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Maximizing Mean Time to the Catastrophic Failure through Burn-In

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Absract

In this paper, the problem of determining optimal burn-in time is considered under a general failure model. There are two types of failure in the general failure model. One is Type I failure (minor failure) which can be removed by a minimal repair and the other is Type II failure (catastrophic failure) which can be removed only by a complete repair. In this model, when the unit fails at its age t, Type I failure occurs with probability $1-p(t)$ and Type II failure occurs with probability $p(t)$, $0 \le p(t) \le 1$. Under the model, the properties of optimal burn-in time maximizing mean time to the catastrophic failure during field operation are obtained. The obtained results are also applied to some illustrative examples.

Keywords : Bathtub shaped failure rate function, change points, general failure model, optimal burn-in

1. Introduction

Burn-in is an engineering method used to eliminate the initial failures in field use. To burn-in a component or system means to subject it to a period of use prior to the time when it is to actually be used, and then only those which survive the burn-in process will be used. Due to the high failure rate in the early stages of component life, burn-in procedure has been widely accepted as a method of screening out failures before systems are actually used in field operations. A general background of this important area of reliability can be found in Kuo and Kuo (1983) and Jensen and Petersen (1982).

Since burn-in is usually costly, to determine the length of the burn-in procedure

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is an important problem. The best time to stop the burn-in process for a given criterion is called the optimal burn-in time. In the literature, certain cost structures have been proposed, and the corresponding problem of finding the optimal burn-in time has been considered. See, for example, Mi (1991), (1994a), (1996), (1997) and Cha (2000). More recently, Cha (2001) considered a burn-in and replacement model which generalizes those of Mi (1994) and Cha (2000). Some other performance-based criterion, for example, the mean residual life criteria, the reliability of a given mission time, or the mean number of failures, have been also considered to determine the optimal burn-in time(See also Mi (1994b), Block et al. (2002)). An excellent survey of research in burn-in can be found in Block and Savits (1997).

In this paper, it is assumed that two types of system failures may occur : one is Type I failure(minor failure) which can be removed by a minimal repair and the other is Type II failure(catastrophic failure) which can be removed only by a complete repair(or a replacement). This model is generally called the general failure model. In the model, when the unit fails, Type I failure occurs with probability $1-p(t)$ and Type II failure occurs with probability $p(t)$, $0 \le p(t) \le 1$, where t is the age at failure of the system. Under the general failure model, we consider the problem of determining the optimal burn-in time maximizing the mean time to the Type II failure in field operation. Some upper and lower bounds for the optimal burn-in time will be given. The obtained results are applied to some illustrative examples.

2. Main Results

Consider the general failure model described in the preceding section. A new system is burned-in for a time b and if the system survives the burn-in, then it is put into field operation. In field operation a system is usually used to accomplish a task or mission. We assume that if a Type I Failure(Minor Failure) occurs during mission time then it can be removed instantly in the field operation by a minimal repair without affecting its mission and thus the repaired device can continue its mission. But, on the other hand, if a Type II Failure(Catastrophic Failure) occurs during its mission time then it means total breakdown thus the failed device should be sent to the repair shop and it cannot continue its mission any more. Then, in this situation, it is very important to determine the burn-in time so that it can maximize the mean time to the Type II failure during filed operation. Hence, in the present paper, we consider this problem.

Denote by the random variable X the lifetime of the system and by $F(t)$ the distribution function of X. Let us assume that X has density function $f(t)$. Then its failure rate $r(t)$ is given by $r(t) = f(t)/\overline{F}(t)$, where $\overline{F}(t) = 1 - F(t)$

is the survivor function of X. Define Y_b as the time length from 0 to the first Type II failure of burned-in system with burn-in time b. Clearly, since the age of the burned-in system is b, the survivor function of Y_b is given by

$$\begin{aligned}\overline{G}_b(t) &= P(Y_b > t) \\ &= \exp\{-\int_0^t p(b+u)r(b+u)du\} \\ &= \exp\{-[\Lambda_p(b+t)-\Lambda_p(b)]\}, \quad \forall t \geq 0,\end{aligned} \tag{1}$$

where $\Lambda_p(t) \equiv \int_0^t p(u)r(u)du$. Then from (1) the mean time to the Type II failure in field operation is given by

$$\begin{aligned}E[Y_b] &= \int_0^\infty \overline{G}_b(t)\,dt \\ &= \int_0^\infty \exp\{-[\Lambda_p(b+t)-\Lambda_p(b)]\}dt \quad (2) \\ &= \exp\{\Lambda_p(b)\}\int_b^\infty \exp\{-\Lambda_p(t)\}dt.\end{aligned}$$

Before obtaining the properties on the optimal burn-in time maximizing $E[Y_b]$ in Eqn. (2), we define eventually non-constant function as follows.

Definition 1.

A function $g(t)$ is eventually non-constant function if for any $t' \geq 0$ there exists $t'' > t'$ such that $g(t') \neq g(t'')$.

The following result gives an upper bound for the optimal burn-in time.

Theorem 1.

Suppose that the function $p(t)r(t)$ is eventually non-constant function and the lifetime distribution function $F(t)$ has a bathtub shaped failure rate function $r(t)$ which has change points $0 \leq t_1 \leq t_2 \leq \infty$. Let the set V be

$$V \equiv \{t : p(u)r(u) \text{ is non-decreasing for all } u \geq t\}$$

and define $v_1 \equiv \inf V$ if the set V is not empty and $v_1 \equiv \infty$ if the set V is empty. Then the optimal burn-in time $b^* \leq v_1$. If, in addition, $p(0) \neq 0$ and

$$r(0) > \frac{1}{p(0)\int_0^\infty \exp\{-\Lambda_p(t)\}dt}$$

then $b^* > 0$.

proof.

When the set V is empty, the result obviously holds. We consider the case when the set V is not empty. Observe that

$$\frac{\partial E[Y_b]}{\partial b} = p(b)r(b)\exp\{\Lambda_p(b)\}\int_b^\infty \exp\{-\Lambda_p(t)\}dt - 1.$$

Then for all $b > v_1$,

$$\begin{aligned} & p(b)r(b)\exp\{\Lambda_p(b)\}\int_b^\infty \exp\{-\Lambda_p(t)\}dt - 1 \\ < & \int_b^\infty p(t)r(t)\exp\{-[\Lambda_p(t)-\Lambda_p(b)]\}dt - 1 \\ = & [-\exp\{-[\Lambda_p(t)-\Lambda_p(b)]\}]_b^\infty - 1 \\ = & 0 \end{aligned} \quad (3)$$

holds since $p(t)r(t)$ is non-decreasing for all $t > v_1$ and is eventually non-constant function. This means that $E[Y_b]$ is strictly decreasing for $b > v_1$. Therefore, we can conclude that $b^* \le v_1$.

For the second part of the theorem, consider the derivative of $E[Y_b]$ evaluated at $b=0$. It is easy to check that

$$\left.\frac{\partial E[Y_b]}{\partial b}\right|_{b=0} = p(0)r(0)\int_0^\infty \exp\{-\Lambda_p(t)\}dt - 1.$$

If we assume that $p(0) \neq 0$ and $r(0) > 1/(p(0)\int_0^\infty \exp\{-\Lambda_p(t)\}dt)$, then $\partial E[Y_b]/\partial b\,|_{b=0} > 0$ holds. This means that $E[Y_b]$ is strictly increasing in a right-hand neighborhood of $b=0$. Therefore $b^* > 0$. ■

Remark 1.

If the Type II failure probability function $p(t)$ is strictly increasing then the function $p(t)r(t)$ is eventually non-constant and the set V in Theorem 1 is not empty. Actually, in this case, the optimal burn-in time b^* has a non-trivial upper bound t_1, i.e., $b^* \le t_1$.

Remark 2.

From Theorem 1, we can see that a large initial failure rate $r(0)$ justifies burn-in, i.e., $b^* > 0$.

If the bathtub shaped failure rate function $r(t)$ has strictly increasing part(i.e., $t_2 < \infty$), then we obtain the following corollary.

Corollary 1.

Suppose that the lifetime distribution function $F(t)$ has a bathtub shaped failure rate function $r(t)$ which has change points $0 \le t_1 \le t_2 < \infty$. If we define

$$W \equiv \{t: p(u) \text{ is non-decreasing for all } u \geq t\}$$

and define $w_1 = \inf W$. Then the optimal burn-in time $b^* \leq \max\{t_1, w_1\}$.

proof.

For all $b > \max\{t_1, w_1\}$, the inequality (3) holds, since the function $p(t)r(t)$ is strictly increasing for $t > \max\{t_1, w_1\}$. Hence the result is readily obtained. ■

We now consider some particular cases of the model.

First let the Type II failure probability function be a constant function, that is, $p(t) = p,\ 0 < p < 1$. In this case, $\Lambda_p(t)$ in Equations (1) and (2) is given by $\Lambda_p(t) = p\int_0^t r(u)du$.

Theorem 2.

Suppose that the lifetime distribution function $F(t)$ has a bathtub shaped failure rate function $r(t)$ which has change points $0 < t_1 \leq t_2 < \infty$ and $p(t) = p,\ 0 < p < 1$, that is, the Type II failure probability function is a constant function of t. Then

(i) the optimal burn-in time satisfies $0 \leq b^* \leq t_1$, and

(ii) if we further assume that $r(\infty) \leq r(0)$, then the optimal burn-in time b^* satisfies $t_0 \leq b^* \leq t_1$, where t_0 is uniquely determined by $r(t_0) = r(\infty)$.

proof.

The result (i) can be obtained without difficulty. Note that, for $0 \leq b < t_0$, it obviously holds that

$$\begin{aligned} &pr(b)\exp\{\Lambda_p(b)\}\int_b^\infty \exp\{-\Lambda_p(t)\}dt - 1 \\ &> \int_b^\infty r(t)\exp\{-[\Lambda_p(t) - \Lambda_p(b)]\}dt - 1 = 0. \end{aligned}$$

This implies that $E[Y_b]$ is strictly increasing for $0 \leq b < t_0$. Hence we have the desired result (ii). ■

Remark 3.

From Theorem 2 we see that, when the lifetime distribution function has a large initial failure rate $r(0)$ which is larger than the supremum of the failure rate in field use, then we also have a lower bound for the optimal burn-in time.

Now let $F(t)$ be exponential, that is its failure rate function is given by $r(t) = \lambda,\ \forall t \geq 0$. In this case from (2),

$$E[Y_b] = \int_0^\infty \exp\{-\lambda\int_b^{b+t} p(u)du\}dt.$$

Corollary 2.

Suppose that the two change points of $r(t)$ satisfy $t_1 = 0$ and $t_2 = \infty$, that is, $F(t)$ is an exponential distribution with $r(t) = \lambda$, $\forall t \geq 0$.

(i) If $p(t)$ is a non-increasing and eventually non-constant function of t, then $b^* = \infty$.

(ii) If $p(t)$ is a non-decreasing and non-constant function of t, then $b^* = 0$.

proof.

If $p(t)$ is a non-increasing and eventually non-constant function of t, then $\int_b^{b+t} p(u)du$ is non-increasing and eventually strictly decreasing in b for each fixed $t > 0$. This means that $E[Y_b]$ is non-decreasing and eventually strictly increasing in b. Hence we have $b^* = \infty$. The result (ii) can be proved similarly ■

3. Numerical Examples

In this section, some illustrative examples are given.

3.1. Example 1

In this example, suppose that the failure rate function of the system is given by

$$r(t) = \begin{cases} 3(t-1)^2 + 1, & 0 \leq t < 1; \\ 1, & 1 \leq t < 6; \\ (t-6)^2 + 1, & t \geq 6. \end{cases}$$

Then the failure rate function of the system is a bathtub shaped failure rate function and two change points are $t_1 = 1.0$ and $t_2 = 6.0$. Assume that the Type II Failure probability is given by $p(t) = 1 - 0.4\exp\{-t\}$. Then note that the function $p(t)$ is strictly increasing and thus by Theorem 1 and Remark 1 in the previous section, an upper bound for the optimal burn-in time is given by $t_1 = 1.0$. Hence it is sufficient to consider only $b \in [0, 1]$ to find the optimal burn-in time b^*. The graph of the mean time to the Type II failure is presented in the following Figure 1. By numerical search, the optimal burn-in time is given by $b^* = 0.797$ and the maximum mean time to the Type II failure is $E[Y_{b^*}] = 1.0865712$.

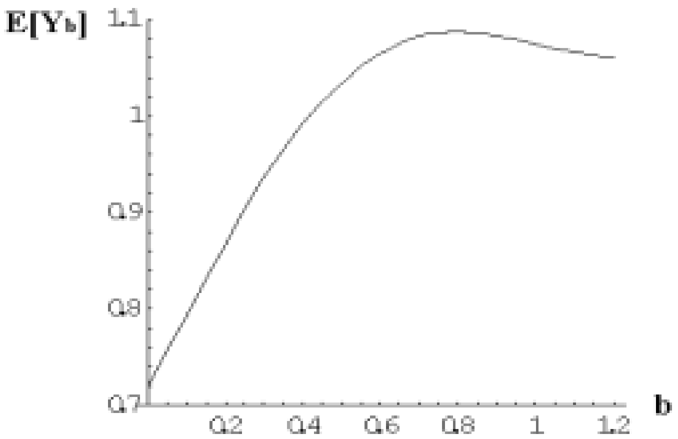

Figure 1. Mean Time to Type II Failure

3.2. Example 2

In this example, suppose that the Type II failure probability function $p(t)$ is a constant function of t and is given by $p(t) = 0.2$. Assume that the failure rate function of the system is given by

$$r(t) = \begin{cases} 3(t-1)^2+1, & 0 \le t < 1; \\ 1, & 1 \le t < 6; \\ -(t-7)^2+2, & 6 \le t < 7; \\ 2, & t \ge 7. \end{cases}$$

Then the failure rate function has two change points $t_1 = 1.0$ and $t_2 = 6.0$ and, furthermore, it holds that $r(0) > r(\infty)$. Then, in this example, t_0 defined in Theorem 2 is given by $t_0 = 1 - 1/\sqrt{3} = 0.42265$. Thus, in this case, it is sufficient to consider $b \in [0.42265, 1.0]$. The following Figure 2 shows the graph of the mean time to the Type II failure. By numerical search, the optimal burn-in time is given by $b^* = 0.684$ and the maximum mean time to the Type II failure is $E[Y_{b^*}] = 3.8503412$.

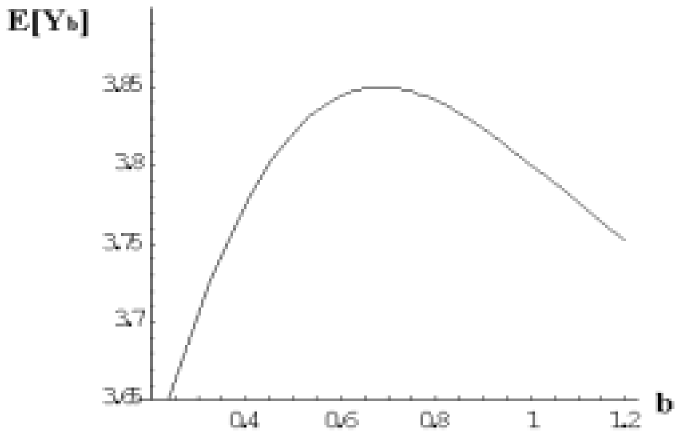

Figure 2. Mean Time to Type II Failure

4. Concluding Remark

In this paper, we have considered the problem of finding optimal burn-in time which maximizes the mean time to the Type II failure in field operation under the general failure model. For some general cases, upper bounds for the optimal burn-in times have been obtained and it has been shown that a large initial failure rate justifies burn-in. When the lifetime distribution function has a large initial failure rate $r(0)$ which is larger than the supremum of the failure rate in field use, then we also have obtained a lower bound for the optimal burn-in time. The obtained results have been applied to some illustrative examples. With the bounds that we have obtained, the optimal burn-in time has been found with ease.

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