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Machine Diagnosis Techniques by a Simplified Calculation Method

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Abstract. Among many dimensional or dimensionless amplitude parameters, kurtosis and ID Factor are said to be sensitive good parameters for machine diagnosis. In this paper, a simplified calculation method for both parameters is introduced when impact vibration arise in the observed data. Compared with the past papers' results, this new method shows a good result which fit well. This calculation method is simple enough to be executed even on a pocketsize calculator and is very practical at the factory of maintenance field. This can be installed in microcomputer chips and utilized as a tool for early stage detection of the failure.

Keywords: impact vibration, probability density function, kurtosis, ID factor, deterioration

1. INTRODUCTION

In the steel making firms that have big equipment, sudden stops by machine failure cause great damages such as shortage of materials to the later processes, delays to due date and increasing idling time.

To prevent these problems, machine diagnosis techniques play important roles. So far, Time Based Maintenance(TBM) technique has constituted the main stream of the machine maintenance, which makes checks for maintenance at previously fixed time. But it has a weak point that it makes checks at scheduled time without taking into account whether the parts still hold good conditions or not. On the other hand, Condition Based Maintenance(CBM) makes maintenance checks watching the condition of machines. So, if the parts still hold good condition beyond its supposed life, the cost of maintenance may be saved because the machine can be used longer than planned. Therefore the use of CBM has become dominant recovery. The latter one needs less cost of parts, less cost of maintenance and leads to lower failure ratio.

Usually when maintenance checks are made, the initial failure probability increases, so it is said that, in general, TBM increases failure ratio compared with CBM.

However, it is mandatory to catch a symptom of the failure as soon as possible of a transition from TBM to CBM is to be made. Many methods are developed and

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examined focusing on this subject. In this paper, we propose a method for the early detection of the failure on rotating machines which is the most common theme in machine failure detection field.

So far, many signal processing methods for machine diagnosis have been proposed (Bolleter, 1998; Hoffner, *et al.*, 1991). As for sensitive parameters, Kurtosis, Bicoherence, Impact Deterioration Factor (ID Factor) were examined (Yamazaki, *et al.*, 1977; Maekawa *et al.*1997; Shao *et al.* 2001; Song *et al.*,1998; Takeyasu, 1989). Maekawa *et al.*(1997) proposed this ID Factor. In this paper, as the index parameters of vibration, we focus on Kurtosis and ID Factor.

Kurtosis is one of the sophisticated inspection parameters which calculates normalized 4th moment of Probability Density Function (PDF). In the industry, there are cases where quick reactions are required on watching the waveform at the machine site.

In this paper, we introduce a simplified machine diagnosis technique to detect failure in early stages. This simplified method enables us to calculate Kurtosis even on a pocketsize calculator and enables us to install it in microcomputer chips.

Also to the calculation of ID Factor, we apply our new method which is introduced at Kurtosis as stated above. We calculate the maximum curvature constant in the Normal Distribution and propose simplified machine diagnosis techniques in the same way as for Kurtosis.

We survey each index of deterioration in section 2. Simplified calculation method of Kurtosis is proposed and compared with the results of other papers in section 3. In section 4, we introduce a simplified calculation method of ID Factor and compare with other papers' data. Section 5 is a summary.

2. FACTORS FOR VIBRATION CALCULATION

In cyclic movements such as those of bearings and gears, the vibration grows larger whenever the deterioration becomes bigger. Also, it is well known that the vibration grows large when the setting equipment to the ground is unsuitable (Yamazaki, 1977). Assume the vibration signal is a function of time as, $x(t)$. Also assume that it is a stationary time series with mean 0. Denote the probability density function of these time series as $p(x)$. Indices for vibration amplitude are as follows.

$$X_{root} = \left[\int_{-\infty}^{\infty} |x|^{\frac{1}{2}} p(x)dx\right]^2 \qquad (1)$$

$$X_{rms} = \left[\int_{-\infty}^{\infty} x^2 p(x)dx\right]^{\frac{1}{2}} \qquad (2)$$

$$X_{abs} = \int_{-\infty}^{\infty} |x| p(x)dx \qquad (3)$$

$$X_{peak} = \lim_{n\to\infty}\left[\int_{-\infty}^{\infty} x^n p(x)dx\right]^{\frac{1}{n}} \qquad (4)$$

These are dimensional indices which are not normalized. They differ by machine sizes or rotation frequencies. Therefore, normalized dimensionless indices are required.

There are four big categories for this purpose.

A. Normalized root mean square value
B. Normalized peak value
C. Normalized moment
D. Normalized correlation among frequency domain

A. Normalized root mean square value

a. Shape Factor : *SF*

$$SF = \frac{X_{rms}}{\bar{X}_{abs}} \qquad (5)$$

($\bar{X}_{abs}$: mean of the absolute value of vibration)

B. Normalized peak value

b. Crest Factor : *CrF*

$$CrF = \frac{X_{peak}}{X_{rms}} \qquad (6)$$

(X_{peak} : peak value of vibration)

c. Clearance Factor : *ClF*

$$ClF = \frac{X_{peak}}{X_{root}} \qquad (7)$$

d. Impulse Factor : *IF*

$$IF = \frac{X_{peak}}{\bar{X}_{abs}} \qquad (8)$$

e. Impact Deterioration Factor : ID Factor / *ID*

This is stated in section 4 which is proposed in Maekawa *et al.* (1997).

$$ID = \frac{X_{peak}}{X_c} \qquad (9)$$

(X_c: vibration amplitude where the curvature of PDF becomes maximum)

C. Normalized moment

f. Skewness : *SK*

$$SK = \frac{\int_{-\infty}^{\infty} x^3 p(x)dx}{\left[\int_{-\infty}^{\infty} x^2 p(x)dx\right]^{\frac{3}{2}}} \quad (10)$$

g. Kurtosis : KT

$$KT = \frac{\int_{-\infty}^{\infty} x^4 p(x)dx}{\left[\int_{-\infty}^{\infty} x^2 p(x)dx\right]^2} \quad (11)$$

D. Normalized correlation in the frequency domain

Bicoherence means the relationship of a function at different points in the frequency domain and is expressed as

$$Bic_{,xxx}(f_1, f_2) = \frac{B_{xxx}(f_1, f_2)}{\sqrt{S_{xx}(f_1)\cdot S_{xx}(f_2)\cdot S_{xx}(f_1+f_2)}} \quad (12)$$

Here

$$B_{xxx}(f_1, f_2) = \frac{X_T(f_1)\cdot X_T(f_2)\cdot X_T^*(f_1+f_2)}{T^{\frac{3}{2}}} \quad (13)$$

means Bispectrum and

$$X_T(t) = \begin{cases} x(t) & (0 < t < T) \\ 0 & (else) \end{cases}$$

T : Basic Frequency Interval

$$X_T(f) = \int_{-\infty}^{\infty} X_T(t) e^{-j2\pi f t} dt \quad (14)$$

$$S_{xx}(f) = \frac{1}{T} X_T(f) X_T^*(f) \quad (15)$$

Range of Bicoherence satisfies

$$0 \le Bic_{,xxx}(f_1, f_2) \le 1 \quad (16)$$

When there exists a significant relationship between frequencies f_1 and f_2, bicoherence is near 1. Otherwise, the value of bicoherence comes close to 0.

These indices are generally used in combination and machine condition is judged totally. Among them, Kurtosis is known to be one of the more superior indexes (Noda, 1987) and numerous researches have been conducted on Kurtosis (Maekawa *et al.*, 1997; Shao *et al.*, 2001; Song *et al.*, 1998).

Judging from the experiment we made in the past, we may conclude that Bicoherence is also a sensitive good index (Takeyasu, 1989, 1989).

In Maekawa *et al.*(1997), ID Factor is proposed as a good index. In this paper, focusing on the indices of vibration amplitude, we introduce a simplified calculation method for Kurtosis as well as ID Factor.

3. SIMPLIFIED CALCULATION METHOD OF KURTOSIS

3.1 Several facts on Kurtosis

KT is transformed into the one for discrete time system as

$$KT_N = \frac{\frac{1}{N}\sum_{i=1}^{N}(x_i - \bar{x}_N)^4}{\left\{\frac{1}{N}\sum_{i=1}^{N}(x_i - \bar{x}_N)^2\right\}^2} \quad (17)$$

$$KT = \lim_{N\to\infty} KT_N$$

where

$$\{x_i\} : i = 1, 2, \cdots, N$$

is the discrete signal data.

Here the variance, the mean, KT, of N amount of data be stated as

$$\sigma_N^2 \ , \ \bar{x}_N \ , \ KT_N$$

and also each of $N+1 \sim N+l$ data as

$$\sigma_{N/l}^2 \ , \ \bar{x}_{N/l} \ , \ KT_{N/l}$$

Therefore

$$\bar{x}_N = \frac{1}{N}\sum_{i=1}^{N} x_i$$

$$\bar{x} = \lim_{N\to\infty} \bar{x}_N$$

As for $\bar{x}_{N/l}$, it is given as

$$\bar{x}_{N/l} = \frac{1}{l}\sum_{i=N+1}^{N+l} x_i$$

KT of N amount of data is denoted as KT_N. Although we assumed the mean to be 0 in section 2, we hold $\bar{x}$ in calculation hereafter. Assume that we get N amount of data and then newly get l amount of data. Assume that

$$\bar{x}_N = \bar{x}_{N+l}$$

where

$$\bar{x}_{N+l} = \frac{1}{N+l}\sum_{i=1}^{N+l} x_i$$

Then, from the above assumption, we get

$$\bar{x}_{N+l} = \bar{x}_N = \bar{x}_{N/l}$$

Utilizing (17), we get

$$KT_{N+l} = \frac{\frac{N}{N+l}\cdot\frac{1}{N}\sum_{i=1}^{N}(x_i-\bar{x}_N)^4 + \frac{l}{N+l}\cdot\frac{1}{l}\sum_{i=N+1}^{N+l}(x_i-\bar{x}_{N/l})^4}{\left\{\frac{N}{N+l}\cdot\frac{1}{N}\sum_{i=1}^{N}(x_i-\bar{x}_N)^2 + \frac{l}{N+l}\cdot\frac{1}{l}\sum_{i=N+1}^{N+l}(x_i-\bar{x}_{N/l})^2\right\}^2} \quad (18)$$

Denote

$$M_N = \frac{1}{N}\sum_{i=1}^{N}(x_i-\bar{x}_N)^4$$

$$M_{N/l} = \frac{1}{l}\sum_{i=N+1}^{N+l}(x_i-\bar{x}_{N/l})^4$$

[Case 1]

The case is where the variance for $1 \sim N+l$ is equal to that for $1 \sim N$ data, as well as their mean values, except for the case where a special peak value arises.

This means,

$$\bar{x}_N = \bar{x}_{N+l}$$

$$\sigma_N^2 = \sigma_{N+l}^2$$

In this case, from (18), we get

$$KT_{N+l} = \frac{N}{N+l}KT_N + \frac{l}{N+l}KT_{N/l} \quad (19)$$

[Case 2]

In this case although the mean value are equal, the variance differ with $1 \sim N$ data and $1 \sim N+l$ data except for the case special peak value arise

Suppose $\sigma^2{}_{N/l}$ is stated as

$$\sigma_{N/l}^2 = \sigma_N^2 + \delta\sigma_N^2$$

From (18), we get

$$KT_{N+l} = \frac{\frac{N}{N+l}M_N + \frac{l}{N+l}M_{N/l}}{\left(\frac{N}{N+l}\sigma_N^2 + \frac{l}{N+l}\sigma_{N/l}^2\right)^2}$$

$$\simeq \frac{\frac{N}{N+l}M_N + \frac{l}{N+l}M_{N/l}}{\sigma_N^4}\cdot\left(1 - 2\cdot\frac{l}{N+l}\frac{1}{\sigma_N^2}\delta\sigma_N^2\right)$$

$$= \frac{N}{N+l}KT_N\left(1 - 2\cdot\frac{l}{N+l}\frac{\delta\sigma_N^2}{\sigma_N^2}\right) + \frac{l}{N+l}\frac{M_{N/l}}{\sigma_N^4} \quad (20)$$

3.2 Several facts on Kurtosis

When the number of failures on bearings or gears arises, the peak value arises cyclically. In the early stage of the defect, this peak signal usually appears clearly. Generally, defects will injure other bearings or gears by contacting the inner covering surface as time passes.

Assume that the peak signal which has S times impact from normal signals arises in each m times sampling. As for determining the sampling interval, the sampling theorem which is well known can be used. But in this paper, we do not pay much attention on this point in order to focus on our proposed theme. Also we assume the 4th moment is the same with that of Case 1 (3.1) except for the case where a special peak value arises. Let $\sigma^2{}_{N/l}$ and $M_{N/l}$ of this case, of $N+1 \sim N+l$ be $\bar{\sigma}^2{}_{N/l}$, $\bar{M}_{N/l}$, then we get

$$\bar{\sigma}_{N/l}^2 = \frac{1}{l}\sum_{i=N+1}^{N+l}(x_i-\bar{x}_{N/l})^2 = \frac{l-\frac{l}{m}}{l}\sigma_N^2 + \frac{\frac{l}{m}}{l}S^2\sigma_N^2 = \sigma_N^2\left(1+\frac{S^2-1}{m}\right) \quad (21)$$

$$\bar{M}_{N/l} = \frac{1}{l}\sum_{i=N+1}^{N+l}(x_i-\bar{x}_{N/l})^4 = \frac{l-\frac{l}{m}}{l}M_{N/l} + \frac{\frac{l}{m}}{l}S^4M_{N/l} = \left(1+\frac{S^4-1}{m}\right)M_{N/l} \quad (22)$$

From these equations, we obtain $\overline{KT}_{N+l}$ as KT_{N+l} of the above case

$$\bar{KT}_{N+l} = \frac{\frac{N}{N+l}M_N + \frac{l}{N+l}\left(1+\frac{S^4-1}{m}\right)M_{N/l}}{\left\{\frac{N}{N+l}\sigma_N^2 + \frac{l}{N+l}\sigma_N^2\left(1+\frac{S^2-1}{m}\right)\right\}^2} = KT_N\frac{\left(1+\frac{l}{N+l}\frac{S^4-1}{m}\right)}{\left(1+\frac{l}{N+l}\frac{S^2-1}{m}\right)^2} \quad (23)$$

We assume that time series are stationary as is stated on page 2. Therefore, even if sample pass may differ, mean and variance are naturally supposed to be the same when the signal is obtained from the same data occurrence point of the same machine.

We consider such case when the impact vibration occurs. Except for the impact vibration, other signals are assumed to be stationary and have the same means and variances. Under this assumption, we can derive the simplified calculation method for machine diagnosis which is a very practical one.

3.3 Numerical Examples

If the system is under normal condition, we may suppose $p(x)$ becomes a normal distribution function. Under this condition, KT is always

$$KT = 3.0$$

Under the assumption of 3.2, let m=12. Considering the case S=2, 3, ..., 6 for 3.2, and setting l=N/10, we obtain Table 1 from the calculation of (23).

Table 1. $\overline{KT}$ by the variation of S

	KT_{N+l}	
S	l=N/10	l=N
2	3.19	4.32
3	4.28	8.28
4	7.09	13.2
5	12.3	17.7
6	20.3	21.3

In Maekawa *et al.*(1997), the waveform is simulated in three cases as (a) normal condition, (b) small defect condition (maximum vibration is two times compared with (a)), (c) big defect condition (maximum vibration is six times compared with (a)).

Here we introduce the following number. Each index is compared with the normal index as follows.

$$Fa = \frac{P_{abn}}{P_{nor}} \quad (24)$$

P_{nor} : Index at normal condition
P_{abn} : Index at abnormal condition

As KT is 3.0 under the normal condition, we divide corresponding items by 3.0 from Table 1 and the results are shown in Table 2.

Table 2. Comparison of Fa

S	*Fa*		$Fa \times 3.0$	
	#	Table 1(l= 0)	#	Table 1(l= N)
2	2.82	1.44	8.46	4.32
6	6.52	7.1	19.6	21.3

#: Data in Maekawa *et al.*(1997)

In a big failure such as S= 6, $\overline{KT}$ value is close to the one in Maekawa *et al.*(1997). In a small failure such as S = 2, our experiment in the past (Takeyasu 1987, 1989) shows that $\overline{KT}$ is between 3.5 ~ 4.5 and the result of Table 2 is a reasonable one. In Shao *et al.*(2001), KT are calculated for the bearings that have small, middle and large defects and the rotation frequency varies(Table 3).

Table 3. KT for each case

	Rotation frequency (rpm)		
	200	500	1700
Small defect	3.5 ~ 4.0*	3.4 ~ 3.9	2.5 ~ 3.8*
Middle defect	6 ~ 7	4.6 ~ 5.6	3.8 ~ 4.6
Large defect	7 ~ 10*	5.7 ~ 8	4 ~ 6*

*: Numerical value stated in Shao *et al.* (2001), else read the score in the graph of Shao *et al.* (2001).

These results can be taken into account, though the definition of defect size does not necessarily coincide.

3.4 Remarks

3.4.1 Remarks on the result of Numerical Examples

Here, we introduce a simplified calculation method for Kurtosis which is one of the most sensitive indices for the failure detection. When we get newly coming l amount of data after normal N amount of data, we can easily calculate KT in a simple way. The result of this simplified calculation method is a reasonable one compared with the results obtained so far.

The steps for the failure detection by this method are as follows.

1. Prepare standard KT Table for each normal or abnormal level
2. Measure peak values by signal data and compare the peak ratio to the normal data
3. Calculate KT by (23)
4. Judge the failure level by the score of KT

Preparing standard KT table for each normal and abnormal level, we can easily judge the failure level only by taking ratio of the peak value to the normal level and calculating KT by (23). This calculation method is simple enough to be carried out even on a pocketsize calculator and is very practical at the factory of maintenance field. This can be installed in microcomputer chips and utilized as the tool for early stage detection of the failure.

3.4.2 Estimation of the peak level by Kurtosis

When Kurtosis is calculated, we can get the peak level by (23) conversely. A case is considered where a machine is being watched and Kurtosis is calculated and sent to the central operating room. In such a case, a converse situation may occur as stated above. Assume that the machine is in irregular condition. Set $N=0$, $l=N$. From (23), we get

$$\bar{KT}_{0+N} = KT_0 \frac{\left(1+\frac{S^4-1}{m}\right)}{\left(1+\frac{S^2-1}{m}\right)^2} \tag{25}$$

Where $KT_0=3.0$. Set

$$\alpha = \frac{KT_0}{\bar{KT}_{0+N}} \tag{26}$$

Then, we can estimate S by the following equation.

$$S \simeq \sqrt{\frac{m-1+\sqrt{(m-1)^2+(m\alpha-1)(m-1)\{m(1-\alpha)-1\}}}{m\alpha-1}} \tag{27}$$

3.4.3 Remarks on the progress of the deterioration

In 3.2, 3.3 we assume that the peak signal which has S times impact from normal signals arises in each m times sampling in order to focus on simplified calculation. But in reality, as deterioration progresses, n-th harmonics may arise and defects may transfer. Therefore many more peaks may arise in the same interval. This implies that when we estimate m, a much smaller one would be estimated if deteriorations are being progressed. We examine this by calculation hereafter. Equation (23) may be much more simply approximated as

$$\begin{aligned} \bar{KT}_{N+l} &\simeq KT_N\left(1+\frac{l}{N+l}\cdot\frac{S^4-1}{m}\right)\left(1-2\frac{l}{N+l}\cdot\frac{S^2-1}{m}\right) \\ &\simeq KT_N\left(1+\frac{l}{N+l}\cdot\frac{S^4-1}{m}-2\frac{l}{N+l}\cdot\frac{S^2-1}{m}\right) \\ &= KT_N\left\{1+\frac{l}{N+l}\cdot\frac{(S^2-1)^2}{m}\right\} \end{aligned} \tag{28}$$

Set the increment of $\overline{KT}_{N+l}$ as $\Delta\overline{KT}_{N+l}$, then we get

$$\frac{\Delta\bar{KT}_{N+l}}{KT_N} = \frac{l}{N+l}\cdot\frac{(S^2-1)^2}{m} \tag{29}$$

As for variance, we can calculate as

$$\begin{aligned} \bar{\sigma}^2_{N+l} &= \frac{N}{N+l}\sigma_N^2 + \frac{l}{N+l}\sigma_N^2\left(1+\frac{S^2-1}{m}\right) \\ &= \sigma_N^2 + \frac{l}{N+l}\sigma_N^2\frac{S^2-1}{m} \end{aligned} \tag{30}$$

Therefore we can get the ratio of the increment as

$$\frac{\Delta\bar{\sigma}^2_{N+l}}{\sigma_N^2} = \frac{l}{N+l}\cdot\frac{S^2-1}{m} \tag{31}$$

Then we get

$$\Delta\xi = \frac{\frac{\Delta\bar{KT}_{N+l}}{KT_{N+l}}}{\left(\frac{\Delta\bar{\sigma}^2_{N+l}}{\sigma_N^2}\right)^2} = \frac{N+l}{l}\cdot m \tag{32}$$

Suppose N is equal to 0, l is equal to N, and the peak arises every m times measurement of sampling, then (32) leads to

$$\Delta\xi = m \tag{33}$$

In the case N is equal to 0, l is equal to N, and peak arises every m/2 times as deterioration progresses, then (32) leads to

$$\Delta\xi = \frac{m}{2} \tag{34}$$

This shows that the estimated m becomes much smaller compared with the initial one and we can identify that the deterioration is progressing. This implies that (32) is a function of m.

Therefore we can estimate the state whether it is the case that added l amount of data have large values everywhere in the interval (case α) or it is the case that m times measurement data increase Kurtosis (case β). Even if the variance and Kurtosis increase on the added l amount of data, we can assume that it is case α when the calculation result of (32) is not the function of m.

If it is possible to keep watching the power spectrum, we may easily identify these situations. However, a spectrum analyzer doesn't always come handy. Even in such cases, our method provides a simple way to detect the condition.

4. ID FACTOR

4.1 About ID Factor

Ideal method to grasp the impact level is to take the ratio of X_{peak} (impact level) and X_n (non impact level) (Maekawa *et al.*, 1997).

In Maekawa *et al.*(1997), the usage of X_c is the proposed instead of X_n, because X_n is hard to get. X_c is amplitude at which the curvature of PDF becomes maximum.

The Curvature of PDF is expressed as

$$C(x) = \frac{p''(x)}{\left\{1+p'(x)^2\right\}^{\frac{3}{2}}} \tag{35}$$

in Mathematical Society of Japan (1985). Impact Deterioration Factor (ID Factor) is defined at (9).

Maekawa *et al.*(1997) demonstrated how this index is effective. Though Maekawa *et al.*(1997) paper's report ends at this point theoretically, we proceed much more concretely.

If $p(x)$ is a normal distribution function, then

$$p(x) = \frac{1}{\sigma\sqrt{2\pi}} exp\Big[-\frac{1}{2}\Big(\frac{x-\bar{x}}{\sigma}\Big)^2\Big] \quad (36)$$

Substituting this to (35), we obtain

$$C(x) = \frac{\frac{1}{\sigma^2}\cdot\frac{1}{\sigma\sqrt{2\pi}}\Big\{\Big(\frac{x-\bar{x}}{\sigma}\Big)^2 - 1\Big\} exp\Big[-\frac{1}{2}\Big(\frac{x-\bar{x}}{\sigma}\Big)^2\Big]}{\Big\{1 + \frac{1}{\sigma^2\sqrt{2\pi}}\Big(\frac{x-\bar{x}}{\sigma}\Big)^2 exp\Big[-\Big(\frac{x-\bar{x}}{\sigma}\Big)^2\Big]\Big\}^{\frac{3}{2}}} \quad (37)$$

We get

$$C'(x) = \frac{1}{\sigma^2\cdot\sigma\sqrt{2\pi}} exp\Big[-\frac{1}{2}\Big(\frac{x-\bar{x}}{\sigma}\Big)^2\Big] \times \frac{1}{\Big\{1 + \frac{1}{2\pi\sigma^2}\Big(\frac{x-\bar{x}}{\sigma}\Big)^2 exp\Big[-\Big(\frac{x-\bar{x}}{\sigma}\Big)^2\Big]\Big\}^{\frac{5}{2}}} \times \frac{x-\bar{x}}{\sigma^2}\Big[3 - \Big(\frac{x-\bar{x}}{\sigma^2}\Big)^2 + \frac{1}{2\pi\sigma^4} exp\Big[-\Big(\frac{x-\bar{x}}{\sigma}\Big)^2\Big] \cdot\Big\{2\Big(\frac{x-\bar{x}}{\sigma}\Big)^4 - 3\Big(\frac{x-\bar{x}}{\sigma}\Big)^2 + 3\Big\}\Big] \quad (38)$$

From

$$C'(X_c) = 0$$

We get

$$X_c \simeq 1.976\sigma \quad (39)$$

When we get C_0 (Constant of Maximum Curvature) for the coefficient of σ of (39), it is expressed as

$$X_c = C_0\sigma \quad (40)$$

4.2 Simplified Calculation Method of ID Factor

Here, we propose a simplified calculation method of ID Factor utilizing above consideration. Generally, when the number of defects arises on bearings or gears, the shape of PDF varies to have flat a shape, i.e., variance grows large.

When peak values arise as assumed in 3.2, under the assumption of case 1 in 3.1, the variance would be the one given in (21). Set the coefficient of $\sigma^2{}_N$ of (21) to be $g(m, S)$.

Then,

$$\bar{\sigma}^2_{N/l} = g(m, S)\sigma^2_N \quad (41)$$

We consider the case when the variance grows big in 4.1. If the variance becomes K^2 times the original one, σ would be replaced by $K\sigma$ in (40) and then expressed as

$$X_c = C_0 K\sigma \quad (42)$$

Comparing with (41), we get

$$K = \sqrt{g(m, S)} \quad (43)$$

Set X_{peak} under normal condition as $X_{peak\ N}$. Also set ID for 1 ~ N amount of data as ID_N, l (N+1 ~ N+l) amount of data $ID_{N/l}$ as is in section 3, then we get

$$ID_N = \frac{X_{peak\ N}}{C_0\sigma_N} \quad (44)$$

$$ID_{0/N} = \frac{S\cdot X_{peak\cdot N}}{C_0\sigma_N\sqrt{g(m,S)}} = \frac{S\cdot X_{peak\cdot N}}{C_0\sigma_N\sqrt{1+\frac{S^2-1}{m}}} = \frac{S}{\sqrt{1+\frac{S^2-1}{m}}} ID_N \quad (45)$$

Set the coefficient part of ID_N as

$$h(m, s) = \frac{S}{\sqrt{1+\frac{S^2-1}{m}}} \quad (46)$$

As $h(m, S)$ is given by the ratio of the abnormal condition ratio to the normal one, it corresponds with Fa in (24).

4.3 Numerical Examples

As is the case with 3.3, we calculate $h(m, S)$ in the case of m = 12, S = 2, 3, ..., 6 (Table 4). In Maekawa *et al.*(1997), *Fa* of the case S = 2 and S = 6 are shown.

The results of numerical examples are quite similar to those of Maekawa *et al.*(1997).

Table 4. Change of ID parameter (l = N)

S	*h(m,S)*	*##*
2	1.79	1.835
3	2.32	
4	2.67	
5	2.89	
6	3.03	3.903

##: Data in Maekawa *et al.*(1997)

4.4 Remarks

4.4.1 Remarks on the result of Numerical Examples

Effectiveness of ID Factor is stated in detail in Maekawa *et al.*(1997). In this paper, we proposed a simplified calculation method of ID Factor. The results of numerical examples are similar to those of Maekawa *et al.*(1997). Using this method, we can calculate ID very simply from (46).

The steps for the failure detection are nearly the same as the case of Kurtosis.

1. Prepare standard ID Factor Table for each normal or

abnormal level

2. Measure peak values by signal data and compare peak ratio to the normal data
3. Calculate $h(m, S)$ (Fa of ID Factor) by (46)
4. Judge failure level by the score of Fa

This calculation is similar to the case of Kurtosis. This calculation method is simple enough to do carried out even on a pocketsize calculator and is very practical at the factory of maintenance field. This can be installed in microcomputer chips and utilized as a tool for early stage detection of the failure.

4.4.2 Estimation of the peak level by ID Factor

By the same reason as 3.4 (2), we explain the estimation method. When ID Factor is calculated, we can get the peak level by (46) conversely. Assume that a machine is in an irregular condition and set $N=0$, $l=N$. From (46), we can estimate S by the following equation.

$$S = \sqrt{\frac{m-1}{m-h^2(m,S)}} \cdot h(m,S) \tag{47}$$

5. CONCLUSIONS

We proposed a simplified calculation method for Kurtosis and ID Factor that are sensitive good indices for failure detection of rotating machine. Compared with results obtained so far, the results of numerical examples of this paper are reasonable. Judging from these results, our method is properly considered to be effective for especially early stage failure detection. This calculation method is simple enough to be executed even on a pocketsize calculator and is very practical at the factory of maintenance field. This can be installed in microcomputer chips and utilized as a tool for early stage detection of the failure.

The effectiveness of this method should be examined in various cases.

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