negligible, results of renewal processes can be applied to resolve the system's maintenance problems, see Ross (1970); if repair time has to be taken into account and the corresponding consecutive repair times constitute another renewal process, results of alternating renewal processes can be applied instead, see Birolini (1985). Minimal repair means that a failed system will function, after repair, with the same rate of failure and the same effective age as at the epoch of the last failure. For a minimal repair model where repair time is assumed negligible, a non-homogeneous Poisson process in which the rate of failure is monotone can provide at least a good first-order model for a deteriorating system, see Ascher and Feingold (1984).

In practice, most repairable systems are deteriorative. Two basic characteristics of a deteriorating system are that because of ageing and irreversible wear, the system's successive operating times decrease and so the system's life is finite; while because it is more difficult and hence takes more time to rectify accumulated wear, the corresponding consecutive repair times increase and finally the system is beyond repair. In this study, a replacement policy N is used to replace a system at the time of the Nth failure since the last system replacement. Based on this understanding, an AP approach, see Leung (2001, 2002) and Leung *et al.* (2002) is considered more relevant, realistic and direct to the modelling of the gearbox maintenance problems that we encounter here than the above-mentioned model (perfect or minimal repair). A definition of an AP is given below.

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Definition: Given a sequence of random variables A_1, A_2,··· if for some real number d, $\{A_n + (n-1)d, n = 1,2,\cdots\}$ forms a renewal process (RP), then $\{A_n, n = 1,2,\cdots\}$ is an AP. d is called the common difference or parameter of the AP.

Three specialisations of an AP mentioned in Leung (2002) are given as follows: If $d \in \left(0, \frac{\mu_A}{n-1}\right]$, where μ_A is the mean of the first random variable A_1, then the AP is called a decreasing AP. If $d < 0$, then the AP is called an increasing AP. If $d = 0$, then the AP reduces to an RP.

Clearly, the positive integer n is limited for a decreasing AP. Moreover, if the value of d is close to its upper bound, we will obtain a short sequence of non-negative random variables. However, such a subtractive process is likely to be useful in a deteriorating system (e.g. a gearbox), which fails rarely (e.g. two or three times) over its usual span of life (e.g. five years). This implicitly means that the system wears out, between two successive failures, to such an extraordinary extent that the corresponding system's successive operating time decreases dramatically.

Therefore, for a deteriorating system, it is reasonable to assume that the successive operating times of the system form a decreasing AP; whereas the corresponding consecutive repair times constitute an increasing AP. However, the replacement times for the system are usually stochastically the same no matter how old the used system is; hence, they will form an RP. This is the motivation behind the introduction of the AP approach.

2. FORMULATION OF THE STUDY

2.1 Background and objectives

The Kowloon Motor Bus (KMB) Company Limited has been providing public transport within Hong Kong over the past 60 years. It owns about 3,300 buses, 20% of which are equipped with air conditioning. Maintenance is essential in order to operate the buses efficiently and safely. Since a gearbox is one of the main components of a bus, KMB is interested in studying the overall operating time and replacement policy for a gearbox. Every time a gearbox fails, it will be taken out of the bus and repaired. Since the operation quality of a gearbox deteriorates owing to ageing and accumulated wear, the operating time decreases and the repair time increases with age and use. Eventually, it is no longer worthwhile to repair a fatigued gearbox and it becomes more economical to replace it with a new one. In this study, seven types of gearbox were examined.

The objectives of this study are to determine the following:

(1) the common difference of the decreasing AP for successive operating times of each type of gearbox,
(2) the common difference of the increasing AP for consecutive repair times of each type of gearbox,
(3) the means of the successive operating times of each type of gearbox,
(4) the means of the consecutive repair times of each type of gearbox,
(5) the mean overall operating time of each type of gearbox, and
(6) the optimal replacement policy that should be adopted for each type of gearbox based on minimising the long-run average cost per day.

2.2 Assumptions and notation

Model assumptions are as follows:

(1) At the beginning, a new gearbox is used. The new gearbox must be operable.
(2) Whenever a gearbox fails, it is repairable.
(3) Let X_n be the operating time after the $(n-1)$th repair. Then $\{X_n, n = 1,2,3,\cdots\}$ forms a decreasing AP with a common difference $a \in \left(0, \frac{\mu_{X_1}}{n-1}\right]$.
(4) Let Y_n be the repair time after the nth failure. Then $\{Y_n, n = 1,2,3,\cdots\}$ forms an increasing AP with a common difference $b < 0$.
(5) The sequences of the operating times $\{X_n, n = 1,2,3,\cdots\}$ and repair times $\{Y_n, n = 1,2,3,\cdots\}$ of a gearbox are independent.
(6) All the gearboxes work under nearly identical conditions.
(7) A gearbox is replaced by a new and identical one when it is no longer economical to repair it.

We will adopt the following notation:

- X_n is the operating time after the $(n-1)$th repair for $n = 1,2\cdots,N$ with $X_0 = 0$.
- Y_n is the repair time after the nth failure for $n = 1,2,\cdots,N$.
- A_n denotes either X_n or Y_n for $n = 1,2,\cdots,N$.
- a is a common difference of a decreasing AP such that $\frac{\mu_{X_1}}{n-1} \geq a > 0$.
- b is a common difference of an increasing AP such that $b < 0$.
- d denotes either a or b.
- μ_{A_n} is the mean of A_n for $n = 1,2,\cdots,N$.
- $\sigma^2_{A_n}$ is the variance of A_n for $n = 1,2,\cdots,N$.
- ε_n is an error term (i.e. a random variable) independently normally distributed with mean 0 and variance denoted by σ^2_ε for $n = 1,2,\cdots,N$.
- c_f is the average cost of a repair per day.
- c_R is the average cost of a replacement.
- $c(N)$ is the long-run average cost per day, a function of the number of failures N.

3. METHODOLOGY

3.1 Data collection

KMB provided gearboxes' failure records from 1983 to 1995 inclusive. Each record includes nine details: depot (there are five depots), gearbox type (there are seven types of gearbox), gearbox number (this is unique for each type of gearbox), bus type, bus number, installation date (this indicates when a gearbox was installed in a bus), outing date (this indicates when a gearbox was taken out of a bus), mileage (the total distance travelled) and reasons for failure. All the yearly data provided are arranged by bus number in ascending order.

3.2 Data manipulation

(1) We rearranged the data sequence in the following order: (i) gearbox types, (ii) gearbox numbers, and (iii) installation dates in ascending order. The numbers of samples of operating time and repair time for seven types of gearbox are tabulated below.

		Numbers of samples of	
Gearbox types	Time periods	operating time	repair time
BENZ	1985-1995	18	10
D/FL	1988-1995	99	52
V	1983-1995	47	30
V2	1986-1995	217	174
V63	1989-1995	13	10
V64G	1989-1995	46	43
ZF	1988-1995	12	9

(2) We screened each set of sample data for each type of gearbox, and exclude extreme values of operating time and repair time.

(3) We determined the operating time and repair time using the following equations:

Operating time after the $(n-1)$th repair
= Completion date for the nth repair
− Installation date after the $(n-1)$th repair

Repair time after the nth failure
= Installation date after the nth repair
− Completion date for the $(n-1)$th repair

3.3 Test the existence of a trend in the data

We test whether the A_n's are identically distributed by checking for the existence of a trend. Laplace's trend test is used for ease of manipulation and interpretation, see Ascher and Feingold (1984).

Null hypothesis H_0: A_n's are identically distributed.

Alternative hypothesis H_1: A_n's are not identically distributed, i.e. there is a trend.

Laplace's test statistic for a time-truncated data set (i.e. when the data are time truncated, the time of the conclusion of observation is fixed and the number of failures is random.) is given by

$$L = \left(\frac{\sum_{n=1}^{N} T_n}{N} - \frac{T}{2} \right) \Big/ \left(T\sqrt{\frac{1}{12\,N}} \right) \qquad (1a)$$

where $T_1,\cdots,T_N$ are the failure times for a process observed in $(0, T]$, and T is the pre-specified time of observation.

Laplace's test statistic for a failure-truncated data set (i.e. when the data are failure truncated, the number of failures is fixed before observation begins and the time of the conclusion of the observation is random.) is given by

$$L = \left(\frac{\sum_{n=1}^{N-1} T_n}{N-1} - \frac{T_N}{2} \right) \Big/ \left[T_N\sqrt{\frac{1}{12\,(N-1)}} \right] \qquad (1b)$$

where $T_n = \sum_{i=1}^{n} A_i$ is the time of the nth failure for $n = 1,2,\cdots,N$, and N is the pre-specified number of failures.

L is approximately distributed as the standard normal for $N \geq 3$, time-truncated data, or $N \geq 4$, failure-truncated data, at the 5% level of significance, see Ascher and Feingold (1985). If $|L| > 1.96$, then H_0 is rejected at the 5% level of significance, i.e. the failure data set $\{A_1,A_2,\cdots,A_N\}$ exhibits a trend.

Rigdon and Basu (1989) have reached the conclusion that "using any model for event times, one clearly indicates the time that data collection started and the time that it ceased. This is necessary so that the appropriate analysis, that is, an analysis based on time-truncated or event-truncated data, can be applied and maximum information can be obtained from the data. For time-truncated data, the time between the last event and the termination of the test contains some information that should not be wasted."

In this study, we have been able to analyse failure-truncated data. The reason is twofold: (1) the procedure for estimating the parameters of an AP proposed in Leung (2002) deals with event-truncated data only and (2) the KMB gearbox data are all failure truncated.

3.4 Estimate parameters d, α and σ_ε^2

We plot A_n against $(n-1)$ for $n = 1,2,\cdots,N$ to see whether there is a linear relationship between them. If there is a linear relationship, the simple linear regression equation can be written as

$$A_n = -d(n-1) + \alpha + \varepsilon_n \quad \text{for } n = 1,2,\cdots,N \qquad (2)$$

where ε_n is an error term that describes the effects on the dependent variable A_n of all factors other than the value of the independent variable $(n-1)$, and ε_n is a random variable independently normally distributed with mean zero and variance denoted by σ_ε^2 for all $n = 1,2,\cdots,N$.

We can estimate the parameters d, α and σ_ε^2 using the simple linear regression method, see Leung (2002). The least-square point estimates $\hat{d}$, $\hat{\alpha}$ and $\hat{\sigma}_\varepsilon^2$ of the parameters d, α and σ_ε^2 are calculated respectively using the following formulae:

$$\hat{d}=\frac{2}{N(N+1)(N-1)}\left[3(N-1)\sum_{n=1}^{N}A_n-6\sum_{n=1}^{N}(n-1)A_n\right] \quad (3)$$

$$\hat{\alpha}=\frac{2}{N(N+1)}\left[(2N-1)\sum_{n=1}^{N}A_n-3\sum_{n=1}^{N}(n-1)A_n\right] \quad (4)$$

and

$$\hat{\sigma}_\varepsilon^2=\frac{\sum_{n=1}^{N}A_n^2-\frac{1}{N}\left(\sum_{n=1}^{N}A_n\right)^2-\hat{d}\left[\left(\frac{N-1}{2}\right)\sum_{n=1}^{N}A_n-\sum_{n=1}^{N}(n-1)A_n\right]}{N-2} \quad (5)$$

3.5 Distinguish an RP from an AP

We test whether the data comes from an AP or an RP, see Leung (2002).

Null hypothesis H_0: $d=0$

Alternative hypothesis H_1: $d\neq 0$

The t-test statistic is given by

$$t=\frac{-\hat{d}\sqrt{(N-1)N(N+1)}}{\sqrt{12}\hat{\sigma}_\varepsilon} \quad (6)$$

t is distributed as a student t with (N–2) degrees of freedom.

If $|t|$ > critical value $t_{N-2,0.025}$, then H_0 is rejected at the 5% level of significance, i.e. the data set $\{A_1,A_2,\cdots,A_N\}$ comes from an AP.

3.6 Estimate the means and variances of A_n's

First, the mean and variance of A_1 are estimated respectively using the relevant estimators with formulae given below, see Leung *et al.* (2002).

If the difference $d<0$, the mean μ_{A_1} and variance $\sigma_{A_1}^2$ of A_1 are estimated respectively by

$$\hat{\mu}_{A_1}=\hat{\alpha} \quad (7)$$

where $\hat{\alpha}$ is determined by equation (4), and

$$\hat{\sigma}_{A_1}^2=\frac{\sum_{n=1}^{N}[A_n+(n-1)\hat{d}]^2-\frac{\left\{\sum_{n=1}^{N}[A_n+(n-1)\hat{d}]\right\}^2}{N}}{N-1} \quad (8)$$

If $d\in\left(0,\frac{\mu_{A_1}}{n-1}\right]$, μ_{A_1} is estimated using the solution of the following equation

$$\ln\left(\frac{\mu_{A_1}}{\hat{\alpha}}\right)-\frac{\hat{\sigma}_\varepsilon^2}{2}\left(\frac{1}{\mu_{A_1}^2}-\frac{1}{\hat{\alpha}^2}\right)=0 \quad (9)$$

where $\hat{\alpha}$ and $\hat{\sigma}_\varepsilon^2$ are determined by equations (4) and (5) respectively, but variance $\sigma_{A_1}^2$ of A_1 is still estimated by equation (8).

If $d=0$, μ_{A_1} and $\sigma_{A_1}^2$ are estimated respectively by

$$\hat{\mu}_{A_1}=\frac{\sum_{n=1}^{N}A_n}{N} \text{ and } \hat{\sigma}_{A_1}^2=\frac{\sum_{n=1}^{N}(A_n-\hat{\mu}_{A_1})^2}{N-1} \quad (10)$$

Having obtained the estimates $\hat{d}$, $\hat{\mu}_{A_1}$, $\hat{\sigma}_{A_1}^2$ of the parameters d, μ_{A_1}, $\sigma_{A_1}^2$ of a gearbox, we compute the averages of the respective estimates for each type of gearbox.

Secondly, the means and variances of A_n (n = 2,3,$\cdots$,N) for each type of gearbox are estimated respectively using the formulae given below, see Leung *et al.* (2002).

$$\hat{\mu}_{A_n}=\hat{\mu}_{A_1}-(n-1)\hat{d} \text{ and } \hat{\sigma}_{A_n}^2=\hat{\sigma}_{A_1}^2 \text{ for } n=2,3,\cdots,N \quad (11)$$

3.7 Determine the optimal replacement policy

Let Z_n be the time between the (n–1)th replacement and the nth replacement with $Z_0=0$, then $\{Z_n, n=1,2,\cdots\}$ forms an RP. Applying known results from renewal theory, see Ross (1970), the long-run (expected or actual) average cost per unit time

$$=\frac{\text{the expected cost incured in a cycle}}{\text{the expected length of a cycle}}$$

where a cycle is the time between two successive replacements.

The long-run average cost per day proposed in Leung (2001) is given by

$$c(N)=\frac{c_f\left\{\frac{N-1}{2}\left[2\mu_{Y_1}-(N-2)b\right]\right\}+c_R}{\frac{N}{2}\left[2\mu_{X_1}-(N-1)a\right]+\frac{N-1}{2}\left[2\mu_{Y_1}-(N-2)b\right]}$$

for $N=1,2,\cdots$. (12)

Since KMB has almost all details (e.g. repair time, replacement time, labour wage, acquisition cost, etc.) of a repair or replacement for accounting purposes, it is not difficult to obtain good estimates of average repair-cost rate c_f and replacement-cost rate c_R. For confidentiality, both figures cannot be disclosed.

The optimal replacement policy is determined by minimising $c(N)$ with respect to N. The following steps show how to determine the optimal replacement policy.

1. Calculate the long-run average cost per day $c(N)$ for N = 1,2,3,$\cdots$ using equation (12).
2. Plot the long-run average cost per day $c(N)$ against N. Then the optimum replacement epoch, shown by the

minimum point of the curve, is determined.

4. ANALYSIS AND FINDINGS

4.1 Results of the estimation of parameters

The results of the hypothesis testing show that all the ratios $\hat{a}$ and $\hat{b}$ are not equal to zero, i.e. each set of data comes from an AP.

1. The estimates $\hat{a}$, $\hat{\mu}_{X_1}$ and $\hat{\mu}_{X_n}$ of the parameters a, μ_{X_1} and μ_{X_n} for the operating times of each type of gearbox are given in Table 1.

In Table 1, all the parameters $\hat{a}$ are larger than zero. This indicates that the operating times of all gearboxes become shorter and shorter, and that the gearboxes will eventually die out. The V gearbox will die out at the faster rate, as it has the highest value for $\hat{a}$. The V gearbox has the highest $\hat{\mu}_{X_1}$ value, which indicates that a V gearbox operates longer than the other gearboxes before the first failure occurs.

2. The estimates $\hat{b}$, $\hat{\mu}_{Y_1}$ and $\hat{\mu}_{Y_n}$ of the parameters b, μ_{Y_1} and μ_{Y_n} for the repair times of each type of gearbox are given in Table 2.

In Table 2, the parameters $\hat{b}$ are larger than zero except for the V63 gearbox. $\hat{b}>0$ indicates that the repair times of the gearboxes decrease and will tend towards zero. The reasons for this phenomenon are as follows:

(a) KMB spends a lot of time on the following when a gearbox first fails, see Leemis (1995):
   (i) Diagnosis time: Time used for fault finding, including adjustment of test equipment, carrying out checks, interpretation of information gained, verification of the conclusions drawn and deciding corrective action.
   (ii) Logistic time: Time used in waiting for spare parts, test gears, additional tools and manpower to be transported to the system.
   (iii) Administrative time: Time used in the allocation of repair tasks, manpower changeover due to demarcation arrangements, official breaks, disputes, etc.

(b) KMB gains repair experience from the first failure, which is used to improve their time management, so repair time decreases.

(c) When a gearbox is taken out of a bus, there is no follow-up tracing of the gearbox and hence we are unable to find exact consecutive repair times.

$\hat{b}<0$ indicates that the repair times for a V63 gearbox increase in number and will tend towards infinity because of ageing and accumulative wear. The damage done to a V63 gearbox becomes more serious at each failure occurring at a later stage; therefore KMB will have to spend a large amount of time repairing it.

Table 1. The estimated values of the parameters a, μ_{X_1} and μ_{X_n}

Gearbox types	$\hat{a}$		$\hat{\mu}_{X_1}$		$\hat{\mu}_{X_n} = \hat{\mu}_{X_1} - (n-1)\hat{a}$
	(in days)	(in years)	(in days)	(in years)	(in years)
BENZ	352.80	0.97	1112.44	3.05	3.05 – (n–1)(0.97)
D/FL	39.01	0.11	294.69	0.81	0.81 – (n–1)(0.11)
V	485.50	1.33	1284.73	3.52	3.52 – (n–1)(1.33)
V2	223.92	0.61	950.27	2.60	2.60 – (n–1)(0.61)
V63	32.60	0.09	461.50	1.26	1.26 – (n–1)(0.09)
V64G	114.13	0.31	634.24	1.74	1.74 – (n–1)(0.31)
ZF	210.60	0.58	732.50	2.01	2.01 – (n–1)(0.58)

Table 2. The estimated values of the parameters b, μ_{Y_1} and μ_{Y_n}

Gearbox types	$\hat{b}$ (in days)	$\hat{\mu}_{Y_1}$ (in days)	$\hat{\mu}_{Y_n} = \hat{\mu}_{Y_1} - (n-1)\hat{b}$ (in days)
BENZ	34.07	85.46	85.46 – (n – 1)(34.07)
D/FL	1.53	53.66	53.66 – (n – 1)(1.53)
V	3.00	86.33	86.33 – (n – 1)(3.00)
V2	2.05	27.99	27.99 – (n – 1)(2.05)
V63	–14.67	7.67	7.67 – (n – 1)(–14.67)
V64G	8.72	40.21	40.21 – (n – 1)(8.72)
ZF	16.80	60.56	60.56 – (n – 1)(16.80)

4.2 Results for seven types of gearbox

V63 gearboxes

1. The mean operating time after the $(n-1)$th repair and the mean overall operating time

Table 3 shows that a V63 gearbox cannot be operated on after the 14th failure. The mean overall operating time of a V63 gearbox is estimated to be

$\sum_{n=1}^{14} \hat{\mu}_{X_n} = 3{,}500$ days ≈ 9.589 years.

Table 3. The estimated means of the successive operating times (in days) for a V63 gearbox

n	1	2	3	4	5
$\hat{\mu}_{X_n}$	462	429	396	364	331
n	6	7	8	9	10
$\hat{\mu}_{X_n}$	229	266	233	168	136
n	11	12	13	14	15
$\hat{\mu}_{X_n}$	103	70	38	5	0

2. The mean repair time after the nth failure

Table 4 shows that the mean repair time of a V63 gearbox increases for each failure. This indicates that KMB has to spend a large amount of time repairing each failure occurring at a later stage.

Table 4. The estimated means of the consecutive repair times (in days) for a V63 gearbox

n	1	2	3	4	5
$\hat{\mu}_{Y_n}$	8	22	37	52	66
n	6	7	8	9	10
$\hat{\mu}_{Y_n}$	81	96	110	125	140
n	11	12	13	14	15
$\hat{\mu}_{Y_n}$	154	169	198	213	228

3. The optimal replacement policy

Figure 1 indicates that a V63 gearbox is replaced at the 5th failure because the minimum average cost per day occurs there. In other words, a V63 gearbox is replaced after its mean overall operating time reaches $\sum_{n=1}^{5} \hat{\mu}_{X_n} = 1{,}982$ days ≈ 5.43 years.

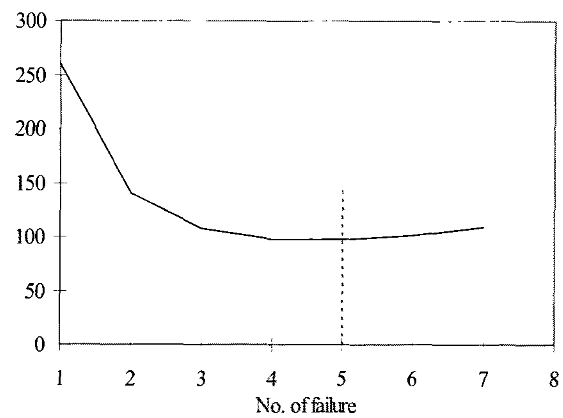

Figure 1. The long-run average cost per day versus the number of failures for a V63 gearbox

BENZ, D/FL, V, V2, V64G, ZF gearboxes

Table 5 summarises the estimated means of the successive operating times and the corresponding repair times, and the estimated mean of the overall operating times for the captioned types of gearbox. The vertical dotted lines in Figures 2 to 7 point out the replacement instants for the respective types of gearbox.

4.3 Summary of results

Based on the preceding two subsections, we can make the following conclusions and recommendations:

Table 5. The estimated means of the successive operating and the corresponding repair times (in days), and the estimated mean of the overall operating times (in years) for the other six types of gearbox

	BENZ		D/FL		V		V2		V64G		ZF	
n	$\hat{\mu}_{X_n}$	$\hat{\mu}_{Y_n}$	$\hat{\mu}_{X_n}$	$\hat{\mu}_{Y_n}$	$\hat{\mu}_{X_n}$	$\hat{\mu}_{Y_n}$	$\hat{\mu}_{X_n}$	$\hat{\mu}_{Y_n}$	$\hat{\mu}_{X_n}$	$\hat{\mu}_{Y_n}$	$\hat{\mu}_{X_n}$	$\hat{\mu}_{Y_n}$
1	1,112	85	295	54	1,285	86	950	28	634	40	733	61
2	760	51	256	52	799	83	726	26	520	31	522	44
3	407	17	217	51	314	80	502	24	406	23	311	27
4	54	0	178	49	0	⋮	279	22	292	14	101	10
5	0	–	139	48	–	⋮	55	20	178	5	0	–
6	–	–	100	46	–	⋮	0	⋮	0	–	–	–
7	–	–	61	44	–	⋮	–	⋮	–	–	–	–
8	–	–	0	⋮	–	⋮	–	⋮	–	–	–	–
Estimated mean of overall operating time	$\sum_{n=1}^{4} \hat{\mu}_{X_n}$ 2,333 days = 6.392 years		$\sum_{n=1}^{7} \hat{\mu}_{X_n}$ 1,246 days = 3.414 years		$\sum_{n=1}^{3} \hat{\mu}_{X_n}$ 2,398 days = 6.570 years		$\sum_{n=1}^{5} \hat{\mu}_{X_n}$ 2,512 days = 6.880 years		$\sum_{n=1}^{5} \hat{\mu}_{X_n}$ 2,030 days = 5.562 years		$\sum_{n=1}^{4} \hat{\mu}_{X_n}$ 1,667 days = 4.567 years	

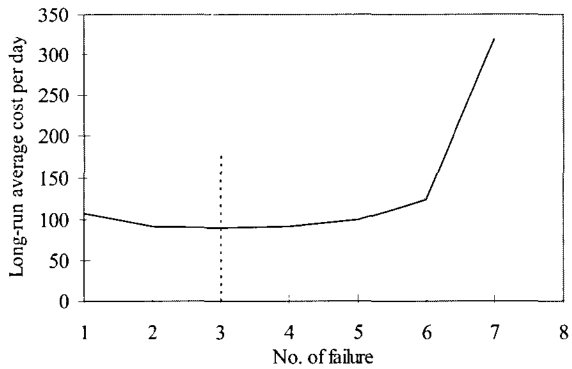

Figure 2. For a BENZ gearbox

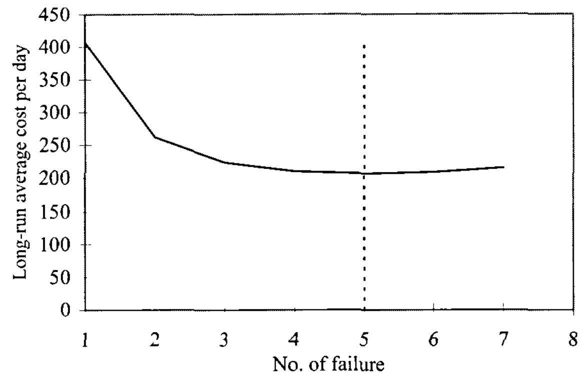

Figure 3. For a D/FL gearbox

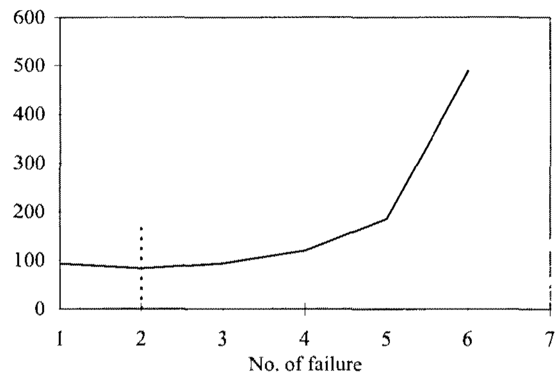

Figure 4. For a V gearbox

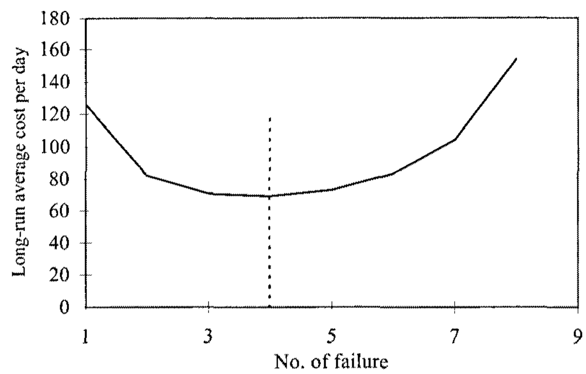

Figure 5. For a V2 gearbox

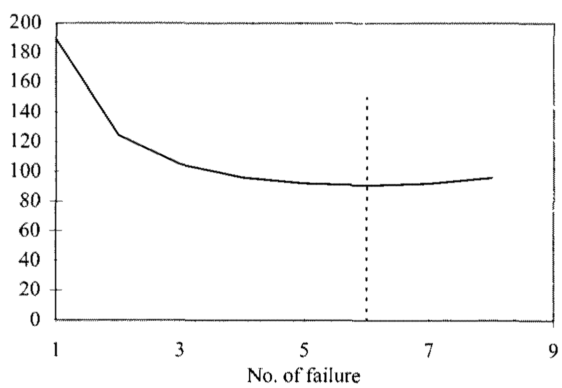

Figure 6. For a V64G gearbox

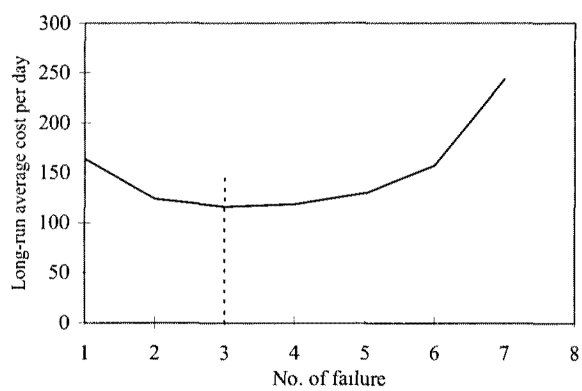

Figure 7. For a ZF gearbox

For objectives 1 and 2

1. Since the estimate $\hat{a}$ of the common difference a for each type of gearbox is greater than zero, the successive operating times form a decreasing AP.
2. Since the estimate $\hat{b}$ of the common difference b for the V63 type of gearbox is smaller than zero, a V63 gearbox's consecutive repair times form an increasing AP. Since the estimate $\hat{b}$ of the common difference b for the other six types of gearbox are greater thanzero, the consecutive repair times form a decreasing AP.

For objectives 3 and 4

We observe from Table 6 that the mean successive operating times for each type of gearbox continuously decrease, the mean consecutive repair times for each type of gearbox continuously decrease except in the case of the

V63 gearbox.

Table 6. The mean operating time is zero after the *n*th failure and the mean repair time is zero after the *n*th repair

Gearbox types	$\mu_{X_n} = 0$ after the *n*th failure	$\mu_{Y_n} = 0$ after the *n*th repair
V63	the 14th failure	not applicable
BENZ	the 4th failure	the 3rd repair
D/FL	the 7th failure	the 34th repair*
V	the 3rd failure	the 28th repair*
V2	the 5th failure	the 13th repair*
V64G	the 5th failure	the 5th repair
ZF	the 4th failure	the 4th repair

* These three numbers can easily be deduced from Table 5. Their interpretation is as follows: If a D/FL gearbox were to survive up to the 35th failure, the corresponding repair time would diminish to zero. Indeed, no practical meanings exist for the estimated mean successive repair times after the 7th failure. Similar reasoning can be applied to the other two.

The repair time decreases because KMB spends more time on diagnosing, logistics and administration at a gearbox's first failure. With this experience, KMB can improve its repair time and so the repair time decreases.

For objectives 5 and 6

From Table 7, we observe the following:

1. The V2 gearbox is recommended for use because it operates for a longer period of time than the others before it needs to be replaced.
2. The D/FL gearbox will die out fastest so is not recommended.

Table 7. The overall operating time and optimal replacement policy for each type of gearbox

Gearbox types	Overall operating time	Optimal replacement policy N at the	Optimal replacement when the overall operating time reaches T
V63	3,500 days ≈ 9.589 years	5th failure	1,982 days ≈ 5.430 years
BENZ	2,333 days ≈ 6.392 years	3rd failure	2,279 days ≈ 6.244 years
D/FL	*1,246 days ≈ 3.414 years*	5th failure	1,085 days ≈ 2.973 years
V	2,398 days ≈ 6.570 years	2nd failure	2,084 days ≈ 5.710 years
V2	2,512 days ≈ 6.880 years	4th failure	*2,457 days ≈ 6.732 years*
V64G	2,030 days ≈ 5.562 years	6th failure	2,030 days ≈ 5.562 years
ZF	1,667 days ≈ 4.567 years	3rd failure	1,566 days ≈ 4.290 years

5. A CONCLUDING REMARK

It is hoped that the paper will be useful for experimenters like engineers who are mainly interested in data analysis rather than in the mathematical development of the procedures (interested reader may refer to Leung (2001, 2002) and Leung *et al.* (2002)). Thus, an attempt is made to state the final results in each case in such a way that they can be immediately applied to data.

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