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On Multipurpose Replacement Policies for the General Failure Model

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Absract

In this paper, various replacement policies for the general failure model are considered. There are two types of failure in the general failure model. One is Type I failure (minor failure) which can be removed by a minimal repair and the other is Type II failure (catastrophic failure) which can be removed only by a complete repair. In this model, when the unit fails at its age t, Type I failure occurs with probability $1-p(t)$ and Type II failure occurs with probability $p(t)$, $0 \le p(t) \le 1$. Under the model, optimal replacement policies for the long-run average cost rate and the limiting efficiency are considered. Also taking the cost and the efficiency into consideration at the same time, the properties of the optimal policies under the Cost-Priority-Criterion and the Efficiency-Priority-Criterion are obtained.

Keywords : Optimal Maintenance Policy; General Failure Model; Long-run Average Cost Rate; Limiting Efficiency

1. Introduction

One important area of interest in reliability theory is the study of various maintenance policies which are performed in order to reduce the occurrence of system failure. There are a large body of work in reliability that deals with the repair or replacement of components and systems. Two of the popular repair types that have been considered in the literature are the minimal repair and the complete

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repair. In complete repair a failed system is repaired as good as new. This means that the lifetime of the repaired system is independent of the lifetime of the original system and has the same distribution function. On the other hand, a minimal repair enables the system to continue its task or work, but does not affect the failure rate of the system; i.e., the failure rate of a system after minimal repair is equal to the failure rate immediately before the failure.

In this paper, we consider various replacement policies for the general failure model. In the general failure model, when the unit fails at its age t, Type I failure occurs with probability $\overline{p}(t) \equiv 1 - p(t)$ and Type II failure occurs with probability $p(t)$, $0 \le p(t) \le 1$. It is assumed that Type I failure is a minor one thus can be removed by a minimal repair, whereas Type II failure is a catastrophic one thus can be removed only by a complete repair. Such models have been considered in the literature. See, for example, Beichelt and Fischer (1980), Nakagawa (1981), Sheu and Griffith (1996) and Sheu (1998). Under the model, the system is completely repaired at its age T or at the time of the first Type II failure, whichever occurs first. For each Type I failure occurring during operation, only minimal repair is done. In the following we call this maintenance rule "policy T" .

In many researches, to determine an optimal policy only economic criterion was considered. However, in some instances, the efficiency criterion is more serious than the economic consideration. For example, when our interest is in studying power supplies for a hospital or electric security systems, or a protection systems, high efficiency is required. In this paper, optimal replacement policies for the long-run average cost rate and those for the limiting efficiency are considered. Also taking the cost and the efficiency into consideration at the same time, the properties of the optimal policies under the Cost-Priority-Criterion, in which optimal policy is determined so that the long-run average cost rate is low enough and the limiting efficiency is maximized, and those under the Efficiency-Priority-Criterion are obtained.

2. Preliminaries

Denote by the random variable X the lifetime of the system and by $F(t)$ the distribution function of X. Let us assume that X has density function $f(t)$. Then its failure rate $r(t)$ is given by $r(t) = f(t)/\overline{F}(t)$, where $\overline{F}(t) = 1 - F(t)$ is the survival function of X. Also denote by the random variable Y the time to the first Type II failure of the system. If we define $G(t)$ to be the distribution function of Y and $\overline{G}(t)$ to be $1 - G(t)$, then, by the results presented in

Beichelt (1993), $\overline{G}(t)$ is given by

$$\overline{G}(t) = \exp\left[-\int_0^t p(u)r(u)du\right], \quad \forall t \geq 0.$$

If we define the random variable Z_t to be $Z_t \equiv \min\{Y, t\}$, where t is a non-negative constant, then the expectation of Z_t is given by

$$\begin{aligned} E(Z_t) &= E(Z_t \mid Y \leq t)P(Y \leq t) + E(Z_t \mid Y > t)P(Y > t) \\ &= \int_0^t u dG(u) + u\overline{G}(u) \\ &= \int_0^t \overline{G}(u)du \end{aligned} \tag{1}$$

Let N_t be the random number of minimal repairs during $(0, Z_t)$ and $P(N_t = k \mid Y = t)$ be the conditional probability of $N_t = k$ given that $Y = t$. Then, from the results of Beichelt (1993), we can see that

$$P(N_t = k \mid Y = t) = \frac{1}{k!}\left(\int_0^{t-} p(u)r(u)du\right)^k \exp\left(-\int_0^{t-} p(u)r(u)du\right), \quad k = 0, 1, 2, \cdots.$$

Hence, on the condition that $Y = t$ the random number N_t is Poisson distributed with the mean

$$E(N_t \mid Y = t) = \int_0^{t-} p(u)r(u)du.$$

Also observe that the conditional expectations of N_t on the condition that $Y < t$ and $Y \geq t$, respectively, are given by

$$E(N_t \mid Y < t) = \int_0^t \int_0^{x-} p(u)r(u)du dG(x)/G(t),$$

and

$$E(N_t \mid Y \geq t) = E(N_t \mid Y = t) = \int_0^{t-} p(u)r(u)du.$$

Hence, it holds that

$$\begin{aligned} E(N_t) &= E(N_t \mid Y < t)G(t) + E(N_t \mid Y \geq t)\overline{G}(t) \\ &= \int_0^t r(u)\overline{G}(u)du - G(t). \end{aligned} \tag{2}$$

3. Optimal Policy for the Long-run Average Cost Rate

In this section, optimal replacement policy for the long run expected cost rate is considered. Let the cost rate(cost per unit repair time) of a minimal repair be C_1, and the cost rate of a replacement be C_2. Hereafter throughout this paper, without loss of generality, we assume that $0<C_1<C_2$ then this means that the cost rate of a replacement is higher than that of a minimal repair. Furthermore, let the reward rate whenever the system is working is r. Similar cost structure was considered in Yeh (1988). We further assume that the mean time consumed for a minimal repair is ν_1 and that for a complete repair is $\nu_2(\nu_2>\nu_1)$. Then from renewal theory the long-run average cost per unit time is given by

$$C(T)\equiv\frac{\textit{the expected cost incurred in a renewal cycle}}{\textit{the expected length of a renewal cycle}}. \tag{3}$$

Hence the long-run average cost per unit time could be obtained by calculating the expected cost incurred in a renewal cycle and the expected length of a renewal cycle.

Lemma 1.

The long-run average cost per unit time $C(T)$ under the policy T is given by

$$C(T)=\frac{[\int_0^T r(t)\overline{G}(t)dt-G(T)]\nu_1\cdot C_1+\nu_2\cdot C_2-r\int_0^T\overline{G}(t)dt}{\int_0^T\overline{G}(t)dt+[\int_0^T r(t)\overline{G}(t)dt-G(T)]\nu_1+\nu_2}. \tag{4}$$

proof.

From (1), we can see that the expected working time in a renewal cycle is

$$\int_0^T\overline{G}(t)dt$$

and, from (2), the expected value of the total time consumed for minimal repairs in a renewal cycle is

$$[\int_0^T r(t)\overline{G}(t)du-G(T)]\nu_1.$$

Therefore, we can obviously see that the long-run average cost per unit time $C(T)$ is given as (4).■

Then, under the following assumptions, the properties of the optimal replacement policy T_C^* which minimizes $C(T)$ can be obtained.

Assumptions

(i) $E(Y)=\int_0^\infty\overline{G}(t)dt>[(C_2-C_1)\nu_2]/(C_1+r)$

(ii) If $p_0\equiv\inf_{t\geq 0}p(t)$ and $p_1\equiv\sup_{t\geq 0}p(t)$, then $0<p_0\leq p_1<1$

(iii) $(1-p(t))r(t)$ is strictly increasing in t

(iv) $\lim_{t\to\infty}r(t)=\infty$

Theorem 1. Suppose that the Assumptions (i)-(iv) hold. Then there exists the unique optimal T_C^*, and it is the unique solution of the following equation :

$$[(C_1+r)\int_0^{T_C^*}\exp\{-\Lambda_p(t)\}dt-(C_2-C_1)\nu_2]\cdot(1-p(T_C^*))r(T_C^*)\cdot\nu_1$$
$$=(C_1+r)\int_0^{T_C^*}r(t)\exp\{-\Lambda_p(t)\}dt\cdot\nu_1+(C_1+r)\cdot\exp\{-\Lambda_p(T_C^*)\}\cdot\nu_1 \qquad (5)$$
$$+(\nu_2-\nu_1)(C_1+r),$$

where $\Lambda_p(t)\equiv\int_0^t p(u)r(u)du$. Furthermore, if we set

$$T_0\equiv\inf\{T:\int_0^T\overline{G}(t)dt\geq[(C_2-C_1)\nu_2]/(C_1+r)\},$$

then the non-trivial lower bound for the optimal T_C^* is given by T_0, that is, $T_C^*\geq T_0$.

proof.

First of all, observe that the cost rate function $C(T)$ in (4) can be rewritten as

$$C(T)=C_1+\frac{-(C_1+r)\int_0^T\exp\{-\Lambda_p(t)\}dt+(C_2-C_1)\nu_2}{\int_0^T\exp\{-\Lambda_p(t)\}dt+[\int_0^T r(t)\exp\{-\Lambda_p(t)\}dt+\exp\{-\Lambda_p(T)\}-1]\nu_1+\nu_2}.$$

Then we can see that

$$C'(T)=\frac{1}{\eta(T)^2}\cdot\phi_1(T),$$

where

$$\eta(T)\equiv\int_0^T\exp\{-\Lambda_p(t)\}dt+[\int_0^T r(t)\exp\{-\Lambda_p(t)\}dt+\exp\{-\Lambda_p(T)\}-1]\nu_1+\nu_2 \qquad (6)$$

and

$$\phi_1(T)\equiv -(C_1+r)\int_0^T r(t)\exp\{-\Lambda_p(t)\}dt\cdot\exp\{-\Lambda_p(T)\}\cdot\nu_1$$
$$-(C_1+r)\cdot\exp\{-2\Lambda_p(T)\}\cdot\nu_1-(\nu_2-\nu_1)(C_1+r)\exp\{-\Lambda_p(T)\}$$
$$+[(C_1+r)\int_0^T\exp\{-\Lambda_p(t)\}dt-(C_2-C_1)\nu_2]\cdot(1-p(T))r(T)\cdot$$
$$\exp\{-\Lambda_p(T)\}\cdot\nu_1.$$

Let us define $\Psi_1(T)$ as

$$\Psi_1(T)\equiv\phi_1(T)\cdot\exp\{\Lambda_p(T)\}.$$

Then, since $\exp\{\Lambda_p(T)\}>0$ for all $T\geq 0$, we can see that the sign of $\Psi_1(T)$ is the same as that of $\phi_1(T)$. Now observe that

$$\Psi_1(0)=-(C_1+r)\nu_1-(\nu_2-\nu_1)(C_1+r)-(C_2-C_1)\nu_2(1-p(0))r(0)\nu_1<0. \qquad (7)$$

Furthermore, by the definition of T_0 it obviously holds that

$$\Psi_1(T)<0 \text{ for } 0\le T\le T_0, \tag{8}$$

and by that fact that

$$\begin{aligned}\int_0^\infty r(t)\exp\{-\int_0^t p(u)r(u)du\}dt &\le \int_0^\infty r(t)\exp\{-p_0\int_0^t r(u)du\}dt \\ &\le -\frac{1}{p_0}\int_0^\infty -p_0 r(t)\exp\{-p_0\int_0^t r(u)du\}dt \\ &= \frac{1}{p_0},\end{aligned}$$

it also holds that

$$\Psi_1(\infty)\equiv \lim_{T\to\infty}\Psi_1(T)=\infty \tag{9}$$

Also observe that

$$\Psi_1{}'(T)>0 \text{ for all } T>T_0, \tag{10}$$

which means that $\Psi_1(T)$ is strictly increasing when $T>T_0$. Then, from (7), (8), (9) and (10), we can see that there exists a unique $T_C{}^*$ which satisfies

$$\Psi_1(T_C{}^*)=0 \tag{11}$$

and that $\Psi_1(T)<0$ for all $T<T_C{}^*$ and $\Psi_1(T)>0$ for all $T>T_C{}^*$. Or equivalently, $\phi_1(T_C{}^*)=0$ and, $\phi_1(T)<0$ for all $T<T_C{}^*$ and $\phi_1(T)>0$ for all $T>T_C{}^*$. Therefore, the optimal replacement policy is unique and given by the unique solution of the equation (11), and moreover, from (8), we can conclude that $T_C{}^*>T_0$.■

4. Optimal Policy for the Limiting Efficiency

In this section, optimal replacement policy for the limiting efficiency is considered. Let the state of the system be given by the binary variable

$$X(t)=\begin{cases}1 & \text{if } \textit{the system is working at time } t\\ 0 & \textit{otherwise},\end{cases}$$

and the function $U(t)$ is defined by

$$U(t)=\int_0^t X(u)du.$$

Then the limiting efficiency, Eff_∞, is defined by

$$Eff_\infty=\lim_{t\to\infty}\frac{E(U(t))}{t},$$

and it can be interpreted as the long-run expected fraction amount of time system is working, and, by renewal theory, it is given by

$$Eff_{\infty} \equiv \frac{\text{the expected time of working in a renewal cycle}}{\text{the expected length of a renewal cycle}}.$$

For our model, under the replacement policy T, the limiting efficiency $Eff_{\infty}(T)$ is determined by

$$Eff_{\infty}(T) = \frac{\int_0^T \exp\{-\Lambda_p(t)\}dt}{\eta(T)}, \tag{12}$$

where $\eta(T)$ is given by (6). Then, under the Assumptions (i)-(iv) described in section 3, the properties of the optimal replacement policy T_E^* which maximizes $Eff_{\infty}(T)$ can be obtained.

Theorem 2. Suppose that the Assumptions (i)-(iv) hold. Then there exists the unique optimal T_E^*, and it is the unique solution of the following equation :

$$\int_0^{T_E^*} r(t)\exp\{-\Lambda_p(t)\}dt \cdot \nu_1 + \exp\{-\Lambda_p(T_E^*)\} \cdot \nu_1 + (\nu_2 - \nu_1) \tag{13}$$
$$= \int_0^{T_E^*} \exp\{-\Lambda_p(t)\}dt \cdot (1-p(T_E^*))r(T_E^*) \cdot \nu_1.$$

proof.
Observe that

$$Eff_{\infty}'(T) = \frac{1}{\eta(T)^2} \cdot \phi_2(T),$$

where

$$\begin{aligned}\phi_2(T) \equiv & \int_0^T r(t)\exp\{-\Lambda_p(t)\}dt \cdot \exp\{-\Lambda_p(T)\} \cdot \nu_1 \\ & + \exp\{-2\Lambda_p(T)\} \cdot \nu_1 + (\nu_2 - \nu_1) \cdot \exp\{-\Lambda_p(T)\} \\ & - \int_0^T \exp\{-\Lambda_p(t)\}dt \cdot (1-p(T))r(T) \cdot \exp\{-\Lambda_p(T)\} \cdot \nu_1.\end{aligned}$$

Let us define $\Psi_2(T)$ as

$$\Psi_2(T) \equiv \phi_2(T) \cdot \exp\{\Lambda_p(T)\}.$$

Then, as in section 3, we can see that the sign of $\Psi_2(T)$ is the same as that of $\phi_2(T)$. Observe that $\Psi_2(T)$ is given by

$$\begin{aligned}\Psi_2(T) = & \int_0^T r(t)\exp\{-\Lambda_p(t)\}dt \cdot \nu_1 \\ & + \exp\{-\Lambda_p(T)\} \cdot \nu_1 + (\nu_2 - \nu_1) \\ & - \int_0^T \exp\{-\Lambda_p(t)\}dt \cdot (1-p(T))r(T) \cdot \nu_1,\end{aligned}$$

and thus we can see that

$$\Psi_2'(T) = -\int_0^T \exp\{-\Lambda_p(t)\}dt \cdot [(1-p(T))r(T)]'\nu_1.$$

Now note that $\Psi_2(0)=\nu_2>0$, $\Psi_2(\infty)\equiv \lim_{T\to\infty}\Psi_2(T)=-\infty$, and $\Psi_2'(T)<0$ for all $T>0$. Therefore, we can see that there exists a unique T_E^* which satisfies

,
$$\Psi_2(T_E^*)=0 \tag{14}$$

and that $\Psi_2(T)>0$ for all $T<T_E^*$ and $\Psi_2(T)<0$ for all $T>T_E^*$. Or equivalently, $\phi_2(T_E^*)=0$ and, $\phi_2(T)>0$ for all $T<T_E^*$ and $\phi_2(T)<0$ for all $T>T_E^*$. Therefore, the optimal replacement policy is unique and is given by the unique solution of the equation (14).■

5. Multipurpose Replacement Policies

In this section, we derive the properties of multipurpose optimal replacement policies under the Cost-Priority-Criterion and the Efficiency-Priority-Criterion. Throughout this section, we suppose that the Assumptions (i)-(iv) described in section 3 hold.

At first, we are interested in determining the optimal policy T_{CP}^* so that the long-run average cost rate is low enough and the limiting efficiency is maximized. In advance observe that $C(0)\equiv \lim_{T\to 0} C(T)=C_2$ and, since

$$\begin{aligned}
\int_0^\infty r(t)\overline{G}(t)\,dt &= \int_0^\infty r(t)\exp\{-\int_0^t p(u)r(u)du\}dt \\
&\geq \int_0^\infty r(t)\exp\{-\int_0^t p_1 r(u)du\}dt \\
&\geq -\frac{1}{p_1}\int_0^\infty -p_1 r(t)\exp\{-\int_0^t p_1 r(u)du\}dt \\
&= [-\frac{1}{p_1}\exp\{-p_1\int_0^t r(u)du\}]_0^\infty \\
&= \frac{1}{p_1}>1,
\end{aligned}$$

it follows that

$$C(\infty) \equiv \lim_{T\to\infty} C(T) = \frac{[\int_0^\infty r(t)\overline{G}(t)dt - 1]\nu_1 C_1 + \nu_2 C_2 - r\int_0^\infty \overline{G}(t)dt}{\int_0^\infty \overline{G}(t)dt + [\int_0^\infty r(t)\overline{G}(t)dt - 1]\nu_1 + \nu_2}$$

$$= C_2 - \frac{(C_2 - C_1)[\int_0^\infty r(t)\overline{G}(t)dt - 1]\nu_1 + (C_2 + r)\int_0^\infty \overline{G}(t)dt}{\int_0^\infty \overline{G}(t)dt + [\int_0^\infty r(t)\overline{G}(t)dt - 1]\nu_1 + \nu_2}$$

$$< C_2.$$

Furthermore, from the results obtained in section 3, it is true that there exists the unique T_C^* which minimizes $C(T)$. Also observe that $C(T)$ strictly decreases for $T \le T_C^*$ and strictly increases for $T \ge T_C^*$.

Now, let us define a set of $T > 0$ such that,

$$S_1(\alpha) \equiv \{T: C(T) \le \alpha\}, \qquad (15)$$

where $C(T)$ is given by (4) and α, $C(T_C^*) < \alpha < C_2$, is the predetermined upper bound of the long-run average cost rate. Then the optimal policy T_{CP}^* is defined by the value which satisfies

$$Eff_\infty(T_{CP}^*) = \max_{T \in S_1(\alpha)} Eff_\infty(T).$$

The property of the optimal policy T_{CP}^* is given as follows.

Theorem 3. Let T_E^* be the unique solution of the equation (13) and suppose that the predetermined upper bound α satisfies $C(T_C^*) < \alpha < C_2$.

(I) Suppose that $\alpha < C(\infty)$. In this case,

(i) if $T_E^* < T_1$, then $T_{CP}^* = T_1$,

(ii) if $T_1 \le T_E^* \le T_2$, then $T_{CP}^* = T_E^*$,

and (iii) if $T_2 < T_E^*$, then $T_{CP}^* = T_2$,

where $T_1 < T_2$ are the two values satisfying $C(T_1) = C(T_2) = \alpha$.

(II) Suppose that $\alpha \ge C(\infty)$. In this case,

(i) if $T_E^* < T_1$, then $T_{CP}^* = T_1$,

and (ii) if $T_E^* \ge T_1$, then $T_{CP}^* = T_E^*$,

where T_1 is the value satisfying $C(T_1) = \alpha$.

proof.

Note that $\lim_{T\to 0} C(T) = C_2$ and $\lim_{T\to\infty} C(T) = C(\infty) < C_2$. Then the following two separate cases are considered.

Case I : $C(T_C^*) < \alpha < C(\infty)$. In this case observe that there exist two values satisfying $C(T_1)=C(T_2)=\alpha$, $T_1 < T_2$ and $S_1(\alpha)=[T_1, T_2]$. Then by the property of $Eff_\infty(T)$, the results are readily obtained.

Case II : $C(\infty)\le \alpha < C_2$. In this case note that there exist only one value satisfying $C(T_1)=\alpha$ and $S_1(\alpha)=[T_1, \infty)$, which lead to the desired result.■

Now the problem is to determine the optimal policy T_{EP}^* so that the limiting efficiency is high enough and the long-run average cost rate is minimized.

In advance observe that $\lim_{T\to 0} Eff_\infty(T)=0$ and

$$\begin{aligned} Eff_\infty(\infty) &\equiv \lim_{T\to\infty} Eff_\infty(T) \\ &= \frac{\int_0^\infty \overline{G}(t)dt}{\int_0^\infty \overline{G}(t)dt+[\int_0^\infty r(t)\overline{G}(t)dt-1]\nu_1+\nu_2}. \end{aligned}$$

Moreover, from the results obtained in section 4, we know that there exists the unique T_E^* which maximizes $Eff_\infty(T)$. Also observe that $Eff_\infty(T)$ strictly increases for $T\le T_E^*$ and strictly decreases for $T\ge T_E^*$.

As in (15) let us define a set of $T>0$ such that,

$$S_2(\beta)\equiv\{T: Eff_\infty(T)\ge\beta\}, \qquad (16)$$

where $Eff_\infty(T)$ is given by (12) and β, $0<\beta< Eff_\infty(T_A^*)$, is the predetermined lower bound for the limiting efficiency. Then the optimal policy T_{EP}^* is defined by the value which satisfies

$$C(T_{EP}^*)=\min_{T\in S_2(\beta)} C(T).$$

The property of the optimal policy T_{EP}^* is given as follows.

Theorem 4. Let T_C^* be the unique solution of the equation (5) and suppose that the predetermined lower bound β satisfies $0<\beta< Eff_\infty(T_E^*)$.

(I) Suppose that $\beta > Eff_\infty(\infty)$. In this case,

(i) if $T_C^* < T_3$, then $T_{EP}^* = T_3$,

(ii) if $T_3\le T_C^*\le T_4$, then $T_{EP}^* = T_C^*$,

and (iii) if $T_4 < T_C^*$, then $T_{EP}^* = T_4$,

where $T_3 < T_4$ are the two values satisfying $Eff_\infty(T_3)=Eff_\infty(T_4)=\beta$.

(II) Suppose that $\beta\le Eff_\infty(\infty)$. In this case,

(i) if $T_C^* < T_3$, then $T_{EP}^* = T_3$,

and (ii) if $T_C^* \geq T_3$, then $T_{EP}^* = T_C^*$,
where T_3 is the value satisfying $Eff_\infty(T_3) = \beta$.

proof.

Note that $\lim_{T \to 0} Eff_\infty(T) = 0$ and $\lim_{T \to \infty} Eff_\infty(T) = Eff_\infty(\infty) < Eff_\infty(T_E^*)$. Then the following two separate cases are considered.

Case I : $\beta > Eff_\infty(\infty)$. In this case observe that there exist two values satisfying $Eff_\infty(T_3) = Eff_\infty(T_4) = \beta$, $T_3 < T_4$ and $S_2(\beta) = [T_3, T_4]$. Then by the property of $C(T)$, the results can be easily obtained.

Case II : $\beta \leq Eff_\infty(\infty)$. In this case note that there exist only one value satisfying $Eff_\infty(T_3) = \beta$ and $S_2(\beta) = [T_3, \infty)$, which also lead to the desired result.■

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