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Students' Growth of Understanding with Technology Experience from the Perspective of Representation[1)]

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The primary purpose of this study was to investigate how and to what extent 'representations' affect the students' understanding and the growth of understanding in a technology [GSP]-based collegiate mathematics classroom. There are three themes related as frames of the study along with this purpose, which are mentioned in the first chapter and extended in the second chapter: technology in mathematics education; images on computer screen - visualization and representation; understanding and growth of understanding. Three research questions guided this study: 1) How do students present each component of representations when they study 'transformations' in a technology [GSP]-based classroom? If there is any difference between the first and second presentation for each component, how are they different?; 2) How and to what extent do representations affect the students' understanding and the growth of understanding in a technology [GSP]-based classroom?; What types of benefits and obstacles are there when students study 'transformations' in a technology [GSP]-based classroom?

Theoretical backgrounds

Technology in mathematics education Over the past decades much investigation has examined the use of technology in mathematics education. Whereas the technology in the past was basically for simple and repeated computation, a different concept about the use of technology now dominates. For example, calculators are highly developed so that we can really explore the mathematical situation rather than simply find an answer by punching in appropriate numbers. We can graph functions, do statistics, use logarithmic and exponential functions, examine trigonometric functions, and so on, which are sometimes tedious without calculators.

Images on computer screen There can be many different perspectives about the use of technology. Among all those various perspectives, I became very much interested in the effects of images on computer screen to students' understanding and the growth of understanding from three pilot studies. Students spent, maybe had to spend, most of their time with GSP in a unique situation

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so that they could successfully complete class activities, projects, assignments, and even tests. In this environment, they spent the majority of their time working when they were involved in learning transformations. These unique learning environments forced students to lean on technology and complexity of contents let students rely on figures on a computer screen.

On the other hand, there is another notion that is similar to visualization, but that contains visualization as the main body in it, which is representation. There are three elements in it: something written on paper (symbols), physical objects (real objects), and arrangement of idea in one's mind (mental object) (Davis, Young, & McLoughlin, 1982; Janvier, 1987) I rephrased these three terms into written representation, pictorial representation, and oral representation respectively for this study. Visualization generally focuses on external or internal constructions, whereas representation has one more element in addition to these, which is 'symbols'. Here, 'symbols' means specifically mathematical notations for definitions, concepts, or the situations having mathematical notions in it.

Understanding and growth of understanding There are many researchers (Ausubel, 1968; Eisenhart et al., 1993; Fuson, Smith, & Lo Cicero, 1997; Hiebert & Carpenter, 1992; Janvier, 1987; Kiser, 1990; Lehrer & Franke, 1992; Stallings & Tascione, 1996) who put high value on understanding in mathematics education. Understanding is one of the main themes, which appears in many areas of researches. Eisenhart et al. (1993) claimed that "teaching mathematics for understanding is one of the hallmarks of current reform efforts in mathematics teacher education" (p. 8). Further, Hiebert and Carpenter (1992) claimed that the goal of research had been to promote learning with understanding (p. 65). Although we do not have the definition of understanding agreed upon and it is much more complex to understand to understand mathematics than might appear (Byers & Herscovics, 1977, p. 24), there have been efforts to figure out the meaning of understanding. Since Richard Skemp's (1978) 'relational understanding and instrumental understanding', there were researchers who attempted to make the meaning of understanding precise (Backhouse, 1978; Buxton, 1977; Byers & Herscovics, 1977).

Methodology

Two college students, Abbey and Emily, were purposefully chosen and voluntarily participated for this study. They had successfully finished the first college geometry course, were able to express what they were doing, and were typical age as college juniors. Both of them liked mathematics and their attitudes were positive toward mathematics. At the time of this study, they were very skillful with the use of GSP. Qualitative methodology was used, especially the case study, which is the most powerful and appropriate method "for intensive, in depth examination" (Goetz & LeCompte, 1984, p. 46). I observed the course, MATH 5210, for 12 weeks in Spring, 2001 and each participant was interviewed nine times. All interviews were audiotaped and videtaped, further all interviews were transcribed for an intensive data analysis. As for data collection, there were five types of methods: descriptive notes, reflective notes, archival data, interviews, and concept maps. The collected data were analyzed using 'constant comparison method' along with analytic induction.

Findings and conclusions

Two diagnostic tests (one in the beginning and the other at the end of this study) were given to each participant in order to compare their representations presented in three different ways: written, pictorial, and verbal. Along with the two diagnostic tests, Abbey and Emily made concept maps in the beginning and at the end of study. Concepts first selected by the researcher and then revised by the instructor were provided so that the two participants could refer to them as they wished.

1. How do students present each component of representations when they study 'transformations' in a technology [GSP]-based classroom? If there is any difference between the first and second presentation for each component, how are they different?

Written representation Although both of them improved their written representations in the second diagnostic test, they did not use written representations much throughout the course. Unless they were asked to use written representations, their reports were in general explanations in words about their discoveries or figures that they constructed.

Pictorial representation Considering the design of the course observed for this study, i.e., there was no textbook used and computer with GSP software was the main tool for studying transformation, we can easily expect the pictorial representation to play an significantly important role throughout the course.

Verbal representation In general, verbal representations came along with pictorial representations. I found that there were two types of verbal representations. One was the direct memorization of a written version of a definition and the other was simply a verbal explanation of pictorial representations or virtual doings in their head. When Abbey and Emily simply recited the definitions that the instructor provided, their concepts were comparatively unstable. After students understood a concept in their heads or/and eyes (written or/and pictorial), they confirmed their understanding by speaking out to themselves or others.

2. How and to what extent do representations affect the students' understanding and the growth of understanding in a technology [GSP]-based classroom?

Except understanding basic concepts of each transformation, it was very hard, due to the complexity of contents, for students to visualize almost all those assignments and class activities in their head. But, because of the technology, GSP, they were using at the moment, they leaned on it whenever they needed. Even for those basics, they depended on GSP for a closer look to some extent. Following the well-organized guidance of the instructor, students could build strong written and pictorial representations at least for each transformation and then the verbal representation came along afterwards. Both Abbey and Emily were able to see each transformation as a unique mathematical being which has a specific mathematical property. Especially each transformation's written representation was often used for extension or for proving their conjectures in the following assignments. Although their basic understanding strongly stood alone, they had to revisit each concept in the process of applying it to new situations. Further, the lack of interpreting the new situation sometimes resulted in their setting up inappropriately, which led them nowhere after hard work. The journey of students' growth began with the harmony of these three factors.

3. What types of benefits and obstacles are there when students study 'transformations' in a technology [GSP]-based classroom?

Obstacles First, images on computer screen restrict learners' logical thinking. Although students constructed basic concepts of each transformation in three different representations, their explorations were dominated by pictorial representations. Second, images on computer screen might lead learners nowhere or to misconception. Stable understanding of basic concepts did not guarantee the success in exploring the mathematical situation.

Benefits First, learners could make an accurate, quick, and solid conjecture with technology and images. The accuracy and quickness that technology brought are the phenomenal features that facilitated students' explorations. Second, the dynamic function of technology plays a significant role. There are many useful functions that GSP provides. In the process of exploring transformations, the dynamic function of GSP was most useful. They could clearly see what was happening as they move figures or defining data. Third, well-constructed figures lead learners to efficient problem solving and psychological relief. GSP provides students with a strong belief and comfort about accuracy as long as it was constructed according to the right process. This gives psychological relief about their finding.

Implications for future research

The perspectives to examine current mathematics classrooms relating to technology may vary. Indeed, we need multiple perspectives. This study mainly focused on geometrical figures on computer screens constructed by the specific software, The Geometer's Sketchpad (GSP). It is still questionable if technology has contributed to enhancing students' understanding in mathematics education as much as it has to the current society, although NCTM (2000) claimed that "technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances students' learning" (p. 24). We do not have enough information to confirm the above statement is true. But, we do have hope and potentials that technology can be efficiently used for enhancing students' understanding mathematics. Teachers, parents, students, administrators, policy makers, and researchers can work together as one group in developing a new curriculum, creating a new structure of contents, etc.

Technology appears in many educational fields let alone mathematics education. As society has achieved many things for human being's better life, technology could contribute to students' better understanding in mathematics education. Now, the issue is not what we have in our hands, rather how we use what we have. I believe the value of technology in education will be heightened by how it is used for students' better understanding not by what we have. The spread of technology into classrooms in increasing numbers will not raise the value of technology in education. But, it is great to have enough and various tools in our mathematics classroom.

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