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Development of Strain-softening Modeling for Interfaces between Geosynthetics

토목섬유 interface의 변형율 연화 모델 개발

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Abstract

Strain-softening model is developed to characterize the interface behavior of geomembrane with geotextile and geosynthetic clay liner(GCL). The model proposed in this research is calibrated by using data from direct shear tests conducted on smooth and textured geomembrane. The research is divided into two regions, pre-peak and post-peak, to take into account of strain-softening effect. Although slight difference between measured and back calculated data is observed under high normal stress, good agreements, in general, are found from back calculations. Especially, good consistency is observed in the case of low normal stress. Based on the results, it can be concluded that the proposed model can be a reasonable constitutive law to figure out the behavior of strain-softening between interfaces of geomembrane. In addition, DSC(Disturbed State Concept) model is also presented for further application in geosynthetic interfaces.

Keywords : Strain-softening, Geosynthetic, Interface shear strength, Modeling, Shear displacement

1. Introduction

Various types of landfill composite liner systems have been developed to prevent leachate migration into the subsurface environment. The use of various types of geosynthetics to construct landfill composite liner systems results in interfaces between different geosynthetics and goesynthetics/soils. However, the shear resistance at soil/geosynthetic and geosynthetic /geosynthetic interfaces is often low, which may give rise to slippage or slope failures of landfills (Koerner and Soong, 2000).

Slope stability is an important consideration in the design of containment systems. Especially, the potential for progressive failure in waste containment systems is an important design consideration (Gilbert *et al.* 1996, Filz *et al.* 2001). Many common interfaces between components in containment systems exhibit strain-softening behavior. However, conventional limit equilibrium methods used widely lack the capability to take into account stress and strain behavior and to compute displacements along

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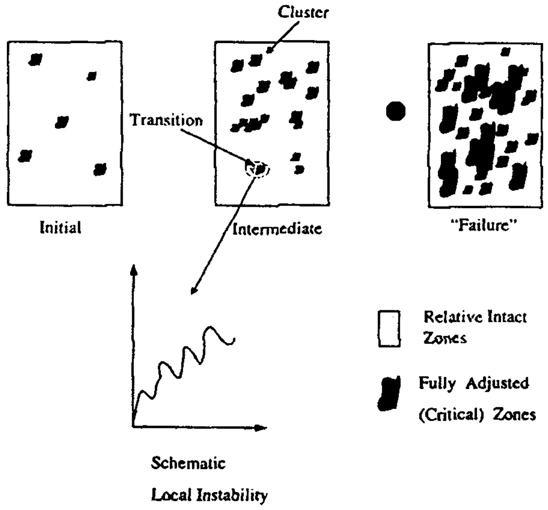
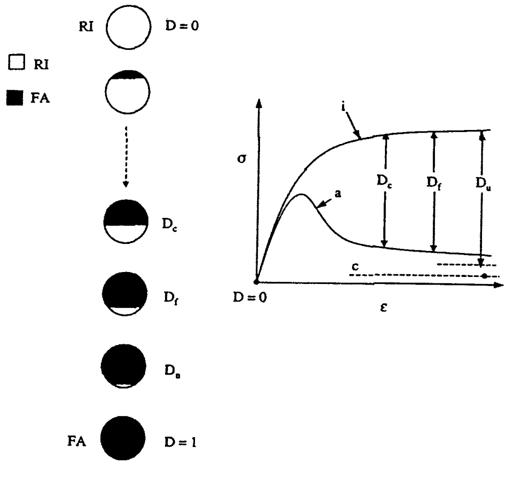


Fig. 1. Relative intact and fully adjusted clusters in DSC (Desai, 2001)



(a) Symbolic Representation of DSC (b) Schematic of Stress-Strain behavior

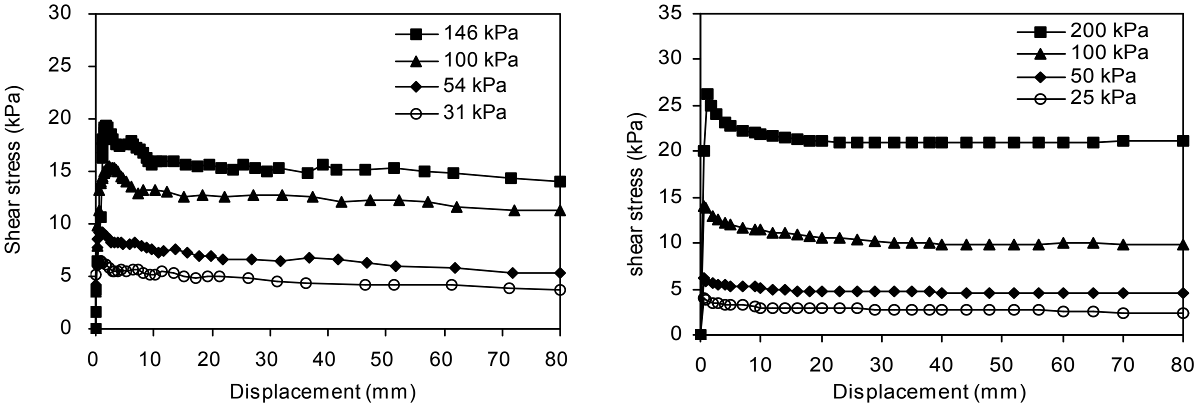
Fig. 2. Representations of DSC (Desai *et al.*, 1998)

the critical shear plane.

Numerical simulation in geoenvironmental engineering practice requires the availability of constitutive models to describe the stress-deformation behavior of the interfaces involved. A few research on the theory of geosynthetic interface modeling for landfill liners has been published (e.g. Gilbert *et al.* 1996, Jenevein *et al.* 1996, Reddy *et al.* 1996, Esterhuizen *et al.* 2001). Reddy *et al.* (1996) conducted analyses using the interface shear-displacement parameters determined from available laboratory direct shear testing data. Esterhuizen *et al.* (2001) suggested that plasticity models are more suitable than elasticity models because they correctly predict sliding displacements as a function of the total shear and normal stresses acting on the interface, and Filz *et al.* (2001) evaluated the practical significance of progressive failure using the constitutive model proposed by Esterhuizen *et al.* (2001).

Desai (2001) proposed DSC (Disturbed State Concept) model which is able to allow for factors such as continuous yielding or hardening, microcracking and softening leading to postpeak degradation, stiffening, and viscous effects. In DSC model, many of the models can be derived as special cases of the DSC and interface can be treated as a deforming material element that is composed of the relative intact (RI) and fully adjusted (FA) parts (Fig. 1). The observed or actual response of the interface is expressed in terms of the responses of parts in the reference states, i.e. RI and FA. The disturbance, D, denotes the deviation of the observed response from those of the reference state. Fig. 2 shows a symbolic and schematic representation of disturbance in the DSC.

In this paper, a constitutive model is proposed for evaluating strain-softening effects on the stability of waste containment systems and the model is verified through comparisons between measured data from direct shear tests and predicted data through back-calculations. This model can be applied in the interfaces involving both smooth geomembrane and textured geomembrane, and will be implemented for finite element analyses of waste containment facility including multiple layered landfill lining/cover systems. In addition, the research on the application of DSC (Disturbed State Concept) model in the geosynthetic interfaces is being conducted. The



(a) Interface shear tests from Seo *et al.* (2002) (b) Interface shear tests from Jones and Dixon (1998)

Fig. 3. Interface shear testing results between S-GM and GT

comparisons between models presented in this paper and DSC models will be followed.

2. Experimental Data for Geosynthetic Interfaces

The constitutive relationships presented for interfaces of geomembrane(GM) and geosynthetic were developed and calibrated in this research, using the results of large-scale direct shear tests by Seo *et al.* (2002), Jones and Dixon (1998), and Triplett and Fox (2001). Two types of interfaces were modeled in one constitutive relationship in the analyses, where one interface involves smooth HDPE (High Density Polyethylene) geomembrane (S-GM) and the other interface includes textured HDPE geomembrane (T-GM).

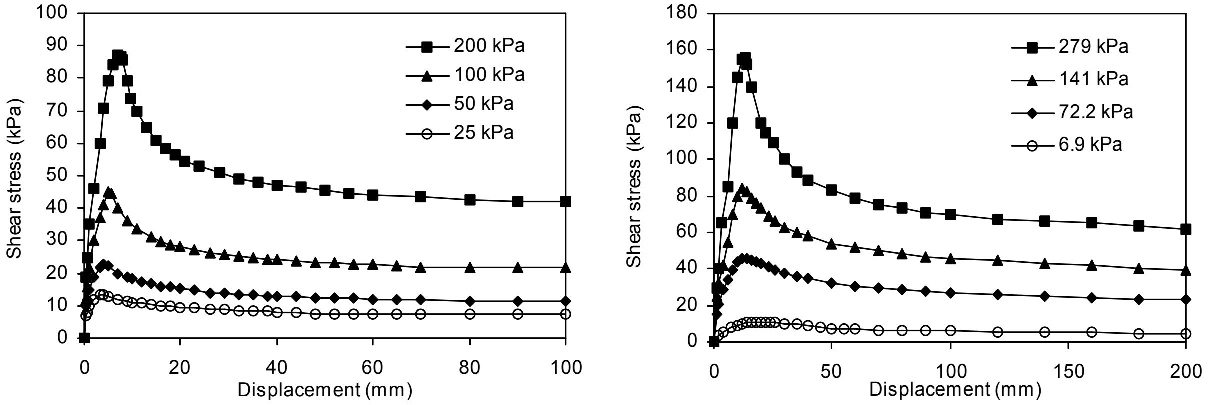
2.1 Interfaces between smooth GM(S-GM) and geotextile(GT)

Two published data were utilized for interfaces involving S-GM (Fig. 3). Seo *et al.* (2002) conducted a large direct shear test (300×300 mm) with a S-GM and nonwoven GT (Fig. 3(a)). Jones and Dixon (1998) also made tests by using large direct shear testing apparatus with the S-GM and 750 g/m^2 PP (Polypropylene) nonwoven geotextile (Fig. 3(b)). A shearing displacement rate of 1.0 mm/min was applied for testing of S-GM and GT interfaces.

The initial shear stiffness of (a) 7,926 kN/m^3 and (b) 12,920 kN/m^3 is approximately the same for all levels of normal stress and the peak shear resistance is usually mobilized with 2.5 mm. As Stark *et al.* (1996) reported that real residual resistance is mobilized within 800 - 1,150 mm from ring shear testing, large displacement shear strength at displacement of 80 mm, in this research, instead of residual value is utilized for analyses. The large displacement shear strength can be reasonable design value because the slope failure in landfill liner interfaces tends to occur within displacements of 100 mm. It can be also seen the peak strength is followed by significant strength reduction , strain-softening, as shear displacement proceeds. The average amount of strength reduction at 80 mm is estimated to be (a) 32% and (b) 29% of peak shear strength respectively.

2.2 Interfaces between textured GM(T-GM) and geotextile(GT)

Two published data were also analyzed for interfaces involving T-GM (Fig. 4). The T-GM used



(a) Interface shear tests for T-GM/nonwoven GT from Jones and Dixon (1998)

(b) Interface shear tests for T-GM/GCL from Triplett and Fox (2001)

Fig. 4. Interface shear testing results between T-GM and geosynthetics

in the testing by Jones and Dixon (1998) was manufactured in Germany, with the texturing produced by impinging hot polyethylene particles on the surface of previously manufactured smooth sheets. Triplett and Fox (2001) performed direct shear tests on large (406×1,067 mm) rectangular GM/Geosynthetic clay liner(GCL), which was composed of GT/clay/GT and reinforced by needle-punching, by using the pullout shear machine described by Fox *et al*. (1997). The GMs used in the testing were a round-dye co-extruded textured products. GCL specimens were hydrated using four-day, two-stage procedure described by Fox *et al*. (1998). For the first stage of hydration, each specimen was hydrated for two days under 1 kPa vertical stress. For the second stage of hydration, normal stress was applied and the specimens were hydrated for additional two days. The nonwoven GT part of GCL contacted with T-GM at the interface of T-GM/GCL (Fig. 4(b)).

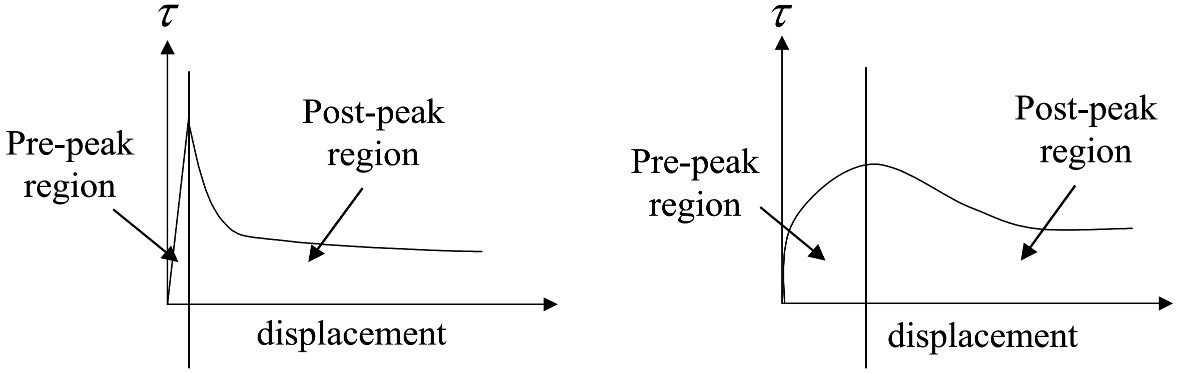
The initial shear stiffness is found to vary with the normal stress employed in the tests (Fig. 4) unlike the test results of S-GM. Each failure occurs at the GM/geosynthetic interface and the nonlinear behavior between shear stress and displacement is observed. The peak shear resistance is usually mobilized with 3.2-18 mm, which implies that more displacement is required to mobilize the peak shear strength than a case with. The large displacement shear strength is also estimated to have (a) 46% and (b) 50% of peak shear strength. The post peak strength loss is substantially greater with a T-GM instead of S-GM.

3. Interface Modeling

The determination of interface shear stress-shear displacement behavior and shear strength parameters is essential for an accurate assessment of the composite liner interface stability. However, it is difficult to establish the available shear resistance along an interface exhibiting strain softening behavior, which make it complex to model the shear stress-displacement relationship. In this paper, the entire region is divided into to two parts; (1) pre-peak region and (2) post-peak region, to take into account of shear strength degradation after peak strength is mobilized (Fig. 5).

3.1 Shear strength determination

For the S-GM/GT interfaces, it is assumed that



(a) Interface involving S-GM (b) Interface involving T-GM

Fig. 5. Generalized shear stress vs. displacement relationship for geosynthetic interface

interface shear strength envelope for the peak and large displacement (80 mm) have a zero cohesion intercept. Two types of failure envelopes, linear or nonlinear failure envelope, are assumed to identify the importance of accurate determination of failure envelopes. Table 1 provides a summary of interface shear strength for S-GM/GT.

where τ= shear strength, σ_n= normal stress, and r^2 = correlation coefficient

The correlation coefficient is the smallest in the case of linear failure envelopes of Seo *et al.* (2002). However, that of Jones and Dixons (1998) for linear failure envelope is the highest among all values.

For T-GM/GT or GCL interfaces, peak and large displacement failure envelopes are approximately linear and characterized using Mohr-Coulomb criterion

$$\tau = c + \sigma_n \tan\delta \qquad (1)$$

where τ= shear stress, c = cohesion, σ_n= normal stress, and δ= friction angle. Table 2 lists peak and

Table 1. Interface shear strength envelopes for interfaces of S-GM/GT

Reference	Linear failure envelope			Nonlinear failure envelope		
Seo *et al.* (2002)	Peak	$\tau_p = \tan 8.2^\circ\ \sigma_n$	$r^2=0.91$	Peak	$\tau_p=0.51\sigma_n^{0.73}$	$r^2=1.00$
	Large displacement	$\tau_l = \tan 5.8^\circ\ \sigma_n$	$r^2=0.97$	Large displacement	$\tau_l=0.16\sigma_n^{0.91}$	$r^2=0.98$
Jones and Dixon (1998)	Peak	$\tau_p = \tan 7.6^\circ\ \sigma_n$	$r^2=1.00$	Peak	$\tau_p=0.18\sigma_n^{0.93}$	$r^2=0.99$
	Large displacement	$\tau_l = \tan 5.9^\circ\ \sigma_n$	$r^2=1.00$	Large displacement	$\tau_l=0.08\sigma_n^{1.05}$	$r^2=1.00$

Table 2. Interface shear strength envelopes for interfaces of T-GM/geosynthetic

Reference	Interface	Peak interface strength (τ_p)			Large displacement interface strength (τ_l)		
		friction angle (°)	cohesion (kPa)	r^2	friction angle (°)	cohesion (kPa)	r^2
Jones and Dixon (1998)	T-GM/ GT	22.9	2.5	1.00	11.3	1.9	1.00
Triplett and Fox (2001)	T-GM/ GCL	28.1	7.7	1.00	11.7	6.6	0.98

where r^2 = correlation coefficient

large displacement strength parameters under the range of normal stress applied in the testing.

3.2 Pre–peak region modeling

As mentioned previously, the modeling is performed, dividing the entire region of displacement-shear stress into two regions, pre-peak region and post-peak region, to describe the strength reduction after peak strength is mobilized. Linear and non linear relationships between displacement and shear stress at the pre-peak region are formulated into one constitutive equation because using two equations for one type of the interface make it complex to implement this model into FEM.

For interfaces involving S-GM, the interface shear stiffness is the same regardless of the level of normal stress. For interfaces with T-GM, however, the initial shear stiffness is dependent on the magnitude of normal stress. The tangent shear modulus value is found to vary with the normal stress employed in the tests. The procedure proposed by Duncan and Chang (1970) is modified in this modeling at pre-peak region. This hyperbolic relationship describes the nonlinear and stress dependent interface shear stress-shear displacement relationship at an interface.

The procedures for identifying the stress-displacement relationship at the interfaces, especially involving T-GM, are as follows.

(1) Approximation of nonlinear stress-displacement relationship by using hyperbolic equation; The hyperbolic equation is considered for interface shear behaviors. The hyperbolic formulation (Kondner, 1963) is given by

$$\tau = \frac{d}{a + bd} \tag{2}$$

where τ = interface shear strength, d = shear displacement, 1/a = initial shear modulus (E_i), and 1/b = ultimate shear strength (τ_{ult}). From the value of b, the failure ratio (R_f) which is defined as the ratio of τ_p/τ_{ult} , is calculated. On the other hand, the failure ratio is evaluated to be zero for interfaces of S-GM as the ultimate shear strength is not converged to one value.

(2) Consideration of stress dependency; Experimental studies show that the relationship between initial shear modulus and normal stress may be expressed as (Janbu, 1963; Reddy *et al.* 1996)

$$E_i = K \gamma_w \left(\frac{\sigma_n}{P_a}\right)^n \tag{3}$$

where E_i = the initial shear modulus, σ_n = normal stress, γ_w = unit weight of water, K = dimensionless shear coefficient, P_a = atmospheric press, and n = modulus exponent. Values of the parameters K and n may be determined readily from the testing results. The values of E_i/γ_w and σ_n/P_a are plotted using logarithmic (log-log) axes. The slope of the best-fit line gives the value of n, and the intercept of the line gives the value of K. However, the value of n is evaluated to be zero at the interfaces with S-GM because shear modulus shows no dependency on normal stress.

(3) Determination of tangent modulus value; The tangent modulus, E_t, may be expressed as

$$E_t = \frac{\partial \tau}{\partial d} \tag{4}$$

Duncan and Chang (1970) proposed that tangent modulus value for any stress condition may be expressed as

$$E_t = K \gamma_w \left(\frac{\sigma_n}{P_a}\right)^n \left(1 - R_f \frac{\tau}{\tau_p}\right)^2 \tag{5}$$

and maintained that this expression for tangent

Table 3. Summary of the hyperbolic interface model parameters at pre-peak region

Interface	R_f	K	n	Reference
S-GM/GT	0	809	0	Seo et al. (2002)
	0	1,318	0	Jones and Dixon (1998)
T-GM/GT	0.88	5,314	0.18	Jones and Dixon (1998)
T-GM/GCL	0.67	1,540	0.65	Triplett and Fox (2001)

modulus may be employed very conveniently in incremental analyses.

The hyperbolic interface model parameters calculated from previous mentioned procedures are summarized in Table 3 for the interfaces of S-GM/GT and T-GM/geosynthetic. Increasing value of n in Table 3 implies that the dependency of tangent modulus on normal stress increases at pre-peak region.

3.3 Post-peak region modeling

The displacement softening model developed by Esterhuizen *et al.* (2001) is used to describe the interface shear behavior at post-peak region in this research. Esterhuizen *et al.* (2001) followed the method outlined by Turnbull and Hvorslev (1967) to model a nonlinear displacement softening behavior for geosynthetic interfaces after peak strength is mobilized.

As strain-softening displacement, d_s (= $d_{pp} - d_p$), defined in Fig. 6 increases, the strength reduction, τ_s, increases until it equals the full difference between peak and large displacement strengths. Fig. 6 provides a generalized stress verse displacement relationship and the definition of parameters used in the modeling, including the definitions of d_s, d_p, and d_{pp}.

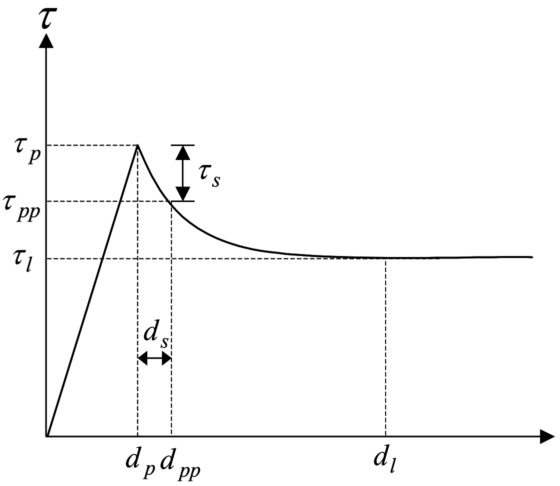


Fig. 6. Definition of parameters used in the modeling

It is assumed that the normalized curves can be approximated as nonlinear hyperbolic relationship between shear strength degradation, τ_s, and strain-softening displacemen, d_s. The procedures for displacement softening modeling used by Esterhuizen *et al.* (2001) are as follows.

(1) Transformation of the initial stress vs. displacement curves into new curves that relate the strength degradation τ_s, $\tau_p - \tau_{pp}$, to the strain- softening shear displacement d_s, $d_{pp} - d_p$, which can be calculated from subtracting the shear displacement at peak shear strength from the large displacement at post-peak region.

(2) Calculation of the strain-softening factor, S, and the displacement ratio, D; S can be calculated by normalizing the post-peak strength degradation by the shear strength degradation from the peak to the large displacement value, $\tau_p - \tau_l$. D also can by calculated by normalizing the strain softening shear displacement, d_s, by the maximum strain softening shear displacement, d_s, that take place at large displacement, d_l.

(3) Plotting the relationship between S and D; The normalized curves can be approximated by a

Table 4. Summary of the hyperbolic interface model parameters at post-peak region

Interface	k	c	Reference
S-GM/GT	14.22	1.08	Seo *et al.* (2002)
	29.17	1.04	Jones and Dixon (1998)
S-GM/Clay	20.00	1.05	Esterhuizen *et al.* (2001)
T-GM/GT	5.82	1.21	Jones and Dixon (1998)
T-GM/GCL	6.51	1.18	Triplett and Fox (2001)

hyperbolic relationship which is described by following equation.

$$S = \frac{D}{\frac{1}{k} + \frac{D}{c}} \tag{6}$$

where k=initial slope of the curve and c=intercept of the horizontal asymptote with the S axis.

(4) Determination of the equation relating to S and D; The Eq.(6) can be determined by using the fact that the curve passes through (1,1) and approximating the testing results. Table 4 shows a hyperbolic parameters of Eq.(6) together with a value provided by Esterbuizen *et al.* (2001) to compare parameters.

The larger values of k are observed for interfaces of S-GM, comparing the values for T-GM, which means that interface shear strength decrease significantly at the early stage after peak strength is mobilized. However, the magnitude of strength degradation are significant for interfaces between T-GM/geosynthetic (Seo *et al.* 2002, Triplett and Fox, 2001).

4. Comparison of results from developed modeling and testing data

The developed stress-displacement relationship may be used very conveniently for incremental analyses of nonlinear behavior. Though the load is generally divided into some increments to estimate the deformations induced by load, the displacements, on the contrary, induce interface shear stress in case of displacement controlled interface shear testing. Therefore, the displacement to be analyzed is divided into a number of increments enough to describe the behavior in detail. For the purposes of analysis, the interface is assumed to behave linearly under each increment of displacement. These steps are repeated until the calculated shear strength reaches the peak shear strength. Once the interface shear strength exceeds the peak strength, the interface shear behavior is tansferred into the post-peak region. Then, the relationship between S and D is applied until the shear behaviors reach large displacement, d_l.

4.1 Interfaces between smooth GM and GT

Fig. 7 and Fig. 8 present the measured and predicted curves using the determined parameters (Table 1, Table 3, and Table 4) at interfaces of S-GM involved. As the smaller incremental displacement at pre-peak region can made a exact prediction, the value of 0.01 mm is selected for incremental displacement, when the limit of increment is 0.01.

Though the overall agreement may be seen to be quite good, it may be also observed that as a result of these approximations there is some difference between the measured and predicted stress-displacement curves in Fig. 7 and Fig. 8. Especially, some discrepancy is found in Fig. 7. The comparison results of nonlinear failure envelope (Fig. 7) demonstrate good agreement between measured and predicted data, which corresponds to the fact that the correlation coefficient of nonlinear failure envelope is bigger than that of linear failure envelope (Table 1). However, good agreement is observed, in general, in Fig. 8 irrespective of linear or nonlinear envelope.

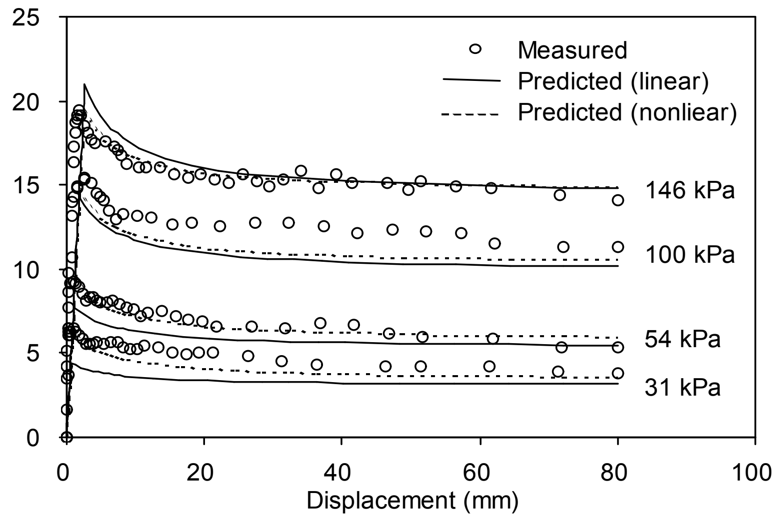


Fig. 7. Comparison of measured data (Seo *et al*, 2002) and predicted data for S-GM/GT interfaces

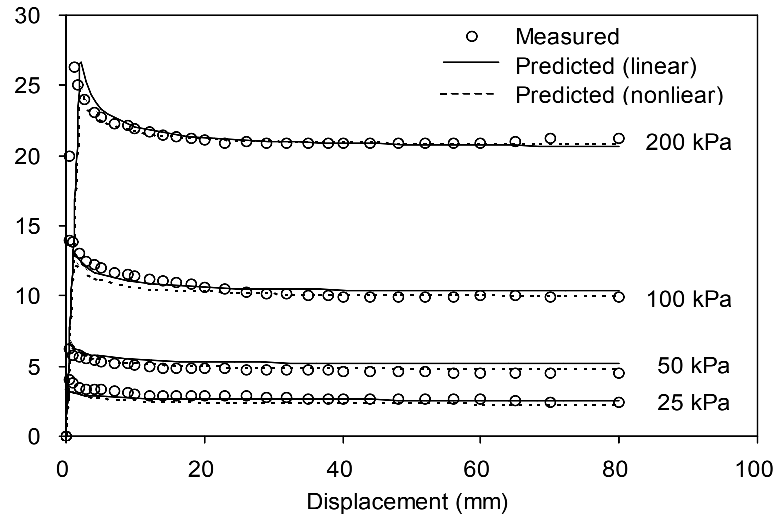


Fig. 8. Comparison of measured data (Jones and Dixon, 1998) and predicted data for S-GM/GT interfaces

It can be concluded from the comparisons that it is much significant to characterize accurately the relation between the normal stress and shear strength for exact prediction. It is very essential, of course, to perform the laboratory tests accurately to evaluate the parameters.

4.2 Interfaces between textured GM and GT

Back-calculation is performed using the same method as is done for the case of S-GM/GT interfaces. Fig. 9 and Fig. 10 display the measured and predicted curves using the determined parameters (Table 2, Table 3, and Table 4). The value of 0.1 mm is selected for incremental displacement because the lower increment causes increasing displacement errors between predicted and measured value, where displacement error means the difference between measured and predicted displacement at peak strength, d_p. Although lower increment predicts peak interface shear strength more accurately, the increasing

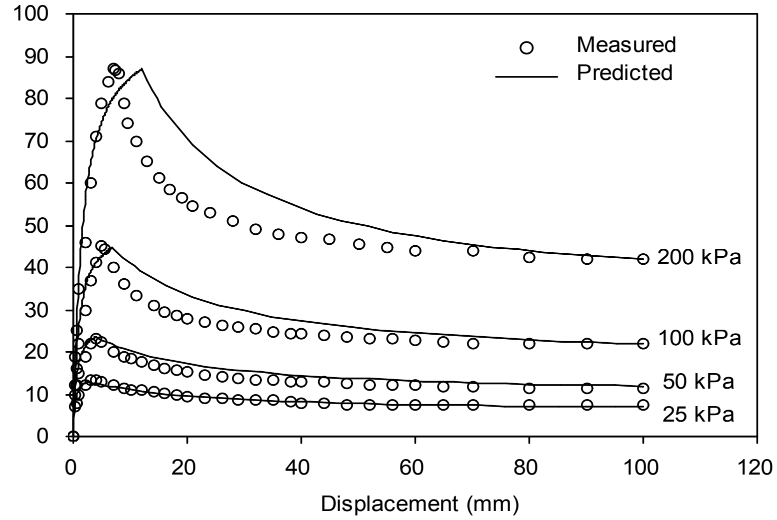


Fig. 9. Comparison of measured data (Jones and Dixon, 1998) and predicted data for T-GM/GT interfaces

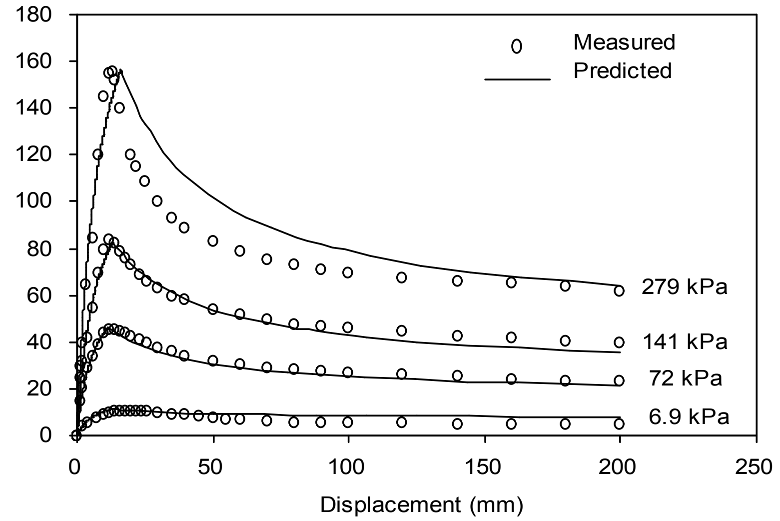


Fig. 10. Comparison of measured data (Triplett *et al.*, 2002) and predicted data for T-GM/GCL interfaces

displacement error gives more influence on the total accuracy.

The predicted shear stresses are compared with measured data in Fig. 9 and 10, which illustrate good agreement between two kinds of results especially on the range of low normal stress. Good consistency for this stress range points out the possibility that this model can be a good constitutive model in the case such as cover soil for landfills, where normal stress is very low. This good consistency is partially attributed to highly correlation between the normal stress and shear strength of testing data.

However, some differences could be seen on high normal stress (≥200kPa), which is caused by some reasons. First, when the value of n which is normal stress dependent parameter is very small, the increasing magnitude of shear strength becomes to be limited on the high normal stress. Second, if the initial slope, k, at post-peak region is greater than average value of k, the interface shear strength decreases dramatically at initial stage of post-peak region. Finally, the increment of displacement has also effect on the accuracy of back-calculation. Therefore, new methods are required to be developed to solve the discrepancy of interface shear behavior between T-GM/geosynthetic on high normal stress.

5. Summary and Conclusions

The simple and efficient model for the interface shear behavior between geosynthetics is proposed. The model proposed in this paper has the advantage that shear strength degradation or strain softening effect along geosynthetic interfaces can be taken into account. The model is calibrated by using four published testing data.

Four relationships are evaluated to develope this strain softening model: (1) the peak strength envelop, (2) the large displacement strength envelop, (3) the hyperbolic or linear equation for pre-peak region, and (4) the hyperbolic strength reduction (strain softening factor) versus displacement ratio relationship.

The back-calculation results based on modeling are generally in good agreement with the experimental results for geomembrane and geosynthetic interfaces. However, it is found that the accuracy of back-calculation is significantly influenced by the relationship between normal stress and peak or large displacement interface shear strength. Good agreement between predicted and measured data is observed especially for low normal stress. This model is expected to be a good constitutive model in a finite-element analysis, which should include strain-softening behavior or slope failure along landfill liner interfaces.

In addition, the research on the application of DSC (Disturbed State Concept) model is being conducted and works of comparison between the results from proposed models and DSC models will be followed.

Notation

The following symbols are used in this paper:

c = cohesion
D = displacement ration
d = displacement (mm)
d_l = displacement at large displacement shear strength (τ_l)
d_p = displacement at peak shear strength (τ_p)
d_{pp} = displacement at post-peak region
d_s = strain-softening displacement ($= d_{pp} - d_p$)
d_s^m = maximum strain-softening displacement ($= d_l - d_p$)
E_i = initial shear modulus (=1/a)
E_t = tangent shear modulus
K = dimensionless shear coefficient
k = initial slope of hyperbolic relationship that relates strain-softening factor, S, to displacement ration, D
r^2 = correlation coefficient
S = strain-softening factor
δ = interface friction angle
σ_n = normal stress
τ = shear strength
τ_l = large displacement shear strength
τ_p = peak shear strength
τ_p' = new peak shear strength determined on back-calculation
τ_{pp} = shear strength at post-peak region
τ_s = shear strength reduction ($= \tau_p - \tau_{pp}$)
τ_{ult} = ultimate shear strength (= 1/b)

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