Journal of Korean
Data & Information Science Society
2003, Vol. 14, No.3 pp. 697~705

Reference Priors in the Normal Distributions with Common Coefficient of Variation

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Abstract

When X and Y have independent normal distributions with equal coefficient of variation, we develop the reference priors for different groups of ordering for the parameters. Propriety of posteriors under reference priors proved. A real example is presented to compare the classical estimator and Bayes estimator.

Keywords : Reference Priors; Normal Distributions; Coefficients of Variations.

1. INTRODUCTION

We consider that X and Y are independent with normal distributions having equal coefficient of variation. That is, let $X=(X_1,\cdots,X_{n_1})$ be a random sample of size n_1 from a normal population with mean μ_1 and variance $\mu_1^2\gamma^2$ and let $Y=(Y_1,\cdots,Y_{n_2})$ be a random sample of size n_2 from a normal population with mean μ_2 and variance $\mu_2^2\gamma^2$. Then the joint probability density function is

$$\begin{aligned} f(x,y\mid\mu_1,\mu_2,\gamma) &= (2\pi)^{-(n_1+n_2)/2}\gamma^{-(n_1+n_2)}\mu_1^{-n_1}\mu_2^{-n_2} \\ &\quad\times\exp\{-\frac{1}{2\gamma^2\mu_1^2}\sum_{i=1}^{n_1}(x_i-\mu_1)^2-\frac{1}{2\gamma^2\mu_2^2}\sum_{i=1}^{n_2}(y_i-\mu_2)^2\}, \end{aligned} \tag{1}$$

where $\mu_1>0$, $\mu_2>0$, $\gamma>0$ and γ is the common coefficient of variation.

The present paper focuses on Bayesian inference for μ_1, μ_2 and γ. The emphasis is on noninformative priors. Although subjective Bayesians are often

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critical of such priors, these priors have clear pragmatic appeal especially when prior information is vague in nature. The most frequently used noninformative prior is Jeffreys' (1961) prior, which is proportional to the positive square of the determinant of the Fisher information matrix. In the one parameter case, Welch and Peers (1963) proved that a one-sided credible interval from Jeffreys' prior matches the corresponding frequentist coverage probability up to $O(n^{-1})$.

In spite of its success in one parameter problems, Jeffreys' prior frequently runs in to serious difficulties in the presence of nuisance parameters. For example, in Neyman-Scott problem, the Jeffrey's prior produces an inconsistent estimator of the error variance, in the multinomial problem it lacks marginalization over nuisance cell probabilities and in estimating the sum of squares of a large number of independent normal means with a common variance, it leads to an unsatisfactory posterior. As an alternative, Ghosh and Mukerjee (1992), and Berger and Bernardo (1989,1992) extended Bernardo's (1979) reference prior approach, giving a general algorithm to derive a reference prior by splitting the parameters into several groups according to their order of inferential importance. This approach is very successful in various practical problems. Quite often reference priors satisfy the matching criterion.

We consider that X and Y are independent with normal distributions having equal coefficient of variation. The assumption of equal coefficient of variation is because the coefficient of variation represents a measure of relative variability and groups can have the same relative variability even if the means and variances of the variable of interest are different. The assumption of homogeneous coefficients of variation is a valid assumption in many types of agricultural, biological and psychological experimentation, because many times the treatment that yields a larger mean also has a larger standard deviation (Lohrding, 1969). Our assumption of equal coefficient of variation will imply that the two means are of equal sign. Thus we can, without loss of generality, assume that the two means are positive (Sinha, Rao and Clement, 1978; Gupta, Ramakrishnan and Zhou, 1999).

The outline of the remaining sections is as follows. In Section 2, we derive Fisher information matrix. We develop the reference priors for different groups of ordering for the parameters. In Section 3, we provide that the propriety of the posterior distribution for the reference priors. In Section 4, a real example is presented to compare the classical estimator and Bayes estimator.

2. THE REFERENCE PRIORS

Bernardo (1979) introduced determining reference priors in two steps. This method has been extended further by Berger and Bernardo (1992) who provided a general algorithm to cover situations with multiple groups ordered in terms of inferential importance. It is possible to have many different ways to order the parameters in order to obtain many different reference priors. For simplicity, the notation $\{\mu_1,(\mu_2,\gamma)\}$ will be used to represent the case where there are two groups, with μ_1 being the most important and μ_2 and γ being of equal importance.

We derive the reference priors for different roups of ordering of (μ_1,μ_2,γ). The log-likelihood function of parameters (μ_1,μ_2,γ) for the model (1) is given by

$$\begin{aligned} l(\mu_1,\mu_2,\gamma) \propto & -n_1\log\mu_1 - n_2\log\mu_2 - (n_1+n_2)\log\gamma \\ & -\frac{1}{2}\gamma^2\mu_1^2\sum_{i=1}^{n_1}(x_i-\mu_1)^2 - \frac{1}{2}\gamma^2\mu_2^2\sum_{i=1}^{n_2}(y_i-\mu_2)^2. \end{aligned} \tag{2}$$

Based on (2), the Fisher information matrix is given by

$$I=\begin{pmatrix} \frac{n_1(1+2\gamma^2)}{\gamma^2\mu_1^2} & 0 & \frac{2n_1}{\gamma\mu_1} \\ 0 & \frac{n_2(1+2\gamma^2)}{\gamma^2\mu_2^2} & \frac{2n_2}{\gamma\mu_2} \\ \frac{2n_1}{\gamma\mu_1} & \frac{2n_2}{\gamma\mu_2} & \frac{2(n_1+n_2)}{\gamma^2} \end{pmatrix}.$$

The reference priors for different groups of ordering of (μ_1,μ_2,γ) are given by as follows.

Theorem 1. The reference prior distributions for different groups of ordering of (μ_1,μ_2,γ) are:

Group ordering	Reference prior	
$\{(\mu_1,\mu_2,\gamma)\}$,	$\pi_1 \propto \mu_1^{-1}\mu_2^{-1}\gamma_3^{-3}(1+2\gamma^2)^{1/2}$,	(3)
$\{\mu_1,\mu_2,\gamma\}$, $\{\mu_2,\mu_1,\gamma\}$, $\{(\mu_1,\mu_2),\gamma\}$,	$\pi_2 \propto \mu_1^{-1}\mu_2^{-1}\gamma^{-1}$.	(4)
$\{\gamma,\mu_1,\mu_2\}$, $\{\gamma,\mu_2,\mu_1\}$, $\{\gamma,(\mu_1,\mu_2)\}$,	$\pi_3 \propto \mu_1^{-1}\mu_2^{-1}\gamma^{-1}(1+2\gamma^2)^{-1/2}$,	(5)
$\{\mu_1,(\mu_2,\gamma)\}$,	$\pi_4 \propto \mu_1^{-1}\mu_2^{-1}\gamma^{-2}(n_1+n_2+2n_1\gamma^2)^{1/2}$,	(6)
$\{(\mu_1,\gamma),\mu_2\}$, $\{(\mu_2,\gamma),\mu_1\}$,	$\pi_5 \propto \mu_1^{-1}\mu_2^{-1}\gamma^{-2}$.	(7)

Proof. We prove only two cases. Others are similar. We use the notation of Berger and Bernardo (1992). The compact subsets were taken to be Cartesian products of sets of the form

$$\mu_1\in[a_1,b_1], \mu_2\in[a_2,b_2], \gamma\in[a_3,b_3].$$

In the limit a_1,a_2,a_3 will tend to 0 and b_1,b_2,b_3 will tend to ∞.

First we will prove that the reference prior for $\{(\mu_1,\mu_2),\gamma\}$ is $\pi_2(\mu_1,\mu_2,\gamma)$

$=\mu_1^{-1}\mu_2^{-1}\gamma^{-2}$. For the derivation of the reference prior, from the Fisher information,

$$h_1=n_1 n_2\mu_1^{-2}\mu_2^{-2}\gamma^{-4}(1+2\gamma^2),\ h_2=2(n_1+n_2)\gamma^{-2}$$

Here, and below, a subscripted K denotes that is constant and does not depend on any parameters but any K may depend on the ranges of the parameters.

Step 1. Note that

$$\int_{a_3}^{b_3} h_2^{1/2}d\gamma=\int_{a_3}^{b_3}[2(n_1+n_2)\gamma^{-2}]^{1/2}d\gamma=[2(n_1+n_2)]^{1/2}\log(b_3/a_3),$$

so $\pi_2^l\{\gamma\mid\mu_1,\mu_2\}=K_1^{-1}\gamma^{-1}$, where $K_1=\log(b_3/a_3)$.

Step 2. Now

$$\begin{aligned}E_1^l\{\log h_1\mid\gamma\} &= \int_{a_3}^{b_3}K_1^{-1}\gamma^{-1}\log\{n_1n_2\mu_1^{-2}\mu_2^{-2}\gamma^{-4}(1+2\gamma^2\}d\gamma\\ &= \log\{\mu_1^{-2}\mu_2^{-2}\}+K_{21}.\end{aligned}$$

It follows that

$$\pi_1^l\{\mu_1,\mu_2\}=\exp[E_1^l\{\log h_1\mid\gamma\}/2]=\mu_1^{-1}\mu_2^{-1}\exp\{K_{21}/2\}.$$

Thus $\pi_1^l\{\mu_1,\mu_2,\gamma\}=K_3^{-1}\mu_1^{-1}\mu_2^{-1}\gamma^{-1}$, where $K_3=\log(b_3/a_3)\exp\{K_{21}/2\}$. The reference prior is thus

$$\pi_1(\mu_1,\mu_2,\gamma)\propto\mu_1^{-1}\mu_2^{-1}\gamma^{-1}.$$

Second we will prove that the reference prior for $\{\mu_1,\mu_2,\gamma\}$ is $\pi_2(\mu_1,\mu_2,\gamma)=\mu_1^{-1}\mu_2^{-1}\gamma^{-1}$. For the derivation of the reference, from the inverse of the Fisher information,

$$h_1=\frac{n_1(n_1+n_2)(1+2\gamma^2)}{\mu_1^2\gamma^2(n_1+n_2+2n_1\gamma^2)},\ h_2=\frac{n_2(n_1+n_2+2n_1\gamma^2)}{(n_1+n_2)\mu_2^2\gamma^2},\ h_3=2(n_1+n_2)\gamma^{-2}.$$

Step 1. Note that

$$\int_{a_3}^{b_3} h_3^{1/2}d\gamma=\int_{a_3}^{b_3}[2(n_1+n_2)\gamma^{-2}]^{1/2}d\gamma=[2(n_1+n_2)]^{1/2}\log(b_3/a_3),$$

so $\pi_3^l\{\gamma\mid\mu_1,\mu_2\}=K_1^{-1}\gamma^{-1}$, where $K_1=\log(b_3/a_3)$.

Step 2. Now

$$\begin{aligned}E_2^l\{\log h_2\mid\mu_1,\mu_2\} &= \int_{a_3}^{b_3}K_1^{-1}\gamma^{-1}\log\{\frac{n_2(n_1+n_2+2n_1\gamma^2)}{(n_1+n_2)\mu_2^2\gamma^2}\}d\gamma\\ &= \log\{\mu_2^{-2}\}+K_{21}.\end{aligned}$$

It follows that

$$\int_{a_2}^{b_2}\exp[E_2^l\{\log h_2\mid\mu_1,\mu_2\}/2]d\mu_2=\exp\{K_{21}/2\}\log(b_2/a_2).$$

Hence $\pi_2^l\{\mu_2,\gamma\mid\mu_1\}=K_2^{-1}\mu_2^{-1}\gamma^{-1}$, where $K_2=\log(b_2/a_2)\log(b_3/a_3)$.

Step 3. Now

$$E^l_1\{\log h_1 \mid \mu_1\} = \int_{a_2}^{b_2}\int_{a_3}^{b_3} K_2^{-1}\mu_2^{-1}\gamma^{-1}\log\{\frac{n_1(n_1+n_2)(1+2\gamma^2)}{\mu_1^2\gamma^2(n_1+n_2+2n_1\gamma^2)}\}d\gamma d\mu_2$$
$$= K_{31} + \log\{\mu_1^{-2}\}.$$

So

$$\int_{a_1}^{b_1}\exp[E^l_1\{\log h_1 \mid \mu_1\}/2]d\mu_1 = \exp\{K_{31}/2\}\log(b_1/a_1).$$

Thus $\pi^l_1\{\mu_1,\mu_2,\gamma\} = K_3^{-1}\mu_1^{-1}\mu_2^{-1}\gamma_3^{-1}$, where $K_3 = \log(\frac{b_1}{a_1})\log(\frac{b_2}{a_2})\log(\frac{b_3}{a_3})$.
The reference prior is thus

$$\pi_1(\mu_1,\mu_2,\gamma) \propto \mu_1^{-1}\mu_2^{-1}\gamma^{-1}.$$

This completes the proof. □

3. PROPRIETY OF POSTERIORS

We investigate the propriety of posteriors for the reference priors in the Theorem 1. The following theorem can be proved.

Theorem 2. The posterior distribution of (μ_1,μ_2,γ) under the reference prior (3), (5) and (7) is proper. But the posterior distribution of (μ_1,μ_2,γ) under the reference prior (4) and (6) is improper.

Proof. We prove only two cases. Others are similar. Firstly we will prove that the posterior distribution of (μ_1,μ_2,γ) under the reference prior (7) is proper. Under the reference prior (7), the joint posterior for μ_1,μ_2,γ given $\boldsymbol{x},\boldsymbol{y}$ is

$$\pi(\mu_1,\mu_2,\gamma \mid \boldsymbol{x},\boldsymbol{y}) \propto \mu_1^{-(n_1+1)}\mu_2^{-(n_2+1)}\gamma^{-(n_1+n_2+2)} \times \exp\{-\frac{\sum_{i=1}^{n_1}(x_i-\mu_1)^2}{2\gamma^2\mu_1^2} - \frac{\sum_{i=1}^{n_2}(y_i-\mu_2)^2}{2\gamma^2\mu_2^2}\}. \quad (8)$$

Integrating with respect to γ in (8), then

$$C_2\mu_1^{-(n_1+1)}\mu_2^{-(n_2+1)}[\frac{\sum_{i=1}^{n_1}(x_i-\mu_1)^2}{\mu_1^2} + \frac{\sum_{i=1}^{n_2}(y_i-\mu_2)^2}{\mu_2^2}]^{-\frac{n_1+n_2+1}{2}}.$$

For $\mu_1 > \frac{3}{2}|\bar{x}|$ and $\mu_2 > \frac{3}{2}|\bar{y}|$,

$$\left[\frac{\sum_{i=1}^{n_1}(x_i-\mu_1)^2}{\mu_1^2}+\frac{\sum_{i=1}^{n_2}(y_i-\mu_2)^2}{\mu_2^2}\right]\geq\frac{n_1}{9}+\frac{n_2}{9}.$$

Thus

$$\int_{\frac{3}{2}|\bar{x}|}^{\infty}\int_{\frac{3}{2}|\bar{y}|}^{\infty}C_2\mu_1^{-(n_1+1)}\mu_2^{-(n_2+1)}\left[\frac{\sum_{i=1}^{n_1}(x_i-\mu_1)^2}{\mu_1^2}+\frac{\sum_{i=1}^{n_2}(y_i-\mu_2)^2}{\mu_2^2}\right]^{-\frac{n_1+n_2+1}{2}}d\mu_1 d\mu_2$$

$$\leq\int_{\frac{3}{2}|\bar{x}|}^{\infty}\int_{\frac{3}{2}|\bar{y}|}^{\infty}C_2\mu_1^{-(n_1+1)}\mu_2^{-(n_2+1)}\left[\frac{n_1}{9}+\frac{n_2}{9}\right]^{-\frac{n_1+n_2+1}{2}}d\mu_1 d\mu_2$$

$$<\infty.$$

For $\mu_1>\frac{3}{2}|\bar{x}|$ and $\mu_2\leq\frac{3}{2}|\bar{y}|$,

$$\left[\frac{\sum_{i=1}^{n_1}(x_i-\mu_1)^2}{\mu_1^2}+\frac{\sum_{i=1}^{n_2}(y_i-\mu_2)^2}{\mu_2^2}\right]\geq\left[\frac{n_1}{9}+\frac{\sum_{i=1}^{n_2}(y_i-\bar{y})^2}{\mu_2^2}\right].$$

Thus

$$\int_{\frac{3}{2}|\bar{x}|}^{\infty}\int_{0}^{\frac{3}{2}|\bar{y}|}C_2\mu_1^{-(n_1+1)}\mu_2^{-(n_2+1)}\left[\frac{\sum_{i=1}^{n_1}(x_i-\mu_1)^2}{\mu_1^2}+\frac{\sum_{i=1}^{n_2}(y_i-\mu_2)^2}{\mu_2^2}\right]^{-\frac{n_1+n_2+1}{2}}d\mu_2 d\mu_1$$

$$\leq\int_{\frac{3}{2}|\bar{x}|}^{\infty}\int_{0}^{\frac{3}{2}|\bar{y}|}C_2\mu_1^{-(n_1+1)}\mu_2^{-(n_2+1)}\left[\frac{n_1}{9}+\frac{\sum_{i=1}^{n_2}(y_i-\bar{y})^2}{\mu_2^2}\right]^{-\frac{n_1+n_2+1}{2}}d\mu_2 d\mu_1$$

$$<\infty.$$

For $\mu_1\leq\frac{3}{2}|\bar{x}|$ and $\mu_2\leq\frac{3}{2}|\bar{y}|$,

$$\left[\frac{\sum_{i=1}^{n_1}(x_i-\mu_1)^2}{\mu_1^2}+\frac{\sum_{i=1}^{n_2}(y_i-\mu_2)^2}{\mu_2^2}\right]\geq\left[\frac{\sum_{i=1}^{n_1}(x_i-\bar{x})^2}{\mu_1^2}+\frac{\sum_{i=1}^{n_2}(y_i-\bar{y})^2}{\mu_2^2}\right].$$

Thus

$$\int_{0}^{\frac{3}{2}|\bar{y}|}\int_{0}^{\frac{3}{2}|\bar{x}|}C_2\mu_1^{-(n_1+1)}\mu_2^{-(n_2+1)}\left[\frac{\sum_{i=1}^{n_1}(x_i-\mu_1)^2}{\mu_1^2}+\frac{\sum_{i=1}^{n_2}(y_i-\mu_2)^2}{\mu_2^2}\right]^{-\frac{n_1+n_2+1}{2}}d\mu_1 d\mu_2$$

$$\leq\int_{0}^{\frac{3}{2}|\bar{y}|}\int_{0}^{\frac{3}{2}|\bar{x}|}C_2\mu_1^{-(n_1+1)}\mu_2^{-(n_2+1)}\left[\frac{\sum_{i=1}^{n_1}(x_i-\bar{x})^2}{\mu_1^2}+\frac{\sum_{i=1}^{n_2}(y_i-\bar{y})^2}{\mu_2^2}\right]^{-\frac{n_1+n_2+1}{2}}d\mu_1 d\mu_2$$

$$=\int_{0}^{\frac{9}{4}\bar{y}^2S_1}\int_{0}^{\frac{9}{4}\bar{x}^2S_2}C_3 z_1^{\frac{n_2-1}{2}}z_2^{\frac{n_1-1}{2}}[z_1+z_2]^{-\frac{n_1+n_2+1}{2}}dz_1 dz_2, \qquad (9)$$

where C_3 is a constant, $S_1=\sum_{i=1}^{n_1}(x_i-\bar{x})^2$ and $S_2=\sum_{i=1}^{n_2}(y_i-\bar{y})^2$. Let $v_1=z_1+z_2$

and $v_2 = \frac{z_1}{z_1+z_2}$. Then the (9) is

$$\int_0^1 \int_0^{\frac{9}{4}(\bar{x}^2 S_2 + \bar{y}^2 S_1)} C_3 v_1^{-\frac{1}{2}} v_2^{\frac{n_2-1}{2}} (1-v_2)^{\frac{n_1-1}{2}} dv_1 dv_2 < \infty.$$

Thus the posterior distribution of μ_1, μ_2 and γ is proper.

Second we will prove that the posterior distribution under the reference prior (4) is improper. Under the reference prior (4), the joint posterior for μ_1, μ_2, γ given x, y is

$$\pi(\mu_1, \mu_2, \gamma \mid x, y) \propto \mu_1^{-(n_1+1)} \mu_2^{-(n_2+1)} \gamma^{-(n_1+n_2+1)} \times \exp\{-\frac{\sum_{i=1}^{n_1}(x_i-\mu_1)^2}{2\gamma^2\mu_1^2} - \frac{\sum_{i=1}^{n_2}(y_i-\mu_2)^2}{2\gamma^2\mu_2^2}\}. \quad (10)$$

Integrating with respect to γ in (10), then

$$C_2 \mu_1^{-(n_1+1)} \mu_2^{-(n_2+1)} [\frac{\sum_{i=1}^{n_1}(x_i-\mu_1)^2}{\mu_1^2} + \frac{\sum_{i=1}^{n_2}(y_i-\mu_2)^2}{\mu_2^2}]^{-\frac{n_1+n_2}{2}},$$

where C_2 is a constant. For $\mu_1 \le \frac{3}{2}|\bar{x}|$ and $\mu_2 \le \frac{3}{2}|\bar{y}|$,

$(\bar{x}-\mu_1)^2 < \max\{\bar{x}^2, (\bar{x}-\frac{3}{2}|\bar{x}|^2)^2\} = t_1$, $(\bar{y}-\mu_2)^2 < \max\{\bar{y}^2, (\bar{y}-\frac{3}{2}|\bar{y}|^2)^2\} = t_2$

Thus

$$\int_0^{\frac{3}{2}|\bar{y}|} \int_0^{\frac{3}{2}|\bar{x}|} C_2 \mu_1^{-(n_1+1)} \mu_2^{-(n_2+1)} [\frac{\sum_{i=1}^{n_1}(x_i-\mu_1)^2}{\mu_1^2} + \frac{\sum_{i=1}^{n_2}(y_i-\mu_2)^2}{\mu_2^2}]^{-\frac{n_1+n_2}{2}} d\mu_1 d\mu_2$$

$$> \int_0^{\frac{3}{2}|\bar{y}|} \int_0^{\frac{3}{2}|\bar{x}|} C_2 \mu_1^{-(n_1+1)} \mu_2^{-(n_2+1)} [\frac{n_1 t_1 + S_1}{\mu_1^2} + \frac{n_2 t_2 + S_2}{\mu_2^2}]^{-\frac{n_1+n_2}{2}} d\mu_1 d\mu_2$$

$$= \int_0^{\frac{9}{4}\bar{y}^2(n_1 t_1 + S_1)} \int_0^{\frac{9}{4}\bar{x}^2(n_2 t_2 + S_2)} C_3 z_1^{\frac{n_2-2}{2}} z_2^{\frac{n_1-2}{2}} [z_1+z_2]^{-\frac{n_1+n_2}{2}} dz_1 dz_2,$$

where C_3 is a constant, $S_1 = \sum_{i=1}^{n_1}(x_i-\bar{x})^2$ and $S_2 = \sum_{i=1}^{n_2}(y_i-\bar{y})^2$. Let $v_1 = z_1+z_2$ and $v_2 = \frac{z_1}{z_1+z_2}$. Then the (13) is

$$\int_0^1 \int_0^{\frac{9}{4}[\bar{x}^2(n_2 t_2 + S_2) + \bar{y}^2(n_1 t_1 + S_1)]} C_3 v_1^{-1} v_2^{\frac{n_2-2}{2}} (1-v_2)^{\frac{n_1-2}{2}} dv_1 dv_2 = \infty.$$

Thus the posterior distribution of (μ_1, μ_2, γ) under the reference prior (4) is improper. This completes the proof. □

4. AN EXAMPLE

The data in Table 1 are taken from Nelson (1990) and represent the hours to failure of 20 motorettes with a new class-H insulation run at $240\ ^{\circ}C$ and $220\ ^{\circ}C$. It has been observed by Nelson (1990) that lognormal distribution adequately fits at the two temperatures. Note that X and Y denote the nature logarithm of the failure times in the Table 1. We thus assume that the below data come from independent normal distributions.

Table 1: Failure Times at the Two Temperatures

X ($240\ ^{\circ}C$)	7.0690	7.0690	7.3271	7.3582	7.3883
	7.4176	7.4176	7.4460	7.4736	7.5771
Y ($220\ ^{\circ}C$)	7.4753	7.7981	7.7981	7.7981	7.7981
	7.7981	8.0417	8.0417	8.0417	8.0417

For the equality of the coefficients of variation, the p value of the score test developed by Gupta and Ma (1996) is almost one under the null hypothesis $H_0: \gamma_1 = \gamma_2$ (Gupta, Ramakrishnan and Zhou, 1999). Therefore we assume equal coefficients of variation.

The maximum likelihood estimate (MLE) and Bayes estimates (BE) with the standard errors in parentheses under square error loss of μ_1, μ_2 and γ are given in Table 2.

Table 2: MLE and Bayes Estimates of μ_1, μ_2 and γ

	$\widehat{\mu}_1$	$\widehat{\mu}_2$	$\widehat{\gamma}$
MLE	7.354238 (0.050324)	7.863381 (0.053808)	0.021644 (0.003424)
BE^1	7.343060 (0.003528)	7.921796 (0.008072)	0.020989 (0.001872)
BE^2	7.343052 (0.002593)	7.921889 (0.006056)	0.020898 (0.001380)
BE^3	7.343048 (0.001906)	7.921939 (0.004607)	0.020848 (0.001016)

BE^1 – reference prior π_3; BE^2 – reference prior π_5; BE^3 – reference prior π_1

From Table 2, it can be seen easily that the Bayes estimators with all three different priors are very comparable to the MLE's. For μ_1, μ_2 and γ, they are almost identical in estimations. This example provides empirical evidence that

Bayes estimators with reference priors are at least as good as non-Bayesian estimators, namely, MLE's, which are asymptotically the best.

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[received date : Jun. 2003, accepted date : Aug. 2003]