Design Criteria and Performance of Space-Frequency Bit-Interleaved Coded Modulations in Frequency-Selective Rayleigh Fading Channels

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Abstract: **In this paper, we investigate design criteria and the performance of the *space-frequency bit-interleaved coded modulation* (SF-BICM) systems in frequency-selective Rayleigh fading channels. To determine the key parameters that affect the performance of SF-BICM, we derive the *pairwise error probability* (PEP) in terms of the determinant of the matrix corresponding to any two codewords. We prove that the bit-interleavers do the function of distributing the nonzero bits uniformly such that two or more nonzero bits are seldom distributed into the symbols that are transmitted in the same frequency bin. This implies that the bit-interleavers transform an SF-BICM system into an equivalent 1-antenna system. Based on this, we present design criteria of SF-BICM systems that maximizes the diversity order and the coding gain. Then, we analyze the performance of SF-BICM for the case of 2-transmit antennas and 2-multipaths by deriving a *frame error rate* (FER) bound. The derived bound is accurate and requires only the distance spectrum of the constituent codes of SF-BICM. Numerical results reveal that the bound is tight enough to estimate the performance of SF-BICM very accurately.**

Index Terms: **Space-time codes, bit-interleaved coded modulation, orthogonal frequency division multiplexing, frequency-selective fading, distance spectrum.**

I. INTRODUCTION

Space-time codes have been introduced as an effective means to increase wireless channel capacity by using multiple antennas in frequency-flat fading channels [1]. Since the usual wireless channel is frequency-selective, space-time codes are not applicable directly to real channels. *Orthogonal frequency division multiplexing* (OFDM) can render a good solution to this problem, as it transforms a frequency-selective channel into parallel correlated frequency-flat channels. There have been reported a large amount of works on the space-time coded OFDM [2]–[4].

Independently of this, *Bit-interleaved coded modulation* (BICM) was introduced as a means of improving the performance of coded modulation over fading channels [5]. It makes the code diversity equal to the smallest number of distinct bits (rather than channel symbols), and offers much better trade-offs between code diversity and trellis complexity than *trellis coded modulation* (TCM) does. The concept of BICM, when it is applied to multiple transmit antenna environment, yields the *space-time bit-interleaved coded modulation* (ST-BICM) [6]–[11]. As BICM divides the code design process into encoder

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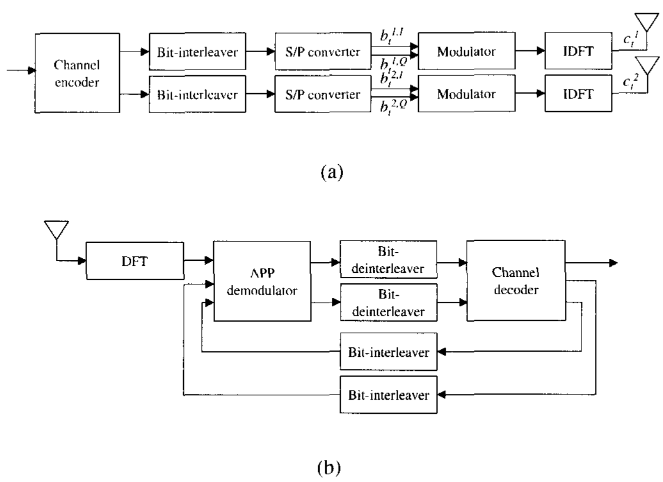

Fig. 1. The block diagram of SF-BICM: (a) Transmitter, (b) receiver.

selection and modulation scheme selection processes, the code design with ST-BICM becomes simpler than that with the standard space-time code.

Biglieri *et al.* [6] analyzed the information theoretic limit of ST-BICM, and showed that ST-BICM achieves ergodic and outage capacities close to those of the general coded modulation. Recently, Park *et al.* [7], [8] showed the desirable property of ST-BICM that it is highly probable that there does not exist overlap of non-zero symbols among the transmitted antennas, which implies that the signal transmitted in each antenna does not interfere with each other. This property enables a simplified performance analysis of ST-BICM systems.

In this paper, we integrate the three components -- space-time codes, interleavers, and OFDM -- to build a new system called *space-frequency BICM* (SF-BICM) and then analyze its performance. To determine the key parameters that affect the performance of SF-BICM, we first derive the *pairwise error probability* (PEP) in terms of the determinant of the distance matrix corresponding to any two codewords. To investigate the effects of the bit-interleavers on those parameters, we prove that they distribute the nonzero bits uniformly such that two or more nonzero bits are seldom distributed into the symbols that are transmitted in the same frequency bin. This enables us to determine the performance of SF-BICM based on that of the equivalent 1-antenna system. In addition, we present simple design criteria of SF-BICM systems that can maximize the diversity order and the coding gain of the system.

This paper is organized as follows: To begin with, we describe the system model in Section II. Then, we derive the PEP in terms of the determinant of the matrix corresponding to any two codewords and, based on this, we present design criteria of SF-BICM systems in Section III. Finally, in Section IV, we derive a new upper bound of the frame error probability and demonstrate by simulations that the bound is tight enough to estimate the performance of SF-BICM directly.

II. SYSTEM MODEL

We consider a baseband communication system with n_T transmit antennas and n_R receive antennas. Fig. 1 (a) shows the block diagram of the transmitter of SF-BICM for the case of $n_T = 2$. The transmitted data are encoded by a rate-$1/n_T$ binary linear channel code, such as a convolutional code or a turbo code. We regard the rate-$1/n_T$ code as n_T rate-1 component codes whose outputs are transmitted over single transmit antenna to avoid the event that all the outputs of the component encoders experience the same fading [12]. n_T independent interleavers bit-interleave n_T encoded sequences separately, and the interleaved sequences are applied to a *serial-to-parallel* (S/P) converter that produces two parallel data sequences. The data sequences are then mapped into QPSK symbols based on the Gray mapping rule. The elements of the signal constellation are contracted such that the average energy of the constellation becomes 1. We define a *space-frequency codeword matrix* of size $n_T \times L$, obtained by arranging the transmitted sequence in an array, as

$$\boldsymbol{c} = \begin{bmatrix} c_1^1 & c_2^1 & \cdots & c_L^1 \\ c_1^2 & c_2^2 & \cdots & c_L^2 \\ \vdots & \vdots & \ddots & \vdots \\ c_1^{n_T} & c_2^{n_T} & \cdots & c_L^{n_T} \end{bmatrix}, \tag{1}$$

for which the ith row $\boldsymbol{c}^i \equiv [c_1^i \; c_2^i \; \cdots \; c_L^i]$ is the data sequence transmitted from the ith transmit antenna, and the kth column $\boldsymbol{c}_k \equiv [c_k^1 \; c_k^2 \; \cdots \; c_k^{n_T}]^T$ is the space-frequency symbol at the kth subcarrier. In the equation, L denotes the number of subcarriers in one OFDM frame. The *inverse discrete Fourier transform* (IDFT) is applied to the symbols before transmission at each antenna. We assume that by using a *cyclic prefix* (CP), the orthogonality of subcarriers is preserved and the intersymbol interference (ISI) between consecutive OFDM symbols is eliminated [13] at the same time.

In this paper, we assume that the channel is modeled as quasi-static and the impulse response from transmit antenna i to receive antenna l is

$$g^{i,l}(\tau) = \sum_{m=1}^{M} h_m^{i,l}\delta(\tau - n_m), \tag{2}$$

where M denotes the number of multipaths and $h_m^{i,l}$'s are the independent complex Gaussian random variables corresponding to the integer delays n_m's, normalized such that

$$\sum_{m=1}^{M} E\left[\left|h_m^{i,l}\right|^2\right] = 1, \quad \text{for all } i, l, \tag{3}$$

where $E[\cdot]$ denotes the expectation operation.

At the receiver, the *discrete Fourier transform* (DFT) is applied to the received signals to obtain the following output for the kth subcarrier and the lth receive antenna

$$r_k^l = \sqrt{E_s}\sum_{i=1}^{n_T} \alpha_k^{i,l} c_k^i + \eta_k^l, \tag{4}$$

where E_s denotes the energy per symbol and η_k^l the noise component of the receive antenna l at subcarrier k, which is an independent sample of the zero-mean complex Gaussian random variable with variance $\frac{N_o}{2}$ per dimension. The coefficient $\alpha_k^{i,l}$ is the fading attenuation for the kth subcarrier from transmit antenna i to receive antenna l, having the expression

$$\alpha_k^{i,l} = \boldsymbol{h}^{i,l}\boldsymbol{w}_k, \tag{5}$$

for [1]

$$\boldsymbol{h}^{i,l} \equiv \left[h_1^{i,l} \; h_2^{i,l} \; \cdots \; h_M^{i,l}\right], \tag{6a}$$

$$\boldsymbol{w}_k \equiv \left[e^{-\frac{j2\pi n_1 k}{L}} \; e^{-\frac{j2\pi n_2 k}{L}} \; \cdots \; e^{-\frac{j2\pi n_M k}{L}}\right]^T. \tag{6b}$$

For decoding, we adopt the iterative demodulation-decoding method in [14] (see Fig. 1 (b)). The *a posteriori probability* (APP) demodulator generates the *log likelihood ratio* (LLR) of channel-encoded bits using noise statistics. The LLR's are deinterleaved and transferred to the channel decoder, such as BCJR decoder [15] or SOVA decoder [16]. The decoder outputs are then re-interleaved and fed back to the APP demodulator. LLR's are iteratively interchanged between the demodulator and the decoder to successively improve the error performance.

III. DESIGN CRITERIA FOR SF-BICM SYSTEMS

SF-BICM systems are composed of a serial concatenation of a linear channel coder, bit-interleavers, modulators, and OFDM as indicated in Fig. 1. Since we employ uniform bit-interleavers, QPSK modulators, and conventional OFDM, the remaining design task is the choice of channel codes. For a design guideline and performance analysis of SF-BICM, we need to analyze the effects of codewords of channel codes on the system performance. So, we first derive an upper bound of the PEP for two given codewords. Then we investigate the property of the distance matrix that is related to the PEP bound, and, based on this, we establish design criteria for SF-BICM systems.

A. Pairwise Error Probability

PEP is the probability that the decoder selects a sequence $\hat{\boldsymbol{c}}$ as an estimate of the transmitted sequence $\boldsymbol{c}$. If an ideal *channel state information* (CSI) is available at the receiver, the PEP takes the expression [13]

$$P_e(\boldsymbol{c}, \hat{\boldsymbol{c}}) = E\left[Q\left(\sqrt{\frac{\gamma_s}{2} d^2(\boldsymbol{c}, \hat{\boldsymbol{c}})}\right)\right], \tag{7}$$

for the tail probability of the Gaussian probability density function $Q(y) \equiv \frac{1}{\sqrt{2\pi}}\int_y^\infty e^{-\frac{1}{2}x^2}dx$, the SNR per symbol $\gamma_s \equiv \frac{E_s}{N_0}$,

[1] Note that j in (6b) denotes $\sqrt{-1}$.

and the squared modified Euclidean distance

$$d^2(\boldsymbol{c}, \hat{\boldsymbol{c}}) = \sum_{l=1}^{n_R}\sum_{k=1}^{L}\left|\sum_{i=1}^{n_T}\alpha_k^{i,l}(c_k^i - \hat{c}_k^i)\right|^2 = \sum_{l=1}^{n_R}\sum_{k=1}^{L}\left|\boldsymbol{\alpha}_k^l\boldsymbol{e}_k\right|^2, \quad (8)$$

for

$$\boldsymbol{\alpha}_k^l \equiv \left[\alpha_k^{1,l}\ \alpha_k^{2,l}\ \cdots\ \alpha_k^{n_T,l}\right], \quad (9a)$$

$$\boldsymbol{e}_k \equiv \left[e_k^1\ e_k^2\ \cdots e_k^{n_T}\right]^T, \quad (9b)$$

where $e_k^i \equiv c_k^i - \hat{c}_k^i$. From (5), we obtain [4]

$$\boldsymbol{\alpha}_k^l = \boldsymbol{h}_l\left(I_{n_T} \otimes \boldsymbol{w}_k\right), \quad (10)$$

where $\boldsymbol{h}_l \equiv \left[\boldsymbol{h}^{1,l}\ \boldsymbol{h}^{2,l}\ \cdots\ \boldsymbol{h}^{n_T,l}\right]$, I_{n_T} is the $n_T \times n_T$ identity matrix, and $\otimes$ denotes the Kronecker product [17]. If we insert (10) to (8), we get

$$d^2(\boldsymbol{c}, \hat{\boldsymbol{c}}) = \sum_{l=1}^{n_R}\tilde{\boldsymbol{h}}_l\boldsymbol{Q}\tilde{\boldsymbol{h}}_l^H, \quad (11)$$

for [2]

$$\boldsymbol{Q} \equiv R_h^{1/2}\sum_{k=1}^{L}\left(I_{n_T} \otimes \boldsymbol{w}_k\right)\boldsymbol{e}_k\boldsymbol{e}_k^H\left(I_{n_T} \otimes \boldsymbol{w}_k\right)^H R_h^{1/2}, \quad (12a)$$

$$R_h \equiv E[\boldsymbol{h}_l^H\boldsymbol{h}_l], \quad (12b)$$

$$\tilde{\boldsymbol{h}}_l \equiv \boldsymbol{h}_l R_h^{-1/2}. \quad (12c)$$

The matrix $\boldsymbol{Q}$ plays the same role of the codeword distance matrix in [7], [18], so it can be called the *generalized codeword distance matrix.*

Let δ_H denote the *symbol-wise Hamming distance*, i.e., the number of subcarriers $k = 1, 2, \cdots, L$, such that $\boldsymbol{e}_k \neq \boldsymbol{0}$. Then, the rank of $\boldsymbol{Q}$, or Δ_H, is bounded above by $\min\{\delta_H, n_T M\}$, since the rank of $\boldsymbol{e}_k\boldsymbol{e}_k^H$ in (12a) is 0 or 1. Let λ_i, $i = 1, 2, \cdots, \Delta_H$, be the positive eigenvalues of $\boldsymbol{Q}$. Then $d^2(\boldsymbol{c}, \hat{\boldsymbol{c}})$ can be rewritten as

$$d^2(\boldsymbol{c}, \hat{\boldsymbol{c}}) = \sum_{i=1}^{\Delta_H}\sum_{l=1}^{n_R}\lambda_i|\beta^{i,l}|^2, \quad (13)$$

where $\beta^{i,l}$'s are independent complex Gaussian random variables with a zero mean and the variance of 1/2 per dimension. If we insert (13) to (7) and apply the alternative form of $Q(y) = 1/\pi\int_0^{\pi/2}e^{-y^2/2\sin^2\theta}d\theta$, we can calculate the PEP in the form [18]–[20]

$$P_e(\boldsymbol{c}, \hat{\boldsymbol{c}}) = \frac{1}{\pi}\int_0^{\pi/2}\prod_{i=1}^{\Delta_H}\left(1 + \frac{\lambda_i\gamma_s}{4\sin^2\theta}\right)^{-n_R}d\theta, \quad (14)$$

which implies that the diversity order is $\Delta_H n_R$ [18]. Since it is not an easy task to find the eigenvalues of $\boldsymbol{Q}$ corresponding to a pairwise error event for a given channel code, we, instead, use a PEP upper bound that depends only on the product of the eigenvalues, not on the individual eigenvalues. Let Δ_p be the product of all the eigenvalues, i.e.,

$$\Delta_p \equiv \prod_{i=1}^{\Delta_H}\lambda_i. \quad (15)$$

If we replace each λ_i with the geometrical mean Δ_p^{1/Δ_H}, we can obtain a tight upper bound to the PEP as follows: [18], [19]

$$P_B(\Delta_H, \Delta_p) = \frac{1}{\pi}\int_0^{\pi/2}\left(1 + \frac{\Delta_p^{1/\Delta_H}\gamma_s}{4\sin^2\theta}\right)^{-\Delta_H n_R}d\theta = J_{(\Delta_H n_R)}\left(\frac{\Delta_p^{1/\Delta_H}\gamma_s}{4}\right), \quad (16)$$

where

$$J_m(c) \equiv [P(c)]^m\sum_{k=0}^{m-1}\binom{m-1+k}{k}[1 - P(c)]^k, \quad (17)$$

for a positive integer m and $P(x) \equiv \frac{1}{2}\left(1 - \sqrt{\frac{x}{1+x}}\right)$, $x \geq 0$. This corresponds to the PEP for the case when one transmit antenna and $\Delta_H n_R$ receive antennas are used in the frequency-flat channel and the squared Euclidean distance between two codewords is Δ_p^{1/Δ_H}. This interpretation gives an insight into the effects of transmit diversity and receive diversity in frequency-selective fading channels. The highest possible diversity order is the product of the number of transmitter antennas, the number of receiver antennas, and the number of multipaths.

If the channel is frequency-flat (i.e., $M = 1$), then $\boldsymbol{w}_k = 1$ and $R_h = I_{n_T}$, so (12a) becomes

$$\boldsymbol{Q} = \sum_{k=1}^{L}\boldsymbol{e}_k\boldsymbol{e}_k^H = (\boldsymbol{c} - \hat{\boldsymbol{c}})(\boldsymbol{c} - \hat{\boldsymbol{c}})^H, \quad (18)$$

which is the case investigated in detail in [7].

B. Distance Matrix

As shown in Fig. 1 (a), the output sequence from each component code of the channel code is bit-interleaved and applied to an S/P converter that produces two parallel bit sequences $b_k^{i,I}$'s and $b_k^{i,Q}$'s, $i = 1, 2, \cdots, n_T$, where i indicates the ith component code. So the S/P-converted bit sequences may be denoted in matrix form by

$$\boldsymbol{\beta} = \left[\ \boldsymbol{b}_1\ \ \boldsymbol{b}_2\ \ \cdots\ \ \boldsymbol{b}_k\ \ \cdots\ \ \boldsymbol{b}_L\ \right] \in \mathbf{Z}_2^{2n_T \times L}, \quad (19)$$

for

$$\boldsymbol{b}_k = \left[\ b_k^{1,I}\ b_k^{1,Q}\ b_k^{2,I}\ b_k^{2,Q}\ \cdots\ b_k^{n_T,I}\ b_k^{n_T,Q}\ \right]^T \in \mathbf{Z}_2^{2n_T}, \quad k = 1, 2, \cdots, L, \quad (20)$$

where $\mathbf{Z}_2$ denotes a binary group comprised of 0 and 1. The modulator at the last stage maps $\boldsymbol{b}_k$ into a space-frequency symbol $\boldsymbol{c}_k$ based on the Gray mapping rule, i.e.,

$$c_k^i = \frac{1}{\sqrt{2}}\left\{(-1)^{b_k^{i,I}} + j(-1)^{b_k^{i,Q}}\right\}. \quad (21)$$

[2]Note that R_h is invertible as we assume $h_m^{i,l}$'s are independent random variables.

The distance matrix $\boldsymbol{D}_k$ between two space-frequency symbols is

$$\boldsymbol{D}_k \equiv \boldsymbol{e}_k \boldsymbol{e}_k^H = \mathrm{diag}\{|e_k^1|^2, |e_k^2|^2, \cdots, |e_k^{n_T}|^2\}, \tag{22}$$

for

$$\begin{aligned} |e_k^i|^2 &= \frac{1}{2}\left\{\left|1-(-1)^{b_k^{i,I}\oplus \hat{b}_k^{i,I}}\right|^2 + \left|1-(-1)^{b_k^{i,Q}\oplus \hat{b}_k^{i,Q}}\right|^2\right\} \\ &= 2d_H\left([b_k^{i,I}\ b_k^{i,Q}], [\hat{b}_k^{i,I}\ \hat{b}_k^{i,Q}]\right), \end{aligned} \tag{23}$$

where $\oplus$ denotes the exclusive OR operation and $d_H(\boldsymbol{x},\boldsymbol{y})$ the Hamming distance between $\boldsymbol{x}$ and $\boldsymbol{y}$. Note that $\boldsymbol{e}_k\boldsymbol{e}_k^H(=\boldsymbol{D}_k)$ in (12a) characterizes the eigenvalues of $\boldsymbol{Q}$. The number of positive eigenvalues of $\boldsymbol{Q}$ (i.e., the rank of $\boldsymbol{Q}$) is less than or equal to δ_H, the number of nonzero $\boldsymbol{D}_k$'s. The distance matrix $\boldsymbol{D}_k$ does not depend on the transmitted codewords but depends only on the Hamming distance. So we can analyze the performance assuming that the space-frequency codeword matrix corresponding to an all-zero codeword is transmitted, without loss of generality.

We assume that the Hamming distance of the output of the ith component code of the channel code is d_i, and define $\boldsymbol{d} \equiv [d_1\ d_2\ \cdots\ d_{n_T}]$ and $\mathrm{sum}(\boldsymbol{d}) \equiv \sum_{i=1}^{n_T} d_i$. By definition, δ_H is equal to the number of nonzero $\boldsymbol{D}_k$'s, that is, the number of nonzero $d_H(\boldsymbol{b}_k, \hat{\boldsymbol{b}}_k)$'s, because $\mathrm{Tr}(\boldsymbol{D}_k) = 2d_H(\boldsymbol{b}_k, \hat{\boldsymbol{b}}_k)$, where $\mathrm{Tr}(\cdot)$ denotes the matrix trace operation. So, if we consider the relation [3]

$$\sum_{k=1}^{L} d_H(\boldsymbol{b}_k, \hat{\boldsymbol{b}}_k) = \mathrm{sum}(\boldsymbol{d}), \tag{24}$$

we find that the maximum number of the nonzero $d_H(\boldsymbol{b}_k, \hat{\boldsymbol{b}}_k)$'s is $\mathrm{sum}(\boldsymbol{d})$. This maximum is obtained, or equivalently, δ_H takes on the maximum value $\mathrm{sum}(\boldsymbol{d})$, if and only if there does not exist any k that makes $d_H(\boldsymbol{b}_k, \hat{\boldsymbol{b}}_k) \geq 2$.

The interleaver has a property which distributes the nonzero bits uniformly such that two or more nonzero bits are seldom distributed into the same column of $\boldsymbol{\beta}$ in (19). This property makes the symbol-wise Hamming distance δ_H maximized with a high probability for a given Hamming distance. This is desirable because the diversity order of PEP is then maximized for a given Hamming distance with a high probability. We can verify this property with the aid of the following theorem.

Theorem 1: For a given $\boldsymbol{d}$ and a frame length L, the probability that δ_H does not take on sum($\boldsymbol{d}$) is $O(L^{-1})$. Equivalently, the probability that there exists a k such that $d_H(\boldsymbol{b}_k, \hat{\boldsymbol{b}}_k) \geq 2$ is $O(L^{-1})$. [4]

Proof: If there exists a k such that $d_H(\boldsymbol{b}_k, \hat{\boldsymbol{b}}_k) \geq 2$, then δ_H takes on a value less than sum($\boldsymbol{d}$) and vice versa. This implies that the event $\delta_H \neq$sum($\boldsymbol{d}$) is equivalent to the complementary event of $d_H(\boldsymbol{b}_k, \hat{\boldsymbol{b}}_k) < 2$ for all k. According to the derivation given in Appendix, the probability that $d_H(\boldsymbol{b}_k, \hat{\boldsymbol{b}}_k) < 2$ for all k is

$$\Pr\{d_H(\boldsymbol{b}_k, \hat{\boldsymbol{b}}_k) < 2,\ k = 1, 2, \cdots, L\} = \prod_{i=1}^{n_T} \frac{\binom{L-\sum_{j=1}^{i-1} d_j}{d_i} 2^{d_i}}{\binom{2L}{d_i}}, \tag{25}$$

so, the probability of the complementary event is

$$\begin{aligned} \Pr\{\delta_H \neq \mathrm{sum}(\boldsymbol{d})\} &= 1 - \prod_{i=1}^{n_T} \frac{\binom{L-\sum_{j=1}^{i-1} d_j}{d_i} 2^{d_i}}{\binom{2L}{d_i}} \\ &= 1 - \prod_{i=1}^{n_T} \prod_{l=0}^{d_i-1} \left(1 - \frac{2\sum_{j=1}^{i-1} d_j + l}{2L - l}\right). \end{aligned} \tag{26}$$

If we apply the following inequality in [8] to (26)

$$1 - \sum_{i=1}^{N} x_i \leq \prod_{i=1}^{N} (1 - x_i), \quad 0 \leq x_i \leq 1, \tag{27}$$

we get

$$\Pr\{\delta_H \neq \mathrm{sum}(\boldsymbol{d})\} \leq 1 - \prod_{i=1}^{n_T} \left(1 - \sum_{l=0}^{d_i-1} \frac{2\sum_{j=1}^{i-1} d_j + l}{2L - l}\right). \tag{28}$$

Applying (27) to (28) again and putting $d_{max} \equiv \max_i\{d_i\}$, we get

$$\begin{aligned} \Pr\{\delta_H \neq \mathrm{sum}(\boldsymbol{d})\} &\leq \sum_{i=1}^{n_T} \sum_{l=0}^{d_i-1} \frac{2\sum_{j=1}^{i-1} d_j + l}{2L - l} \\ &\leq \frac{n_T(n_T-1)d_{max}^2 + n_T d_{max}(d_{max}-1)/2}{2L - d_{max} + 1}, \end{aligned} \tag{29}$$

which is of order $O(L^{-1})$. This completes the proof. □

[3]Note that the left-hand side of the equation is the sum of the number of nonzero bits in each column of $\boldsymbol{\beta}$ and the right-hand side is the sum of the number of nonzero bits in each row of $\boldsymbol{\beta}$.

[4]We write $f(x) = O(g(x))$ if there exist positive constants M and x_0 such that $|f(x)| < Mg(x)$ for all $x > x_0$.

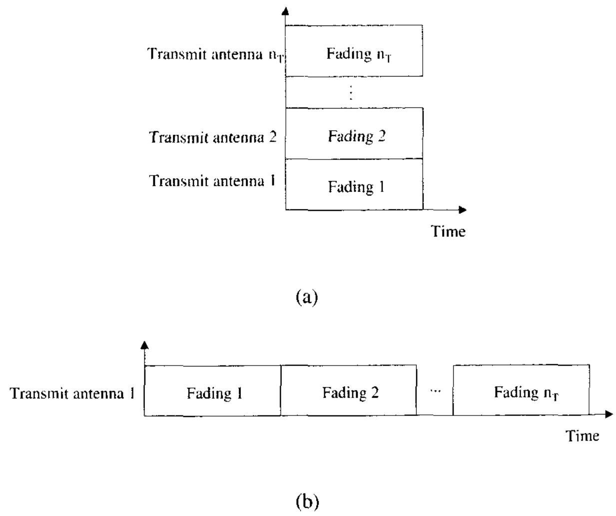

Fig. 2. Fading channels: (a) Transmit diversity channels, (b) equivalent fading channels.

The probability that δ_H does not take on sum($\boldsymbol{d}$) is inversely proportional to the frame length L. Since sum($\boldsymbol{d}$) is the maximum value that δ_H can take, Theorem 1 implies that the probability of the maximum symbol-wise Hamming distance approaches 1 as L becomes large.

If $d_H(\boldsymbol{b}_k, \hat{\boldsymbol{b}}_k) < 2$ for all k, then the PEP would be the same as the case when the signal transmitted in each antenna does not interfere each other. So if the frame length is sufficiently long, then $d_H(\boldsymbol{b}_k, \hat{\boldsymbol{b}}_k)$ becomes less than 2 with a probability close to 1. This implies that the bit-interleavers transform the n_T-antenna space-frequency code into an equivalent 1-antenna system where n_T IDFT-output sequences are transmitted serially (i.e., not simultaneously).

Fig. 2 depicts the transmit diversity channels in comparison with the equivalent fading channels. In the case of the transmit diversity channels, a superposition of n_T transmitted signals is received at each receive antenna. In contrast, in the case of the equivalent fading channels, n_T sequentially transmitted signals are received at each receive antenna and the throughput reduces by the ratio $1/n_T$. Based on this transmit diversity channel-to-equivalent fading channel transformation, we can predict the performance of SF-BICM from that of the equivalent 1-antenna system.

C. Design Criteria

Theorem 1 guarantees that $d_H(\boldsymbol{b}_k, \hat{\boldsymbol{b}}_k) < 2$ for all k with a high probability. If $d_H(\boldsymbol{b}_k, \hat{\boldsymbol{b}}_k) = 1$, the distance matrix $\boldsymbol{D}_k$ in (22) becomes diag$\{0, \cdots, 0, 2, 0, \cdots, 0\}$, that is, only one diagonal element takes on a nonzero value. Let K_i denote the set of instance k such that $|e_k^i|^2 = 2$. If we assume $d_H(\boldsymbol{b}_k, \hat{\boldsymbol{b}}_k) < 2$ for all k, then $|K_i| = d_i$, where $|K_i|$ denotes the number of elements in set K_i.

If we rewrite (12a) in matrix form, we get

$$\boldsymbol{Q} = R_h^{1/2} \begin{bmatrix} \boldsymbol{Q}_1 & \boldsymbol{0} & \cdots & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{Q}_2 & \cdots & \boldsymbol{0} \\ \vdots & \vdots & \ddots & \vdots \\ \boldsymbol{0} & \boldsymbol{0} & \cdots & \boldsymbol{Q}_{n_T} \end{bmatrix} R_h^{1/2}, \tag{30a}$$

for

$$\boldsymbol{Q}_i \equiv 2 \sum_{k \in K_i} \boldsymbol{w}_k \boldsymbol{w}_k^H. \tag{30b}$$

The diversity order is determined by Δ_H, or the rank of $\boldsymbol{Q}$, and Δ_H can be determined by evaluating the rank of $\boldsymbol{Q}_i$ as follows:

Theorem 2: For a given number of multipaths M, and the number of elements in set K_i, the generalized codeword distance matrix $\boldsymbol{Q}$ has the rank

$$\Delta_H = \sum_{i=1}^{n_T} \min(|K_i|, M). \tag{31}$$

Proof: Since R_h in (30a) is invertible, the rank of $\boldsymbol{Q}$ is the same as that of $R_h^{-1/2} \boldsymbol{Q} R_h^{-1/2}$. Since the eigenvalues of $R_h^{-1/2} \boldsymbol{Q} R_h^{-1/2}$ are the roots of the characteristic equation $\det(\lambda I_{n_T M} - R_h^{-1/2} \boldsymbol{Q} R_h^{-1/2}) = \prod_{i=1}^{n_T} \det(\lambda I_M - \boldsymbol{Q}_i) = 0$, the set of the eigenvalues of $R_h^{-1/2} \boldsymbol{Q} R_h^{-1/2}$ is the union of the sets of the eigenvalues of $\boldsymbol{Q}_i$, $i = 1, 2, \cdots, n_T$. Therefore, the number of positive eigenvalues of $R_h^{-1/2} \boldsymbol{Q} R_h^{-1/2}$ is equal to the sum of the number of positive eigenvalues in each $\boldsymbol{Q}_i$, $i = 1, 2, \cdots, n_T$. Therefore, to prove the theorem, it suffices to prove that rank $\boldsymbol{Q}_i = \min(|K_i|, M)$.

We rewrite $\boldsymbol{Q}_i$ in (30b) as

$$\boldsymbol{Q}_i = 2 \boldsymbol{W}_i \boldsymbol{W}_i^H, \tag{32}$$

where $\boldsymbol{W}_i \equiv \left[\boldsymbol{w}_{k_1} \boldsymbol{w}_{k_2} \cdots \boldsymbol{w}_{k_{|K_i|}} \right]$ for the elements $k_1, k_2, \cdots, k_{|K_i|}$ of set K_i. If we carry out row and column permutation on the $L \times L$ DFT matrix $\boldsymbol{F}$, we can get the permuted matrix $\tilde{\boldsymbol{F}}$

$$\tilde{\boldsymbol{F}} = \begin{bmatrix} \boldsymbol{W}_i & \boldsymbol{S} \\ \boldsymbol{T} & \boldsymbol{U} \end{bmatrix}, \tag{33}$$

where $\boldsymbol{S}$, $\boldsymbol{T}$ and $\boldsymbol{U}$ consist of the remaining elements of the DFT matrix $\boldsymbol{F}$. Since $\boldsymbol{F}$ has full rank, the permuted version $\tilde{\boldsymbol{F}}$ also has full rank. $\boldsymbol{W}_i$ is a submatrix of a leading principal submatrix of $\tilde{\boldsymbol{F}}$. A matrix has full rank if and only if all the leading principle submatrices have full rank [23]. Therefore, $\boldsymbol{W}_i$ has full rank, $\min(|K_i|, M)$. Since $\boldsymbol{W}_i$ has full rank, $\boldsymbol{Q}_i$ has the rank $\min(|K_i|, M)$. □

Since $\boldsymbol{Q}$ is an $n_T M \times n_T M$ matrix, the maximum possible number of positive eigenvalues is $n_T M$. If $|K_i| \geq M$, $i = 1, 2, \cdots, n_T$, then Δ_H becomes $n_T M$ by Theorem 2. But $|K_i|$ is equal to d_i if $d_H(\boldsymbol{b}_k, \hat{\boldsymbol{b}}_k) < 2$ for all k, which is highly probable according to Theorem 1. Therefore, if $d_i \geq M$, $i = 1, 2, \cdots, n_T$, then Δ_H becomes $n_T M$ with a high probability, that is, the diversity order becomes the maximum value $n_T n_R M$ with a high probability. When such maximum diversity order is achieved, $\boldsymbol{Q}$ has full rank and Δ_p is equal to the determinant of $\boldsymbol{Q}$. As a larger Δ_p yields a lower PEP in (16), it is desirable to arrange the system parameters to make det($\boldsymbol{Q}$) large. This observation leads us to the following design criteria for SF-BICM systems .

Design Criteria for SF-BICM Systems:

(1) In order to achieve the maximum diversity order $n_T n_R M$, the minimum Hamming weight d_i of each component code should be greater than or equal to the number of the multipaths M. If there are some minimum Hamming weights d_i's less than M, the achievable diversity order drops to $n_R \sum_{i=1}^{n_T} \min(d_i, M)$.

(2) Once the maximum diversity order $n_T n_R M$ is achieved, the minimum determinant of $\boldsymbol{Q}$, or $\prod_{i=1}^{n_T} \det(\boldsymbol{Q}_i)$, should be maximized over all pairs of distinct codewords.

IV. FER PERFORMANCE OF SF-BICM SYSTEMS

We consider the SF-BICM system with two transmit antennas and two multipaths (i.e., $n_T = 2$ and $M = 2$). Based on Theorem 1, we assume that L is sufficiently large and $d_H(\boldsymbol{b}_k, \hat{\boldsymbol{b}}_k) < 2$ for all k and $|K_i| = d_i$, $i = 1, 2$. We also assume that each minimum Hamming weight of the component codes is greater than or equal to M. We derive the performance bound based on these assumptions first and then examine some numerical results of the SF-BICM system.

A. Approximate Performance Bound

To evaluate the performance of the 2-transmit antenna, 2-multipath SF-BICM system, we need to compute the determinant of $\boldsymbol{Q}$ in (12a) for $n_T = 2$ and $M = 2$ for a given interleaver, then determine the upper bound of the PEP in (16), and finally take the average of the PEP over all possible interleavers. However, this process requires a large amount of computation and memory. So, we switch the order of the operations in such a way that we compute the average of the determinant of $\boldsymbol{Q}$ over all possible interleavers first and then substitute it for Δ_p in (16). Since the PEP upper bound in (16) is a convex function of Δ_p, this switched operation yields a value lower than the upper bound by Jensen's inequality [21], i.e.,

$$E[P_B(\Delta_H, \Delta_p)] \geq P_B(\Delta_H, E[\Delta_p]). \quad (34)$$

As the resulting PEP bound may be less than the true value in extreme cases, it is rather an approximate bound than an upper bound.

We first consider the term $\boldsymbol{Q}_i$ in (30b). By applying (6b), we get

$$\boldsymbol{Q}_i = 2\begin{bmatrix} d_i & \sum_{k\in K_i} e^{-\frac{j2\pi(n_1-n_2)k}{L}} \\ \sum_{k\in K_i} e^{\frac{j2\pi(n_1-n_2)k}{L}} & d_i \end{bmatrix}, \quad (35a)$$

whose determinant is

$$\det(\boldsymbol{Q}_i) = 4d_i^2 - 4\left|\sum_{k\in K_i} e^{-\frac{j2\pi(n_1-n_2)k}{L}}\right|^2. \quad (35b)$$

Let k_i, $i = 1, 2, \cdots, d_i$ (with $k_1 < k_2 < \cdots < k_{d_i}$) be the elements of K_i and denote $\theta_p \equiv \frac{2\pi(n_1-n_2)k_p}{L}$ for $p = 1, 2, \cdots, d_i$. If we assume $L >> d_i$, then θ_p's can be regarded as independent uniformly distributed random variables, ranging from 0 to 2π. Then $E\left[e^{-j(\theta_p-\theta_q)}\right] = 0$ for $p \neq q$, and applying this to (35b), we obtain

$$E\left[\det(\boldsymbol{Q}_i)\right] = 4d_i^2 - 4E\left[\left|\sum_{p=1}^{d_i} e^{-j\theta_p}\right|^2\right] = 4d_i^2 - 4d_i. \quad (36)$$

Returning this to (30a), we get

$$E\left[\Delta_p\right] = E\left[\det(\boldsymbol{Q})\right] = \det(R_h)\prod_{i=1}^{2}\left[4d_i(d_i-1)\right]. \quad (37)$$

If we substitute $E\left[\Delta_p\right]$ for Δ_p in (16), we can obtain an approximate PEP bound. The above equation means that as the product of the Hamming weights of the component codes becomes large, the average of det($\boldsymbol{Q}$) increases, thereby yielding better performances. Therefore, for the case of $n_T = 2$ and $M = 2$, the second design criterion in the previous section may be rewritten as follows: Once the maximum diversity order is achieved, $\prod_{i=1}^{n_T} d_i(d_i-1)$ should be maximized.

We apply the union bounding technique to obtain an upper bound of FER for a *maximum likelihood* (ML) decoding of SF-BICM. [5] Then, the FER of SF-BICM with QPSK signaling can be expressed by [13]

$$P(e) \leq 2n_T rL\sum_{\boldsymbol{d}} a_{\boldsymbol{d}} P_B\left(n_T n_R M, \det(R_h)\det(\boldsymbol{Q})\right), \quad (38)$$

where r denotes the code rate of the binary channel code, $a_{\boldsymbol{d}}$ the number of codewords whose Hamming weight from the ith component code is d_i, $i = 1, 2, \cdots, n_T$, and $P_B(\Delta_H, \Delta_p)$ the PEP upper bound in (16). [6] When $n_T = 2$ and $M = 2$, we obtain an approximate FER performance bound based on the approximate PEP bound incorporating (37), i.e.,

$$P(e) \approx 4rL\sum_{\boldsymbol{d}} a_{\boldsymbol{d}} P_B\left(4n_R, \det(R_h)\prod_{i=1}^{2}\left[4d_i(d_i-1)\right]\right). \quad (39)$$

Table 1. Distance spectrum of the convolutional code with generators (7,5) in octal.

d_1	d_2	a_d	d_1	d_2	a_d
2	3	1	4	5	10
2	4	1	4	6	15
2	5	1	4	7	21
2	6	1	6	2	1
2	7	1	6	3	5
2	8	1	6	4	15
2	9	1	6	5	35
2	10	1	8	2	1
4	2	1	8	3	7
4	3	3	10	2	1
4	4	6	10	3	9

B. Numerical Results

We now examine some numerical FER performances of the 2-transmit antenna, 2-multipath SF-BICM system through simulations. We use the newly derived bound in (39) to evaluate the FER bounds and compare them with the simulation results. As we need only the distance spectrum of the binary channel codes to calculate the above FER bounds, the FER bounds are applicable to any binary linear codes, such as convolutional codes, turbo codes, and others. In evaluating the FER bound, we use the truncated distance spectrum, as the FER upper bounds may be satisfactorily approximated by taking into account the codewords for which the product of Hamming distances is less than or equal to a predetermined value. We take 30 as this predetermined value. By modifying the algorithm in [24], we can obtain the resulting number of simple error events of the convolutional code with the generators (7,5) in octal expression as listed in Table 1.

In the simulations, we take the available bandwidth of 1 MHz and use 256 carrier tones (i.e., L=256) for OFDM modulation. This corresponds to a subchannel sparation of 3.9 kHz and the OFDM frame duration of 256 μs. To each frame, we add a cyclic

[5] It is not tractable to obtain the performance bound of the communication systems employing iterative decoding, so we derive the bound of SF-BICM based on ML decoding. Though the performance of iterative decoding is not guaranteed to converge to the ML performance, it has been empirically known to approach closely [22].

[6] For the case of time-invariant codes (e.g., convolutional codes), the sets of the simple error events that start at different times are identical if the edge effect is ignored. Since the number of input bits to the code is $2n_T rL$, there are $2n_T rL$ error events for a simple error event pattern. So, the FER bound based on the union bound is $2n_T rL$ times the first error probability. In contrast, for the case of turbo codes, the sets of the simple error events that start at different times are not identical and the effect of $2n_T rL$ is incorporated in the parameter $a_{\boldsymbol{d}}$.

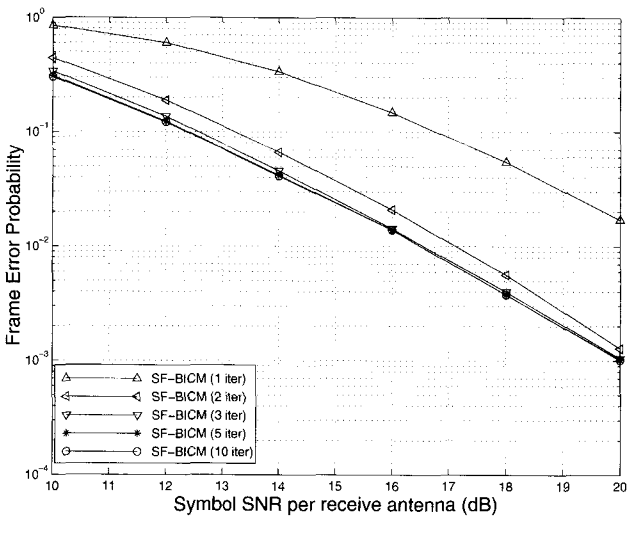

Fig. 3. Frame error probability versus the number of iterations for SF-BICM whose channel code is a 4-state convolutional code ($n_T = 2$, $M = 2$).

prefix of 40 μs duration to combat the effects of intersymbol interference. Therefore, the information rate is $2 \times 256/296 = 1.73$ bits/sec/Hz.

We use random interleavers as the bit interleavers and employ iterative demodulation-decoding method with perfect *channel state information* (CSI) at the receiver [14]. We plot the resulting FER performance curves with respect to the symbol SNR per receive antenna, $n_T E_s/N_o$. We also evaluate through simulations the FER performance of the equivalent 1-antenna system with the same generators. For this, we employ Viterbi decoding to obtain ML performances.

Fig. 3 plots the FER performance for different numbers of iterations of SF-BICM. From the figure, we observe that there is a large improvement in FER between the first iteration and the second. The FER improvement from the second to the third iteration is still considerable, but not as dramatic as the first improvement. Performance improves with increasing iterations but the amount of the improvement diminishes and nearly no improvement is observed beyond the fifth iteration. So we may regard the performance after the fifth iteration as the final.

Fig. 4 plots the resulting performance of SF-BICM whose channel code is the 1/2-rate convolutional code with the generators (7,5) in octal expression. The channel of Fig. 4 (a)–(c) is a 2-ray equal-power delay profile and that of Fig. 4 (d) is a 2-ray delay profile with the power ratio of 3:1. The delay spread of channel is 10 μs for Fig. 4 (a), (b), and (d) but 40 μs for Fig. 4 (c). The number of receive antennas is one for Fig. 4 (a), (c), and (d) but two for Fig. 4 (b). Overlaid in each graph is the performance of the equivalent 1-antenna system ($n_T = 2$, $M = 2$). We observe from the four figures that the performance of SF-BICM closely approaches that of the equivalent 1-antenna system after five iterations. We also observe that the FER bounds are tight enough to estimate the performance of SF-BICM with sufficient accuracy for the FER range between 10^{-4} and 10^{-3}.

In case no interleaver is employed, the delay spread is supposed to affect the performance significantly [2]. However, we observe, in Fig. 4 (a) and (c), that the delay spread does not affect the performance if interleavers are included. It is not a surprising result because the FER bounds in (39) is independent of the delay spread. This is because the random interleavers spread non-zero bits uniformly, causing an averaging effect on the performance and, consequently, nullifying the delay spreading effect.

The determinant of $\boldsymbol{Q}$ in (37) is proportional to the determinant of the channel gain autocorrelation matrix R_h. For a given fixed trace of R_h, $\det(R_h)$ becomes maximum when the diagonal elements are all equal, that is, the channel has an equal-power delay profile. Comparing Fig. 4 (a) and (d), we can confirm that the SF-BICM in an equal-power delay profile outperforms that in an unequal-power delay profile with the power ratio of 3:1.

V. CONCLUDING REMARKS

So far, we have explored SF-BICM systems in frequency-selective fading channels. We have established SF-BICM by combining bit-interleavers and OFDM and examined its design criteria that maximize the diversity order and coding gain. We have unveiled the effects of bit-interleavers by showing that it transforms an n_T-antenna SF-BICM system into an equivalent 1-antenna system. By exploiting this property, we were able to analyze the performance of SF-BICM very easily.

We have derived the PEP in terms of the determinant of the matrix $\boldsymbol{Q}$ and have proved that the allocated bits in each antenna seldom coincide. So, the PEP would be the same as the case when the signal transmitted in each antenna does not interfere each other. This property implies that the bit-interleavers transform an n_T-antenna SF-BICM into an equivalent 1-antenna system where each IDFT output sequence is transmitted serially. Based on the PEP, we have determined design criteria for SF-BICM.

We have analyzed the performance of SF-BICM using the fact that the performance of SF-BICM is equivalent to that of the equivalent 1-antenna coded system. We have derived a simple FER bound that is accurate and requires only the distance spectrum of the constituent codes of SF-BICM systems.

If we summarize the findings out of the analyses and numerical results, we get the following: First, the bit-interleavers uniformly spread the nonzero bits from the channel code, so that the allocated bits in each antenna seldom coincide. Second, δ_H takes on sum($\boldsymbol{d}$) with a high probability, which approaches 1 for a large L. Third, we can obtain the performance bound by substituting $n_T M$ for Δ_H and $\det(R_h)\det(\boldsymbol{Q})$ for Δ_p, respectively, and this bound is tight enough to use in predicting the performance of SF-BICM directly. Fourth, SF-BICM systems reduce to the equivalent 1-antenna system for a large value L. Fifth, the performance of SF-BICM is irrelevant of the delay spread.

APPENDIX

Derivation of Equation (25)

For $d_H(\boldsymbol{b}_k, \hat{\boldsymbol{b}}_k) < 2$, the number of possible locations of nonzero d_1 bits in $\boldsymbol{\beta}$ is $\binom{L}{d_1}$ and the number of different ordering

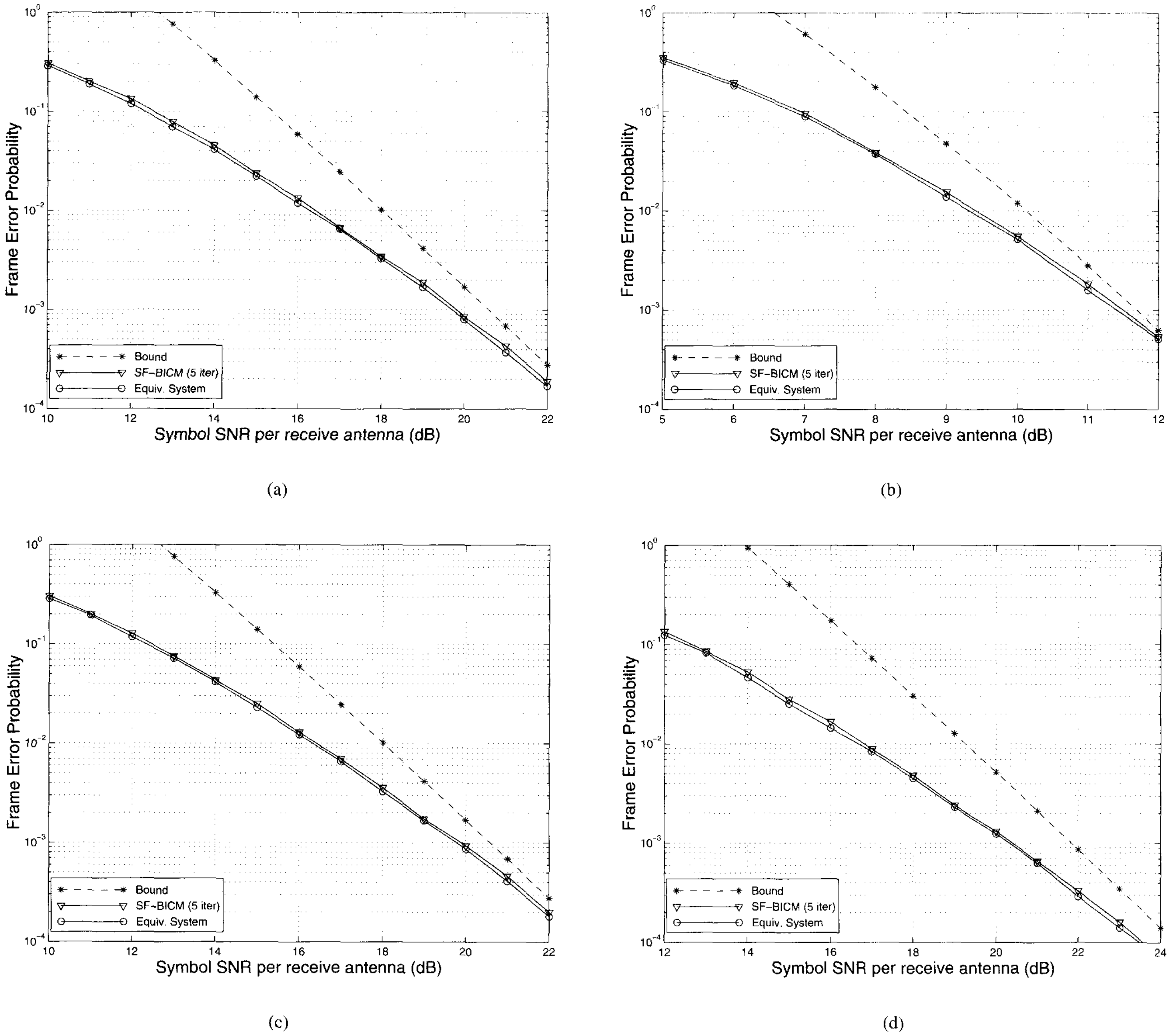

Fig. 4. Frame error probability of SF-BICM whose channel code is a 4-state convolutional code ($n_T = 2$, $M = 2$): (a) One receive antenna and two equal-power taps with the delay spread 10 μs, (b) two receive antennas and two equal-power taps with the delay spread 10 μs, (c) one receive antenna and two equal-power taps with the delay spread 40 μs, (d) one receive antenna and two taps with the delay spread 10 μs and the power ratio of 3:1.

of a nonzero bit in selected locations is 2^{d_1}, since we assume a QPSK modulation. Likewise, the number of possible locations of d_2 columns in the submatrix of $\boldsymbol{\beta}$ that exclude d_1 columns is $\binom{L-d_1}{d_2}$ and the number of different ordering of a nonzero bit in selected locations is 2^{d_2}. Repeating this procedure, we find that the number of all possible $\boldsymbol{\beta}$'s is $\prod_{i=1}^{n_T} \binom{L-\sum_{j=1}^{i-1} d_j}{d_i} 2^{d_i}$. As we assume uniform interleaving, the number of possible permutation becomes $\prod_{i=1}^{n_T} \binom{2L}{d_i}$. Therefore, we obtain equation (25).

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