Journal of Korean
Data & Information Science Society
2003, Vol. 14, No.4 pp. 1055~1065

Notes on the Comparative Study of the Reliability Estimation for Standby System with Exponential Lifetime Distribution

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Abstract

We shall propose maximum likelihood, Bayesian and generalized maximum likelihood estimation for the reliability of the two-unit hot standby system with exponential lifetime distribution that switch is perfect. Each estimation will be compared numerically in terms of various mission times, parameter values and asymptotic relative efficiency through Monte Carlo simulation.

Key Words and Phrases: Reliability, Standby system, Maximum likelihood, Bayesian estimation, Generalized maximum likelihood, Asymptotic relative efficiency, Monte Carlo simulation.

1. Introduction

The two-unit standby redundant system configuration is a form of paralleling where only one component is in operation; if the operating component fails, the another component is brought into operation, and the redundant configuration continues to function. Depending failure characteristic, standby redundancy is classified into three types. Hot standby system, where each component has the same failure rate regardless of whether it is standby or in operation; Cold standby system, where components do not fail when they are in standby; Warm standby system, where a standby component can fail but it has a smaller failure rate than the principal component.

Reliability computations for a two-unit standby redundant systems with

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constant

failure rate are found by Osaki and Nakagawa(1971). Fujii and Sandoh(1984) considered the Bayesian estimation for reliability of a two-unit hot standby redundant system. Kaput and Garg(1990) considered the technique of Markov renewal process to obtain various reliability measures for a two-unit standby system with perfect switch and Shen and Xie(1991) considered the effect of standby redundancy at the system and the component level.

The classical statistical estimation procedure, for example maximum likelihood estimation, have been applied to many situations. But recently there are many cases in which the Bayesian methods and generalized maximum likelihood estimation are frequently used. The main contribution of this paper is to propose some Bayesian estimators and generalized maximum likelihood estimators and to compare them with maximum likelihood estimator in the sense of asymptotic relative efficiency(ARE) for the reliability of standby system.

In this paper, we shall find maximum likelihood estimator(MLE), generalized maximum likelihood estimator(GMLE) and Bayesian estimator for reliability of a two-unit hot standby system with perfect switch. Also we compare these estimators by ARE of GMLE and Bayesian estimator for MLE through generating random number of the proposed estimators and numerical integration.

2. Reliability for Standby System

We consider an exponential distribution of lifetime governed by the probability density function

$$f(t \mid \lambda) = \begin{cases} \lambda e^{-\lambda t}, 0 < t < \infty \\ 0, otherwise \end{cases} \tag{2.1}$$

Many authors have utilized the exponential distribution because of its wide applicability in statistical inferences and reliability engineering(Saunders & Mann (1985) and Bain & Engelhart (1987)).

Here we shall consider the Bayesian approach of the estimation for the two-unit hot standby system reliability with perfect switch in an exponential distribution. In the two-unit hot standby system with perfect switch, we shall assume the following;

1. The system consists of two independent and identically distributed units and a switch.
2. One unit serves as a hot standby when the other is in use.
3. The switch is instantaneous when the one in use fails.
4. The times to failure of both units in use and standby are independent and exponentially distributed with the failure rate λ.
5. The unit and the switch are independent.

6. The switch is failure free.

Then the reliability for a two-unit hot standby system at specified mission time t_o is given by

$$R(t_0)=e^{-\lambda t_0}(1+\lambda t_o),\ t_o>0. \tag{2.2}$$

3. The Method of Reliability Estimation

3.1 The Method of Maximum Likelihood Estimation

Let $T_1,\cdots,T_n$ be a simple random sample from an exponential distribution with failure rate λ and $G=\Sigma_{k=1}^{n} T_k$ be the total test time under the given mission time t_0. If the total test time G is accumulated on all items including those that failed and those that did not fail prior to test termination. Then the total test time G is as follows;

$$G=\Sigma_{i=1}^{R} T_i+(n-R)t_0, \tag{3.1}$$

where R is the number of failures.

In this case maximum likelihood estimator(MLE) for the failure rate λ as follows;

$$\hat{\lambda}=\frac{R}{G}. \tag{3.2}$$

By the invariance property of MLE, the MLE of standby system is as follows;

$$\widehat{R_M}(t_0)=e^{-\hat{\lambda} t_0}(1+\hat{\lambda} t_0),\ t_0>0. \tag{3.3}$$

3.2 The Method of Bayesian Estimation

Now we shall consider Bayesian estimation of reliability (2.2) under the squared error loss. Let the random variable of failure rate λ be Λ with prior probability density function(p.d.f.) $\pi(\lambda)$. Then the Bayesian estimator $\widetilde{R}(t_0)$ of $R(t_0)$ is posterior mean because the loss function is squared error loss.

First we assume that Λ has an uniform distribution U(0,β) with p.d.f.

$$\pi_U(\lambda \mid \beta)=\begin{cases}\frac{1}{\beta},\ 0<\lambda<\beta,\\ 0,\ \text{otherwise}.\end{cases} \tag{3.4}$$

Then the posterior p.d.f. of Λ given the total test time G is

$$g_U(\lambda \mid G, \beta) = \frac{G^{n+1}\lambda^n e^{-\lambda G}}{\Gamma(n+1, \beta G)}, 0 < \lambda < \beta, \tag{3.5}$$

where $\Gamma(a, z)$ represents the standard incomplete gamma function.

Hence the Bayesian estimator $\widetilde{R}_U(t_0)$ for the system reliability $R(t_0)$ is

$$\widetilde{R}_U(t_0) = \frac{1}{\Gamma(n+1, \beta G)} \left(\frac{G}{G+t_0}\right)^{n+1} \Gamma(n+1, \beta(G+t_0)) + \frac{t_0}{(G+t_0)} \cdot \Gamma(n+2, \beta(G+t_0)) \ . \tag{3.6}$$

Second we assume that Λ has a gamma distribution GAM(α, β) with p.d.f.

$$\pi_G(\lambda \mid \alpha, \beta) = \frac{1}{\Gamma(\alpha)\beta^\alpha} \lambda^{\alpha-1} e^{-\lambda/\beta}, 0 < \lambda < \infty. \tag{3.7}$$

Then the posterior p.d.f. of Λ given the total test time G is

$$g_G(\lambda \mid G, \alpha, \beta) = \frac{1}{\Gamma(\alpha+n)\left(\frac{\beta}{\beta G+1}\right)^{\alpha+n}} \lambda^{\alpha+n-1} e^{-\lambda\left(\frac{\beta G+1}{\beta}\right)}, \quad 0 < \lambda < \infty. \tag{3.8}$$

Hence the Bayesian estimator $\widetilde{R}_G(t_0)$ for the system reliability $R(t_0)$ is

$$\widetilde{R}_G(t_0) = \left(\frac{\beta G+1}{\beta G+\beta t_0+1}\right)^{\alpha+n} \left\{1 + \frac{\beta t_0(\alpha+n)}{\beta G+\beta t_0+1}\right\} . \tag{3.9}$$

Third we assume that Λ has an inverted gamma distribution IGAM(α, β) with p.d.f.

$$\pi_{IG}(\lambda \mid \alpha, \beta) = \frac{\beta^\alpha}{\Gamma(\alpha)} \left(\frac{1}{\lambda}\right)^{\alpha+1} e^{-\beta/\lambda}, \alpha, \beta, \lambda > 0. \tag{3.10}$$

Then the posterior distribution of Λ given G is

$$g_{IG}(\lambda \mid t) = \frac{\lambda^{n-\alpha-1} e^{-(\lambda G + \beta/\lambda)}}{2(\beta/G)^{(n-\alpha)/2} K(n-\alpha, 2\sqrt{(\beta G)})}, \tag{3.11}$$

where $K(n, x)$ is a modified Bessel function of the second kind of order n.

Hence the Bayesian estimator $\widetilde{R}_{IG}(t_0)$ for the system reliability $R(t_0)$ is

$$\widetilde{R}_{IG}(t_0) = \delta_n (I_1 + t_0 I_2) \ , \tag{3.12}$$

where

$$\delta_n = \frac{1}{2} \frac{(G/\beta)^{(n-\alpha)/2}}{K(n-\alpha, 2\sqrt{\beta G})}, \quad I_1 = 2(\beta/(t_0+G))^{(n-\alpha)/2} \times K(n-\alpha, 2\sqrt{\beta(t_0+G)}$$

and $I_2 = 2(\beta/(t_0+G))^{(n-\alpha+1)/2} \times K(n-a+1, 2\sqrt{\beta(t_0+G)}$.

Fourth we assume that Λ has a truncated gamma distribution TGAM(α, β, λ_T) with p.d.f.

$$\pi_{TG}(\lambda; \alpha, \beta, \lambda_T) = \frac{1}{\Gamma(\alpha, \lambda_T/\beta)\beta^{\alpha}} \lambda^{\alpha-1} e^{-\lambda/\beta}, 0 < \lambda < \lambda_T. \tag{3.13}$$

Then the posterior distribution of Λ given G is

$$g_{TG}(\lambda \mid t) = \frac{\lambda^{\alpha+n-1} e^{-\lambda(\beta G+1)/\beta}}{\Gamma(\alpha+n, \lambda_T(G+1/\beta))\{\beta/(\beta G+1)\}^{\alpha+n}}, 0 < \lambda < \lambda_T. \tag{3.14}$$

Hence the Bayesian estimator $\widetilde{R_{TG}}(t_0)$ for the system reliability is

$$\widetilde{R_{TG}}(t_0) = \left(\frac{\beta G+1}{\beta G+\beta t_0+1}\right)^{\alpha+n} \frac{1}{\Gamma(n+\alpha, \lambda_T(G+1/\beta))} \{\Gamma(n+\alpha, \lambda_T(G+t_0+1/\beta)) + \frac{\beta t_0}{(\beta G+\beta t_0+1)} \Gamma(\alpha+n+1, \lambda_T(G+t_0+1/\beta))\}. \tag{3.15}$$

3.3 The Method of Generalized Maximum Likelihood Estimation

Now we shall consider GMLE of reliability (2.2). Let the random variable of failure rate λ be Λ with prior p.d.f. $\pi(\lambda)$. Then GMLE $\widehat{R_{GMLE}}(t_0)$ of $R(t_0)$ is MLE that is replaced λ by $\hat{\lambda}$, which maximizes the posterior distribution $g(\lambda)$ in reliability (2.2).

First we assume that Λ has an uniform distribution U(0,β).

Then MLE of Λ is

$$\widehat{\lambda_U} = \frac{n}{G}. \tag{3.16}$$

Hence GMLE $\widehat{R_U}(t_0)$ for the system reliability $R(t_0)$ is

$$.\widehat{R_U}(t_0) = e^{-\hat{\lambda}_U t_0}(1+\hat{\lambda}_U t_0) \tag{3.17}$$

Second we assume that Λ has a gamma distribution GAM(α, β).

Then MLE of Λ is

$$\widehat{\lambda_G} = \frac{\beta(\alpha+n-1)}{\beta G+1}. \tag{3.18}$$

Hence GMLE $\widehat{R_G}(t_0)$ for the system reliability $R(t_0)$ is

$$.\widehat{R_U}(t_0) = e^{-\hat{\lambda}_U t_0}(1+\hat{\lambda}_U t_0) \tag{3.19}$$

Third we assume that Λ has an inverted gamma distribution IGAM(α, β).

Then MLE of Λ is

$$\widehat{\lambda_{IG}} = ((n-\alpha-1) + \sqrt{(n-\alpha-1)^2 + 4\beta G})/2G. \tag{3.20}$$

Hence GMLE $\widehat{R_{IG}}(t_0)$ for the system reliability $R(t_0)$ is

$$. \widehat{R_{IG}}(t_0) = e^{-\hat{\lambda}_{IG} t_0}(1 + \hat{\lambda}_{IG} t_0) \tag{3.21}$$

Fourth we assume that Λ has a truncated gamma distribution TGAM(α, β, λ_T).
Then MLE of Λ is the same as the $\widehat{\lambda_{IG}}$ in (3.20).
Hence GMLE $\widehat{R_{TG}}(t_0)$ of the system reliability $R(t_0)$ is the same as the GMLE .$\widehat{R_G}(t_0)$

4. Conclusion

Tables 1.1 through 4.3 show the simulated values for the asymptotic relative efficiency(ARE) of the proposed reliability estimators for MLE under the two-unit hot standby system with perfect switch when $\lambda = (1\times10^{-1}, 3\times10^{-1}, 5\times10^{-1})$, sample size n=30, various values of specified mission time t_0 and simulations were replicated 500 times. We can know from the Table 1, the Bayesian estimator with respect to a uniform prior distribution on failure rate λ is more efficient than the generalized maximum likelihood estimator(GMLE) when the mission time t_0 and the parameter β of the uniform prior distribution decrease together. We can know from the Table 2 and Table 3, the GMLE with respect to a gamma prior distribution is more efficient than the Bayesian estimator under the given mission times and parameters, specially on the Table 3 the two estimators are more efficient as the mission time is increasing. We can know from the Table 4, the Bayesian estimator with respect to the truncated gamma prior distribution is more efficient than GMLE as the mission time and the parameter β of the truncated gamma prior distribution decrease together. Also the another Bayesian method such that the noninformative or nonparametric approach are remained for future works.

[Table 1.1] The simulated ARE's of GMLE and Bayesian estimator for MLE on system reliability under the UNIF(0,β) prior on λ($\lambda = 1\times 10^{-1}$).

t_0	$R(t_0)$	UNIF(0, 0.5)		UNIF(0, 1.0)		UNIF(0, 2.0)	
		ARE ($\widehat{R}_U, \widehat{R}_M$)	ARE ($\widetilde{R}_U, \widehat{R}_M$)	ARE ($\widehat{R}_U, \widehat{R}_M$)	ARE ($\widetilde{R}_U, \widehat{R}_M$)	ARE ($\widehat{R}_U, \widehat{R}_M$)	ARE ($\widetilde{R}_U, \widehat{R}_M$)
1.0	0.9953	12.8042	163.6325	12.6841	20.9659	12.6745	11.6334
1.5	0.9898	12.3001	47.1137	12.0454	11.6385	12.2169	11.2159
2.0	0.9825	11.9461	22.4000	11.8604	10.8987	11.8519	10.8822
2.5	0.9735	11.5682	14.4605	11.5277	10.5850	11.5994	10.6483
3.0	0.9631	11.3063	11.5538	11.1827	10.2676	11.0307	10.1337
3.5	0.9513	10.9171	10.3660	11.3977	10.4449	11.1005	10.1836

[Table 1.2] The simulated ARE's of GMLE and Bayesian estimator for MLE on system reliability under the UNIF(0,β) prior on λ($\lambda = 3\times 10^{-1}$).

t_0	$R(t_0)$	UNIF(0, 0.5)		UNIF(0, 1.0)		UNIF(0, 2.0)	
		ARE ($\widehat{R}_U, \widehat{R}_M$)	ARE ($\widetilde{R}_U, \widehat{R}_M$)	ARE ($\widehat{R}_U, \widehat{R}_M$)	ARE ($\widetilde{R}_U, \widehat{R}_M$)	ARE ($\widehat{R}_U, \widehat{R}_M$)	ARE ($\widetilde{R}_U, \widehat{R}_M$)
1.0	0.9631	11.1261	438.9539	11.1515	25.2502	11.1201	10.2398
1.5	0.9246	10.7008	128.5394	10.6852	12.1493	10.8393	9.9302
2.0	0.8781	10.9639	60.6149	10.7881	10.2268	10.8972	9.9484
2.5	0.8266	11.0814	37.2803	11.1732	10.2258	10.8424	9.8775
3.0	0.7725	11.1644	26.9329	10.9246	9.9633	11.5899	10.5050
3.5	0.7174	11.7995	22.2534	11.9290	10.7972	11.8050	10.6855

[Table 1.3] The simulated ARE's of GMLE and Bayesian estimator for MLE on system reliability under the UNIF(0,β) prior on λ($\lambda = 5\times 10^{-1}$).

t_0	$R(t_0)$	UNIF(0, 0.5)		UNIF(0, 1.0)		UNIF(0, 2.0)	
		ARE ($\widehat{R}_U, \widehat{R}_M$)	ARE ($\widetilde{R}_U, \widehat{R}_M$)	ARE ($\widehat{R}_U, \widehat{R}_M$)	ARE ($\widetilde{R}_U, \widehat{R}_M$)	ARE ($\widehat{R}_U, \widehat{R}_M$)	ARE ($\widetilde{R}_U, \widehat{R}_M$)
1.0	0.9098	10.781	16969.723	11.0377	40.7465	10.9221	10.1901
1.5	0.8266	10.7966	3136.2412	11.0191	18.4381	11.0162	10.0305
2.0	0.7358	12.0480	898.5046	11.6423	13.8400	11.4322	10.3726
2.5	0.6446	12.5541	346.8146	12.6742	13.2360	12.8235	11.5853
3.0	0.5578	14.6422	151.4469	13.1932	13.1993	13.4849	12.2892
3.5	0.4779	13.5962	79.8968	13.9724	13.8358	14.9411	13.8403

[Table 2.1] The simulated ARE's of GMLE and Bayesian estimator for MLE on system reliability under GAM(α,β) prior on λ($\lambda = 1\times10^{-1}$).

t_0	$R(t_0)$	GAM(1,2)		GAM(1,3)		GAM(1,5)	
		ARE $(\widehat{R}_G, \widehat{R}_M)$	ARE $(\widetilde{R}_G, \widehat{R}_M)$	ARE $(\widehat{R}_G, \widehat{R}_M)$	ARE $(\widetilde{R}_G, \widehat{R}_M)$	ARE $(\widehat{R}_G, \widehat{R}_M)$	ARE $(\widetilde{R}_G, \widehat{R}_M)$
1.0	0.9953	13.8707	12.6931	13.0264	11.9436	12.8515	11.7880
1.5	0.9898	12.8795	11.8045	12.4828	11.4518	12.2715	11.2643
2.0	0.9825	12.6044	11.5491	12.0886	11.0920	11.9712	10.9881
2.5	0.9735	11.8091	10.8339	11.6439	10.6874	11.6094	10.6566
3.0	0.9631	11.9208	10.9186	11.4655	10.5180	11.3558	10.4205
3.5	0.9513	11.6242	10.6454	11.1720	10.2469	11.2170	10.2874

[Table 2.2] The simulated ARE's of GMLE and Bayesian estimator for MLE on system reliability under GAM(α,β) prior on λ($\lambda = 3\times10^{-1}$).

t_0	$R(t_0)$	GAM(1,2)		GAM(1,3)		GAM(1,5)	
		ARE $(\widehat{R}_G, \widehat{R}_M)$	ARE $(\widetilde{R}_G, \widehat{R}_M)$	ARE $(\widehat{R}_G, \widehat{R}_M)$	ARE $(\widetilde{R}_G, \widehat{R}_M)$	ARE $(\widehat{R}_G, \widehat{R}_M)$	ARE $(\widetilde{R}_G, \widehat{R}_M)$
1.0	0.9631	11.9651	10.9586	11.6150	10.6504	11.7091	10.7321
1.5	0.9246	11.4980	10.5059	11.1171	10.1752	11.1198	10.1761
2.0	0.8781	11.0518	10.0828	10.8705	9.9264	11.1079	10.1304
2.5	0.8266	11.3222	10.2957	10.8650	9.8952	10.9386	9.9581
3.0	0.7725	12.0400	10.8836	11.8762	10.7400	11.3315	10.2771
3.5	0.7174	11.9872	10.8444	12.1720	10.9886	11.9367	10.7981

[Table 2.3] The simulated ARE's of GMLE and Bayesian estimator for MLE on system reliability under GAM(α,β) prior on λ($\lambda = 5\times10^{-1}$).

t_0	$R(t_0)$	GAM(1,2)		GAM(1,3)		GAM(1,5)	
		ARE $(\widehat{R}_G, \widehat{R}_M)$	ARE $(\widetilde{R}_G, \widehat{R}_M)$	ARE $(\widehat{R}_G, \widehat{R}_M)$	ARE $(\widetilde{R}_G, \widehat{R}_M)$	ARE $(\widehat{R}_G, \widehat{R}_M)$	ARE $(\widetilde{R}_G, \widehat{R}_M)$
1.0	0.9098	12.1549	11.0647	11.3819	10.3934	11.3249	10.3428
1.5	0.8266	11.4697	10.4199	11.5413	10.4728	11.4560	10.3997
2.0	0.7358	12.1535	10.9738	11.5035	10.4341	11.5935	10.5025
2.5	0.6446	13.6399	12.2460	13.4921	12.1574	13.1787	11.8777
3.0	0.5578	14.2973	13.0125	13.1965	12.0259	13.8846	12.6303
3.5	0.4779	16.1036	14.8327	14.1602	13.1457	14.0470	13.0675

[Table 3.1] The simulated ARE's of GMLE and Bayesian estimator for MLE on system reliability under IGAM(α,β) prior on λ(λ= 1×10^{-1}).

t_0	$R(t_0)$	IGAM(1, 0.01)		IGAM(1, 0.03)		IGAM(1, 0.05)	
		ARE ($\widehat{R}_{IG}$, $\widehat{R}_M$)	ARE ($\widetilde{R}_{IG}$, $\widehat{R}_M$)	ARE ($\widehat{R}_{IG}$, $\widehat{R}_M$)	ARE ($\widetilde{R}_{IG}$, $\widehat{R}_M$)	ARE ($\widehat{R}_{IG}$, $\widehat{R}_M$)	ARE ($\widetilde{R}_{IG}$, $\widehat{R}_M$)
5	0.9953	15.5141	14.0333	15.4395	13.9135	15.4126	13.8357
10	0.9898	14.8651	13.4397	14.8943	13.3873	15.1077	13.4931
15	0.9825	14.4264	13.0287	14.5712	13.0532	14.5900	12.9696
20	0.9735	13.9430	12.5781	14.1603	12.6458	14.3996	12.7272
25	0.9631	13.7974	12.4201	13.7965	12.2804	13.6563	12.0256
30	0.9513	13.5348	12.1643	14.0252	12.4156	14.0230	12.2473

[Table 3.2] The simulated ARE's of GMLE and Bayesian estimator for MLE on system reliability under IGAM(α,β) prior on λ(λ= 3×10^{-1}).

t_0	$R(t_0)$	IGAM(1, 0.01)		IGAM(1, 0.03)		IGAM(1, 0.05)	
		ARE ($\widehat{R}_{IG}$, $\widehat{R}_M$)	ARE ($\widetilde{R}_{IG}$, $\widehat{R}_M$)	ARE ($\widehat{R}_{IG}$, $\widehat{R}_M$)	ARE ($\widetilde{R}_{IG}$, $\widehat{R}_M$)	ARE ($\widehat{R}_{IG}$, $\widehat{R}_M$)	ARE ($\widetilde{R}_{IG}$, $\widehat{R}_M$)
5	0.9631	13.5331	12.2406	13.8093	12.4308	13.8860	12.4506
10	0.9246	13.5141	12.1514	13.4609	12.0341	13.6270	12.1033
15	0.8781	14.2792	12.7202	14.0808	12.4534	13.9803	12.2667
20	0.8266	14.9385	13.1878	14.2750	12.5229	14.5883	12.6367
25	0.7725	16.5442	14.4395	16.1146	13.8977	15.7206	13.4121
30	0.7174	17.5534	15.2059	15.6846	13.5301	15.5565	13.2001

[Table 3.3] The simulated ARE's of GMLE and Bayesian estimator for MLE on system reliability under IGAM(α,β) prior on λ(λ= 5×10^{-1}).

t_0	$R(t_0)$	IGAM(1, 0.01)		IGAM(1, 0.03)		IGAM(1, 0.05)	
		ARE ($\widehat{R}_{IG}$, $\widehat{R}_M$)	ARE ($\widetilde{R}_{IG}$, $\widehat{R}_M$)	ARE ($\widehat{R}_{IG}$, $\widehat{R}_M$)	ARE ($\widetilde{R}_{IG}$, $\widehat{R}_M$)	ARE ($\widehat{R}_{IG}$, $\widehat{R}_M$)	ARE ($\widetilde{R}_{IG}$, $\widehat{R}_M$)
5	0.9098	13.9238	12.0342	14.1789	11.5610	15.7399	11.5229
10	0.8266	15.5795	12.9501	15.9697	12.0769	17.6296	11.2730
15	0.7358	17.6402	14.0794	18.5519	12.8354	21.0442	11.5193
20	0.6446	19.7714	15.1906	23.0949	14.5351	29.6457	12.9977
25	0.5578	26.2039	19.2430	27.8214	16.2552	31.5660	13.0699
30	0.4779	27.4519	20.6480	31.5035	18.8438	36.5660	14.0000

[Table 4.1] The simulated ARE's of GMLE and Bayesian estimator for MLE on system reliability under TGAM(α,β, λ_T) prior on λ(λ= 1×10^{-1}).

t_0	$R(t_0)$	TGAM(1,2,0.1)		TGAM(1,3,0.1)		TGAM(1,5,0.1)	
		ARE ($\widehat{R_{TG}}$, $\widehat{R_M}$)	ARE ($\widetilde{R_{TG}}$, $\widehat{R_M}$)	ARE ($\widehat{R_{TG}}$, $\widehat{R_M}$)	ARE ($\widetilde{R_{TG}}$, $\widehat{R_M}$)	ARE ($\widehat{R_{TG}}$, $\widehat{R_M}$)	ARE ($\widetilde{R_{TG}}$, $\widehat{R_M}$)
1.0	0.9953	13.8707	163.9244	13.0264	163.8562	12.8515	163.7567
1.5	0.9898	12.8795	47.2668	12.4828	47.2085	12.2715	47.1570
2.0	0.9825	12.6044	22.4715	12.0886	22.3974	11.9712	22.3870
2.5	0.9735	11.8091	14.5150	11.6439	14.4117	11.6094	14.3802
3.0	0.9631	11.9208	11.6841	11.4655	11.6124	11.3558	11.5967
3.5	0.9513	11.6242	10.5584	11.1720	10.7185	11.2170	10.4440

[Table 4.2] The simulated ARE's of GMLE and Bayesian estimator for MLE on system reliability under TGAM(α,β, λ_T) prior on λ(λ= 3×10^{-1}).

t_0	$R(t_0)$	TGAM(1,2,0.3)		TGAM(1,3,0.3)		TGAM(1,5,0.3)	
		ARE ($\widehat{R_{TG}}$, $\widehat{R_M}$)	ARE ($\widetilde{R_{TG}}$, $\widehat{R_M}$)	ARE ($\widehat{R_{TG}}$, $\widehat{R_M}$)	ARE ($\widetilde{R_{TG}}$, $\widehat{R_M}$)	ARE ($\widehat{R_{TG}}$, $\widehat{R_M}$)	ARE ($\widetilde{R_{TG}}$, $\widehat{R_M}$)
1.0	0.9631	11.9651	86.2951	11.6150	86.1459	11.7091	86.0728
1.5	0.9246	11.4980	29.6355	11.1171	29.6469	11.1198	29.4130
2.0	0.8781	11.0518	17.0743	10.8705	16.8028	11.1079	16.7346
2.5	0.8266	11.3222	13.1363	10.8650	13.1220	10.9386	13.1276
3.0	0.7725	12.0400	12.2587	11.8762	11.9144	11.3315	12.1103
3.5	0.7174	11.9872	11.9481	12.1720	11.7310	11.9367	11.7940

[Table 4.3] The simulated ARE's of GMLE and Bayesian estimator for MLE on system reliability under TGAM(α,β, λ_T) prior on λ(λ= 5×10^{-1}).

t_0	$R(t_0)$	TGAM(1,2,0.5)		TGAM(1,3,0.5)		TGAM(1,5,0.5)	
		ARE ($\widehat{R_{TG}}$, $\widehat{R_M}$)	ARE ($\widetilde{R_{TG}}$, $\widehat{R_M}$)	ARE ($\widehat{R_{TG}}$, $\widehat{R_M}$)	ARE ($\widetilde{R_{TG}}$, $\widehat{R_M}$)	ARE ($\widehat{R_{TG}}$, $\widehat{R_M}$)	ARE ($\widetilde{R_{TG}}$, $\widehat{R_M}$)
1.0	0.9098	12.1549	41.0054	11.3819	40.9384	11.3249	40.8616
1.5	0.8266	11.4697	18.6745	11.5413	18.7557	11.4560	18.6480
2.0	0.7358	12.1535	14.2388	11.5035	13.7693	11.5935	14.0364
2.5	0.6446	13.6399	13.9028	13.4921	13.0654	13.1787	12.5973
3.0	0.5578	14.2973	14.1198	13.1965	14.5193	13.8846	13.7300
3.5	0.4779	16.1036	14.5966	14.1602	14.2070	14.0470	13.7731

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[received date : Sep. 2003, accepted date : Nov. 2003]