Two-stage ML-based Group Detection for Direct-sequence CDMA Systems

Stefano Buzzi and Marco Lops

Abstract: **In this paper a two-stage maximum-likelihood (ML) detection structure for group detection in DS/CDMA systems is presented. The first stage of the receiver is a linear filter, aimed at suppressing the effect of the unwanted (i.e., out-of-group) users' signals, while the second stage is a non-linear block, implementing a ML detection rule on the set of desired users signals. As to the linear stage, we consider both the decorrelating and the minimum mean square error approaches. Interestingly, the proposed detection structure turns out to be a generalization of Varanasi's group detector, to which it reduces when the system is synchronous, the signatures are linearly independent and the first stage of the receiver is a decorrelator. The issue of blind adaptive receiver implementation is also considered, and implementations of the proposed receiver based on the LMS algorithm, the RLS algorithm and subspace-tracking algorithms are presented. These adaptive receivers do not rely on any knowledge on the out-of-group users' signals, and are thus particularly suited for rejection of out-of-cell interference in the base station.**

Simulation results confirm that the proposed structure achieves very satisfactory performance in comparison with previously derived receivers, as well as that the proposed blind adaptive algorithms achieve satisfactory performance.

I. INTRODUCTION

The Code Division Multiple Access (CDMA) technique, implemented with the Direct Sequence (DS) modulation format, has established itself as the leading technology for the realization of the physical layer of several third-generation wireless cellular networks, both terrestrial [1], [2], and satellite-based [3]. Indeed, the DS/CDMA technique appears to offer superior performance with respect to the conventional frequency- and/or time-division multiple access systems, especially when coupled with the adoption of advanced multiuser receivers at the demultiplexing stage [4]. Since the seminal paper by Verdù [5], who showed that it is the thermal noise and *not* the multiaccess interference (MAI) which rules the ultimate performance levels attainable in a CDMA communication system, huge research efforts have been devoted to the study of multiuser receivers, and an impressive progress has been experienced in the last decade. Among the most prominent results of such an activity, we cite here the synthesis of linear, reduced complexity, multiuser detectors (see [4] for an updated survey), the design of blind adaptive multiuser detection algorithms [6], [7], and the synthesis of narrowband interference resistant receivers [8], [9].

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In this context, *group detection*, introduced by Varanasi in [10] to jointly demodulate the information bits stream from a certain subset, say $\mathcal{G}$, of the K transmitting users of a DS/CDMA system, provides a unifying approach to multiuser detection. In fact, both the optimum multiuser detector and some suboptimal single-user detectors may be regarded as special cases of the detectors derived in [10], [11] which are based on the application of the generalized likelihood ratio test (GLRT) strategy. Group detection is also a very powerful tool for the communication designer since it allows to take into account the complexity constraints in a very flexible way. In fact, by partitioning the K active users in subsets of smaller cardinality, it is possible to trade performance levels for complexity. However, the reason why group detection has recently gained more and more attention is its suitability for the design of advanced wireless cellular CDMA-based networks. Indeed, due to the explosive growth of multimedia applications, nowadays networks are required to transport several kinds of data like, for instance, voice, packet data, multimedia e-mail, low-resolution video, etc., each with its own required quality of service and data rate [12], [13]. A very simple way to accommodate a high-rate data stream over a lower-rate CDMA network amounts to assigning to the high-rate user many signature waveforms, so that it is able to transmit several information bits in parallel, in just one bit interval. This technique, commonly referred to as *multicode*, has been incorporated in many standard proposals for the implementation of the third generation wireless networks air interface [1]. Thus, in a multirate CDMA network implemented with a multicode access technique, a high-rate bit stream is multiplexed onto multiple low-rate signals: the mobile receiver of a high-rate user is thus interested in decoding multiple bits at each signaling interval, i.e., it is to behave like a group detector. Moreover, in a multicell environment, a base station is to jointly detect the bits from the users present in its own covered area, while not being interested in demodulating the signals originating from the out-of-cell users. In this context, group detectors appear much more suited than classical multiuser detectors, as they permit to take into account the presence of the (unwanted) inter-cell interference, which may be up to 40% of the total interference [14].

Due to the strong relevance of these applications, many research efforts have been recently devoted also to blind adaptive group detection, i.e., to the design of group detectors capable of being implemented with as limited as possible prior knowledge on the interference structure, but capable of achieving performance close to that of their non-blind counterparts. In this context, we cite here the work [15], wherein a semi-blind implementation of the detector in [10] has been presented, and the works [16], [17], wherein linear group detectors are derived, and the tools of subspace tracking theory are applied so as to come up with blind adaptive implementations.

Following on such a track, in this work we consider an asynchronous DS/CDMA system and present a new group detection structure. More precisely, the main contributions of the present paper are summarized as follows.

- We present here a two-stage group detector, where suppression of the unwanted out-of-group users' signals is accomplished in the first stage, which is linear, while data detection for the users in the group takes place in the second stage of the receiver.
- In order to design the first stage of the receiver, we introduce a minimum mean output energy (MOE) cost function, which is borrowed from previous studies in the area of beamforming, and which does represent a generalization of the classical MOE cost functions introduced in [6], [7] with reference to plain multiuser detection. Interestingly, this cost function leads to receivers which represent the generalization to the group detection framework of the classical decorrelating and MMSE linear multiuser receivers.
- We present three blind adaptive implementations, based on the least-mean-squares (LMS), the recursive-least-squares (RLS) and the projection approximation subspace tracking with deflation (PASTd) algorithms, of the first stage of the proposed group detector. The result is thus a blind adaptive group detector, which may be implemented with no knowledge on the out-of-group interfering users and with no need for known training sequences.

With regard to the performance assessment, we give sample curves in order to contrast the error probability of the proposed receiver with that of existing detection structures, as well as plots of the convergence dynamics of the receiver adaptive implementations.

This paper is organized as follows. Section II contains the system model and the problem formulation, while in Section III the synthesis of the new group detector is outlined. Section IV is devoted to the performance assessment, while in Section V the problem of blind adaptive receiver implementation is addressed. Finally, concluding remarks are given in Section VI.

II. SYSTEM MODEL

Let us consider an asynchronous DS/CDMA network with K active users. The baseband equivalent of the received signal is written as

$$r(t) = \sum_{k=0}^{K-1} A_k e^{j\phi_k} \sum_{m=-P}^{P} b_k(m) s_k(t - mT_b - \tau_k) + w(t) \ . \quad (1)$$

In the above expression, $A_k e^{j\phi_k}$ is the complex amplitude of the k-th user signal, accounting for the transmitted energy and the channel propagation effects; $\{b_k(m)\}_{m=-P}^{+P}$ represents the bits stream of the $k-$th user, modeled as a sequence of independent and identically distributed binary variates, each taking on values in the set $\{-1; 1\}$; $T_b = NT_c$ is the bit interval of the network, with T_c the chip-interval and N the processing gain; $[\tau_0, \cdots, \tau_{K-1}]$ is a set of delays, taking on values in the set $[0, T_b)$; $2P + 1$ is the transmitted frame length; $s_k(t)$ is the *signature* assigned to the $k-$th user, expressed as

$$s_k(t) = \sum_{n=0}^{N-1} c_{kn} u_{T_c}(t - nT_c) \ ,$$

with $\{c_{kn}\}_{n=0}^{N-1}$ the $k-$th spreading code, and $u_{T_c}(\cdot)$ a unit-height rectangular pulse supported on the interval $[0, T_c]$; finally, $w(t)$ is the thermal noise term, modeled as a sample function from a complex white Gaussian process with Power Spectral Density (PSD) $2\mathcal{N}_0$.

The received signal (1) may be conveniently re-cast to separate the useful terms and the interferers. To fix the ideas, let us assume that the users to be demodulated are the first G (with $G < K$), i.e., $\mathcal{G} = \{0, 1, \cdots, G-1\}$, and that $\tau_0 = \cdots = \tau_{G-1} = 0$, i.e., the users of interest transmit synchronously. This last assumption is made so as to simplify notation. However, it is naturally met in a multirate/multicode CDMA system, and it also models a Wideband-CDMA [1], [2] third-generation system, wherein asynchronous intercell operations have been planned. Nonetheless, what follows can be easily extended also to the case that the system is fully asynchronous. In order to take a decision on the bits transmitted by the users in $\mathcal{G}$ in the p-th signaling interval, $\boldsymbol{b}_G(p) = [b_0(p), \cdots, b_{G-1}(p)]^T$ (with $(\cdot)^T$ denoting transpose), we project the received waveform $r(t)$ onto the unit vectors of the $NM-$dimensional orthonormal system

$$\left\{ \frac{1}{\sqrt{T_{OS}}} u_{T_{OS}}(t - pT_b - iT_{OS}) \right\}_{i=0}^{NM-1} , \quad (2)$$

with $T_{OS} = T_c/M$ and M a positive integer. Notice that in defining the system (2) we have accounted for possible signal-space oversampling in the sense specified in [18]. In particular, choosing $M > 1$, although resulting in some complexity increase, allows reducing the representation error due to the system asynchrony, and, eventually, achieving better performance. It is also worth pointing out that the oversampling is beneficial since we are considering here rectangular chip-pulses. If, instead, the system employs band-limited Nyquist chip-pulses in place of the rectangular waveform $u_{T_c}(\cdot)$, then the received waveform may be obviously converted to discrete-time with no loss of information through a low-pass filtering followed by a sampler at the Nyquist rate. Notice also that, even though in this paper rectangular chip-pulses are considered, the following derivations apply with no modification to the case that band-limited chip-pulses are employed.

Projecting the waveform (1) onto the orthonormal set (2) yields the following $NM-$dimensional vector of observables:

$$\begin{aligned} \boldsymbol{r}(p) = & \textstyle\sum_{k=0}^{G-1} A_k e^{j\phi_k} b_k(p) \boldsymbol{s}_k^0 + \sum_{k=G}^{K-1} A_k e^{j\phi_k} \\ & \cdot \textstyle\sum_{m \in \{-1,0\}} b_k(p+m) \boldsymbol{s}_k^m + \boldsymbol{w}(p) \quad (3) \\ = & \textstyle\sum_{k=0}^{G-1} A_k e^{j\phi_k} b_k(p) \boldsymbol{s}_k^0 + \boldsymbol{z}(p) + \boldsymbol{w}(p) \ . \end{aligned}$$

In (3) the first term on the right-hand-side (RHS) represents the contribution from the bits to be decoded, while $\boldsymbol{z}(p)$ and $\boldsymbol{w}(p)$ represent the contribution from the multiaccess interference (MAI) and the thermal noise, respectively. In particular,

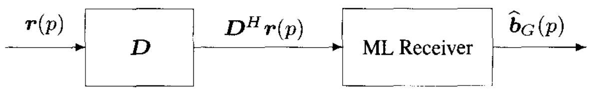

Fig. 1. Block-scheme of the group detector.

the interference suffered by the users of interest in an asynchronous CDMA network can be regarded as equivalent to that generated in a synchronous system by a number of fictitious users employing the signatures $\left\{\{s_k^i\}_{i\in\{-1,0\}}\right\}_{k=G}^{K-1}$. Finally, $\boldsymbol{w}(p)$ is a white complex Gaussian vector with covariance matrix $2\mathcal{N}_0\boldsymbol{I}_{NM}$, with $\boldsymbol{I}_{NM}$ denoting the identity matrix of order NM.

III. DETECTOR DESIGN

In order to decode the bits $\boldsymbol{b}_G(p)$, we resort to the two-stage architecture depicted in Fig. 1. First the observables are fed to a linear filter, represented by the $NM \times G$-dimensional matrix $\boldsymbol{D}$ and aimed at suppressing MAI, and then the output of the linear filter is forwarded to a non-linear block implementing a ML decision rule with respect to the G users of interest only. Assuming that the matrix $\boldsymbol{D}$ has rank G, the proposed detection rule can be thus written as (4) at the bottom of this page with $(\cdot)^H$ denoting conjugate transpose, $\|\cdot\|$ denoting the norm in the NM-dimensional Euclidean space $\mathcal{C}^{NM}$ (over the complex field $\mathcal{C}$) and x_k denoting the k-th entry of $\boldsymbol{x}_G$. Otherwise stated, we assume that the MAI contribution (i.e., the term $\boldsymbol{D}^H\boldsymbol{z}(p)$) at the output of the linear filter is vanishingly small and implement the ML decision rule on the new observables *neglecting* the MAI contribution. Notice that the computational complexity of the rule (4) is $\mathcal{O}(2^G) + \mathcal{O}(NMG)$ per bit-interval.

In order to derive the filter $\boldsymbol{D}$, we introduce two MOE cost functions, whose constrained minimization leads to "group-versions" of the decorrelating and MMSE linear multiuser receivers. To this end, we assume that the vectors in the set $\{\boldsymbol{s}_k^0\}_{k=0}^{G-1}$ are linearly independent.

A. The Decorrelating Design Strategy

First of all, we recall that, if $G = 1$, i.e., if one were interested in decoding just one user, a decorrelating linear detector, say $\boldsymbol{d}_0$, could be obtained as the NM-dimensional vector solving the following constrained minimization problem [7]:

$$\min E\left[\left|\boldsymbol{d}_0^H(\boldsymbol{r}(p) - \boldsymbol{w}(p))\right|^2\right], \qquad \text{subject to } \boldsymbol{d}_0^H\boldsymbol{s}_0^0 = 1. \tag{5}$$

Here, since we are actually interested in decoding the information bits from G users, we extend such an approach and require that the $NM \times G$-dimensional matrix $\boldsymbol{D}$ solve the following constrained minimization problem:

$$\begin{cases} \min E\left[\left\|\boldsymbol{D}^H(\boldsymbol{r}(p) - \boldsymbol{w}(p))\right\|^2\right], \\ \text{subject to } \boldsymbol{D}^H\boldsymbol{S}_G = \boldsymbol{K}, \end{cases} \tag{6}$$

wherein $\boldsymbol{S}_G$ is an $NM \times G$-dimensional matrix containing on its columns the signatures from the users to be decoded $\{\boldsymbol{s}_0^0, \cdots, \boldsymbol{s}_{G-1}^0\}$, and $\boldsymbol{K}$ is a $G \times G$ matrix, which we assume to have full-rank. Letting $\boldsymbol{R} = E\left[(\boldsymbol{r}(p) - \boldsymbol{w}(p))(\boldsymbol{r}(p) - \boldsymbol{w}(p))^H\right]$ be the (rank-deficient) covariance matrix of the thermal noise-free observables $\boldsymbol{r}(p) - \boldsymbol{w}(p)$, the solution to (6) is shown to be written as

$$\boldsymbol{D} = \boldsymbol{R}^+\boldsymbol{S}_G\left(\boldsymbol{S}_G^H\boldsymbol{R}^+\boldsymbol{S}_G\right)^{-1}\boldsymbol{K}^H, \tag{7}$$

wherein $(\cdot)^+$ denotes the Moore-Penrose generalized inverse. Some remarks are now in order about the found solution. First of all, notice that the matrix $\boldsymbol{D}$ has rank G, thus implying the applicability of the decision rule (4). Additionally, substituting expression (7) into the decision rule (4), it is seen that the choice of the constraint matrix $\boldsymbol{K}$ is irrelevant, as any full-rank matrix leads to the same receiver, and one can set $\boldsymbol{K} = \left(\boldsymbol{S}_G^H\boldsymbol{R}^+\boldsymbol{S}_G\right)$, so that solution (7) can be given in the following simplified form:

$$\boldsymbol{D} = \boldsymbol{R}^+\boldsymbol{S}_G. \tag{8}$$

Consider now the condition, usually met in DS/CDMA systems with carefully chosen signature sequences and with values of K not close to the processing gain N, that the vectors in the set

$$\left\{\{\boldsymbol{s}_k^i\}_{i\in\{-1,0\}}\right\}_{k=G}^{K-1} \bigcup \{\boldsymbol{s}_k^0\}_{k=0}^{G-1}$$

are linearly independent. Then, we have that

$$\boldsymbol{R}^+\boldsymbol{s}_l^0 = \frac{\boldsymbol{s}_l^{0\perp}}{A_l^2\left\|\boldsymbol{s}_l^{0\perp}\right\|^2}, \qquad l = 0, \cdots, G-1,$$

with $\boldsymbol{s}_l^{0\perp}$ denoting the projection of $\boldsymbol{s}_l^0$ onto the orthogonal complement to the subspace spanned by the signatures $\left\{\{\boldsymbol{s}_k^i\}_{i\in\{-1,0\}}\right\}_{k=G}^{K-1} \bigcup \{\boldsymbol{s}_k^0\}_{k=0,k\neq l}^{G-1}$. As a consequence, expression (8) can be rewritten as (9) at the bottom of next page with $\boldsymbol{S}_G^\perp$ denoting an $NM \times G$-dimensional matrix containing on its l-th column the vector $\boldsymbol{s}_{l-1}^{0\perp}/\|\boldsymbol{s}_{l-1}^{0\perp}\|^2$. It can be also shown that, under the linear independence assumption, the filter (9) is capable of nullifying the external interference, i.e., $\boldsymbol{D}^H\boldsymbol{z}(p) = \boldsymbol{0}$, thus implying that the decision rule (4) is the

$$\widehat{\boldsymbol{b}}_G(p) = \arg\min_{\boldsymbol{x}_G\in\{+1,-1\}^G}\left[\left(\boldsymbol{r}(p) - \sum_{k=0}^{G-1}A_k e^{j\phi_k}x_k\boldsymbol{s}_k^0\right)^H \boldsymbol{D}(\boldsymbol{D}^H\boldsymbol{D})^{-1}\boldsymbol{D}^H\left(\boldsymbol{r}(p) - \sum_{k=0}^{G-1}A_k e^{j\phi_k}x_k\boldsymbol{s}_k^0\right)\right], \tag{4}$$

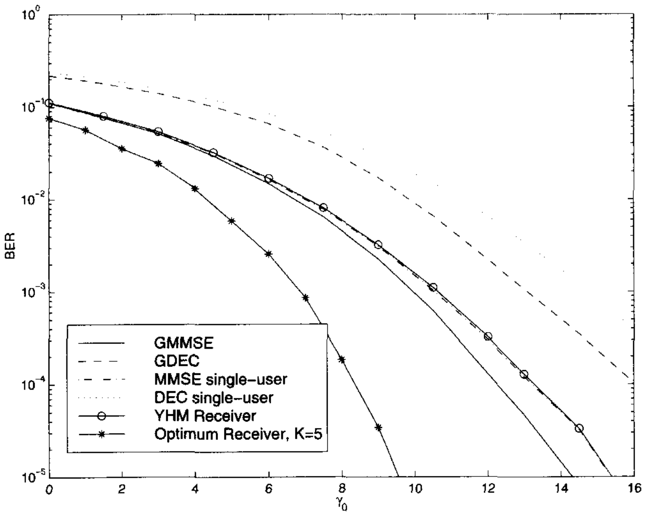

Fig. 2. Bit-Error-Rate versus γ_0 for $K = 15$ and $G = 5$.

optimal processing for the new observables (i.e., $\boldsymbol{D}^H \boldsymbol{r}(p)$). Additionally, if a synchronous system with $M = 1$ (no oversampling) is considered, the proposed receiver ends up coincident with the group detector derived by Varanasi in [10], wherein however a different design strategy (i.e., the GLRT) had been adopted; the proof of this equivalence is reported in the Appendix. Thus, the proposed detection structure, coupled with a decorrelating design strategy, can be thought of as a generalization, in particular to the case of asynchronous systems with finite observation interval, of the receiver in [10]. Additionally, it is also worth pointing out that both Varanasi's receiver and the proposed decorrelating group detector are similar to the projection receiver independently derived in [19]. Notice, however, that the proposed receiver is much more general than Varanasi's receiver, since it may operate also in a close-to-saturation scenario, wherein the out-of-group users' signatures span entirely the space $\mathcal{C}^{NM}$. Indeed, in this situation, Varanasi's receiver cannot be used since the orthogonal complement of the subspace spanned by the undesired signatures is empty, while the proposed decorrelating strategy is still applicable, as demonstrated by the results, illustrated later, shown in Fig. 5.

B. The MMSE Design Strategy

Besides the decorrelating detector, another very popular multiuser receiver is the linear MMSE detector. In order to obtain a "group-version" of the plain linear MMSE multiuser receiver, here we require that the matrix $\boldsymbol{D}$ solve the following constrained minimization problem:

$$\begin{cases} \min E\left[\left\|\boldsymbol{D}^H \boldsymbol{r}(p)\right\|^2\right] , \\ \text{subject to } \boldsymbol{D}^H \boldsymbol{S}_G = \boldsymbol{K} . \end{cases} \tag{10}$$

Problem (10) resembles the linear constrained minimum variance (LCMV) criterion [6], [20], which finds its main applications in the field of array signal processing [21]. The corresponding solution is shown to be written as

$$\boldsymbol{D} = \boldsymbol{R}_{\boldsymbol{rr}}^{-1} \boldsymbol{S}_G \left(\boldsymbol{S}_G^H \boldsymbol{R}_{\boldsymbol{rr}}^{-1} \boldsymbol{S}_G\right)^{-1} \boldsymbol{K}^H , \tag{11}$$

with $\boldsymbol{R}_{\boldsymbol{rr}} = E\left[\boldsymbol{r}(p)\boldsymbol{r}^H(p)\right]$ denoting the observables covariance matrix. Once again, substituting the above solution into the decision rule (4) it is easily seen that the full-rank constraint matrix $\boldsymbol{K}$ is irrelevant, and hereafter we let $\boldsymbol{K} = \left(\boldsymbol{S}_G^H \boldsymbol{R}_{\boldsymbol{rr}}^{-1} \boldsymbol{S}_G\right)$ so that solution (11) has the simplified form

$$\boldsymbol{D} = \boldsymbol{R}_{\boldsymbol{rr}}^{-1} \boldsymbol{S}_G . \tag{12}$$

Notice also that, unlike the previous case, the MAI contribution at the output of the linear filter $\boldsymbol{D}$, i.e., the term $\boldsymbol{D}^H \boldsymbol{z}(p)$, is now non-zero. As a consequence, the ML decision rule (4) is no longer optimal for the new observables, since it does not take into account the presence of the external interference. However, this is not expected to cause a remarkable performance impairment; indeed, on one hand, in a power-controlled scenario, the interfering signals strength at the output of the MMSE filter is very limited, on the other hand, should the interfering signals become increasingly large, they would be totally suppressed.

IV. SIMULATION RESULTS

In order to test the effectiveness of the proposed group detectors, we have considered a DS/CDMA system employing Gold codes with processing gain $N = 31$ and with perfect power control (i.e., $A_0 = \cdots = A_{K-1}$). Notice that this is by no way a limiting assumption, since both the ZF-based and the MMSE-based group detectors are near-far resistant and their performance is slightly perturbed in near-far situations. In particular, the performance of the ZF-based receiver is independent of the amplitudes of the signals from the out-of-group users.

First of all, we carry out a performance comparison with other multiuser detectors; in particular, in Fig. 3, we represent the average Bit-Error-Rate (BER) for the users in $\mathcal{G}$ versus the received energy contrast per bit $\gamma_0 = (A_0^2 T_b)/(2\mathcal{N}_0)$ for the following detectors:

- the proposed group-detector with an MMSE-based linear filter (GMMSE);

$$\boldsymbol{D} = \left[\frac{\boldsymbol{s}_0^{0\perp}}{A_0^2\left\|\boldsymbol{s}_0^{0\perp}\right\|^2} \; \frac{\boldsymbol{s}_1^{0\perp}}{A_1^2\left\|\boldsymbol{s}_1^{0\perp}\right\|^2} \; \cdots \; \frac{\boldsymbol{s}_{G-1}^{0\perp}}{A_{G-1}^2\left\|\boldsymbol{s}_{G-1}^{0\perp}\right\|^2}\right] \begin{bmatrix} A_0^{-2} & 0 & \cdots & 0 \\ 0 & A_1^{-2} & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & A_{G-1}^{-2} \end{bmatrix}^{-1} = \boldsymbol{S}_G^{\perp} , \tag{9}$$

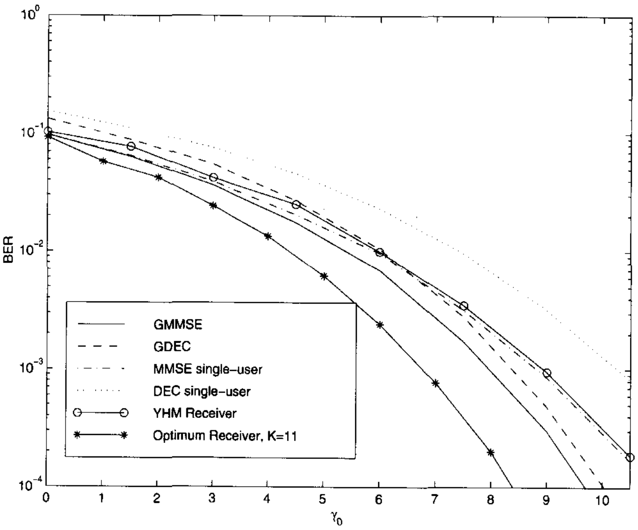

Fig. 3. Bit-Error-Rate versus γ_0 for $K = 15$ and $G = 11$.

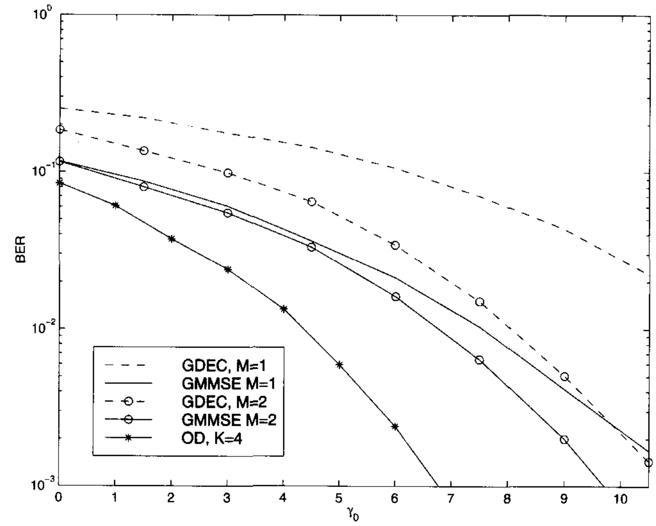

Fig. 4. Bit-Error-Rate versus γ_0 for $K = 15$ and $G = 11$.

- the proposed group-detector with a decorrelating-based linear filter (GDEC);
- the non-adaptive[1] version of the linear group-detector proposed by Yu and Host-Madsen in [16] (YHM);
- the conventional single-user decorrelating detector;
- the conventional single-user MMSE detector.

We have assumed $K = 15$, no oversampling (i.e., $M = 1$), while G is set equal to 5. The plots have been obtained through a Montecarlo counting procedure and are the result of an average over a set of 40 random realizations of the MAI delays. The results clearly show that the proposed detection strategy is effective in rejecting multiuser interference. In particular, it is seen that, as expected, the GMMSE detector achieves the best performance; conversely, the GDEC receiver performance lies in between that of the single user decorrelating detector and that of the single-user MMSE receiver. This fact can be easily justified by noticing that the GDEC receiver totally rejects the received signal projection onto the subspace spanned by the unwanted $K - G$ interfering users, thus causing an useless noise enhancement. Additionally, it is also seen that the YHM receiver and the single-user MMSE receiver are, in the considered scenario, practically equivalent. For comparison purposes, we also report the performance of the minimum error probability optimum detector (OD) for $K = G = 5$, i.e., for the case that no MAI is present at the receiver antenna. Of course, such a curve represents the ultimate performance level that any group detector may possibly achieve. The scenario considered in Fig. 4 is the same as that in Fig. 3 except that G is now set equal to 11. Once again, it is seen that the GMMSE detector achieves the best performance; the relevant difference is that, due to the reduced number of the external interfers $K - G$, the GDEC receiver turns out to outperform the other detection structures. Interestingly, the YHM receiver is slightly outperformed by the single-user MMSE receiver, while, due to the dramatic noise enhancement effects, the single user decorrelating detector achieves the worst performance. Other simulation results, not shown here for sake of brevity, have confirmed that the GMMSE receiver always achieves the best performance, whatever the considered scenario is. As expected, in a near-far situation the gap existing between the GDEC and the GMMSE receivers is greatly reduced, and the same happens for the gap between the single-user decorrelating and MMSE receivers.

[1] More precisely, we consider the receiver developed in [16] under the assumption that the covariance matrix of the received signal is either known or perfectly estimated at the receiver.

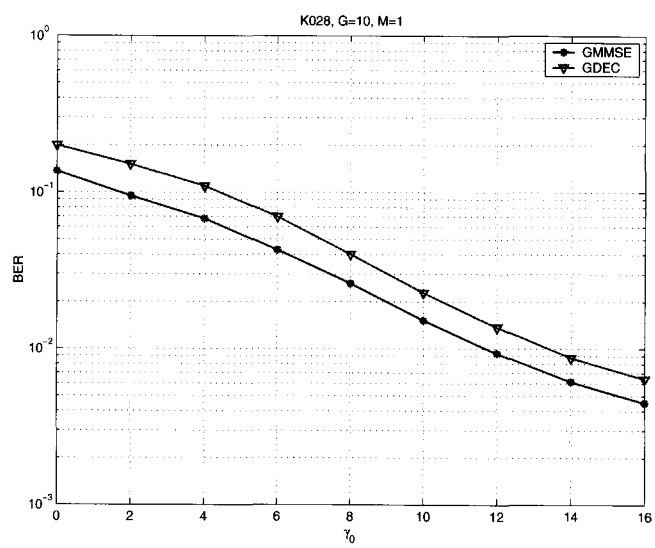

Fig. 5. Bit-Error-Rate of the proposed detectors versus received energy contrast per bit for $K = 28$, $G = 10$, and $M = 1$. The results have been averaged over 50 independent realizations of the interfering users.

Let us now focus on the assessment of the effect of the oversampling ratio M on the system performance. In Fig. 4, we have represented the system BER versus γ_0 for the GDEC and the GMMSE receivers for two values of M. Also in this case the plots are the result of an average over 40 random realizations of the MAI delays, and we have assumed $K = 15$ and $G = 4$. For comparison purposes, we also report the performance of the OD detector for $K = G = 4$. It is seen that, as already highlighted and discussed in [18], oversampling the signal space (i.e., letting $M > 1$) is beneficial to the system performance. Additionally, results confirm the superiority of the MMSE approach with respect to the ZF strategy; indeed, even though in the MMSE case the ML decision rule (4) is not optimal, the MMSE approach attains much superior performance, as it avoids the huge noise enhancement introduced by the ZF strategy. It is also seen that the performance gap between the MMSE receiver and the ZF detector significantly reduces for increasing M. This fact can be justified by noticing that increasing M corresponds to increasing the dimensionality of the observables space, and, eventually, to reducing the undesired effects of the noise enhancement (see also [18])[2].

Finally, in Fig. 5 we consider a close-to-saturation scenario. More precisely, an asynchronous system with no oversampling ($M = 1$), $K = 28$ and $G = 10$ is considered. Notice that in this situation the signatures of the out-of-group users span $\mathcal{C}^N$, thus implying that Varanasi's group detector cannot be used (it achieves .5 error probability). Conversely, the curves shown in the figure demonstrate that the proposed receivers achieve an acceptable performance level even in a such adverse situation.

V. BLIND ADAPTIVE RECEIVERS IMPLEMENTATION

Implementing the first stage of the proposed receivers requires full knowledge of the interference structure, i.e., the number of users, their signatures, amplitudes and relative delays. In practice, however, especially in base-to-mobile communication links, it is not plausible to assume that the receiver can acquire such information so that blind adaptive receivers are needed. Accordingly, in what follows we deal with the problem of blind adaptive implementation of the first stage of the newly proposed receiver. In our context, the term "adaptive" means that no knowledge on the MAI is assumed, i.e., only the spreading codes, the timing offsets and the channel gains for the users in $\mathcal{G}$ are assumed to be known[3], while the term "blind" means that no use of periodical training sequences is needed.

Regarding classical blind adaptive multiuser detection, well-established filtering algorithms which have been successfully applied to cope with such an issue are the LMS, the RLS [20], and the Projection Approximation Subspace Tracking with deflation (PASTd) algorithm [7], [22]. In what follows we show that these algorithms may be applied to our context to blindly implement the first-stage of the proposed receiver. We restrict our attention to the implementation of the MMSE-based detector, which has been shown to achieve a better performance than the ZF-based receiver.

To begin with, we start by considering the LMS, or stochastic-gradient, algorithm. Generalizing the approach of [6] to the group-detection context, we decompose the filter $\boldsymbol{D}$ in the following two components:

$$\boldsymbol{D} = \boldsymbol{D}_0 + \boldsymbol{X} \ , \tag{13}$$

wherein $\boldsymbol{D}_0 = \boldsymbol{S}_G(\boldsymbol{S}_G^H \boldsymbol{S}_G)^{-1}$ and $\boldsymbol{X}$ is a matrix whose column span is orthogonal to the column span of $\boldsymbol{S}_G$. Note that the above decomposition ensures that the constraint in (10) (with $\boldsymbol{K} = \boldsymbol{I}_G$) is satisfied, thus implying that the adaptive algorithm task is to update in a recursive fashion the matrix $\boldsymbol{X}$ only. Accordingly, at the n-th symbol interval, the LMS algorithms selects the estimate of the matrix $\boldsymbol{X}$, say $\boldsymbol{X}(n)$, according to the rule

$$\boldsymbol{X}(n) = \boldsymbol{X}(n-1) - \mu \boldsymbol{r}^{\perp}(n)\boldsymbol{r}^H(n)\boldsymbol{X}(n-1) \ , \tag{14}$$

wherein μ is the algorithm step size and

$$\boldsymbol{r}^{\perp}(n) = \boldsymbol{r}(n) - \boldsymbol{S}_G(\boldsymbol{S}_G^H \boldsymbol{S}_G)^{-1}\boldsymbol{S}_G \boldsymbol{r}(n)$$

is the projection of the data onto the orthogonal complement of the span of the matrix $\boldsymbol{S}_G$. This projection ensures that, for any n, the span of $\boldsymbol{X}(n)$ and $\boldsymbol{S}_G$ are kept orthogonal. Obviously, the estimate of the filter $\boldsymbol{D}$ at the n-th iteration, say $\boldsymbol{D}_b(n)$, is obtained as

$$\boldsymbol{D}_b(n) = \boldsymbol{D}_0 + \boldsymbol{X}(n) \ .$$

Notice that the computational complexity of the iteration (14) is $\mathcal{O}(NMG)$.

Besides the LMS procedure, another popular adaptive filtering algorithm is the RLS algorithm. Given the observables $\{\boldsymbol{r}(i)\}_{i=0}^n$, the exponentially windowed RLS algorithm selects, at the n-th iteration, the $NM \times G$-dimensional matrix, $\boldsymbol{D}_b(n)$ say, which solves the following constrained minimization problem:

$$\begin{cases} \min \sum_{i=0}^{n} \lambda^{n-i} \left\| \boldsymbol{D}_b^H(n)\boldsymbol{r}(i) \right\|^2 \ , \\ \text{subject to} \quad \boldsymbol{D}_b^H(n)\boldsymbol{S}_G = \boldsymbol{I}_G \ , \end{cases} \tag{15}$$

wherein λ, the "forgetting factor," is a constant slightly smaller than unity aimed at ensuring the tracking capability of the algorithm. The solution to the above problem can be shown to be written as

$$\boldsymbol{D}_b(n) = \boldsymbol{R}_{\boldsymbol{rr}}^{-1}(n)\boldsymbol{S}_G \left(\boldsymbol{S}_G^H \boldsymbol{R}_{\boldsymbol{rr}}^{-1}(n) \boldsymbol{S}_G \right)^{-1} \ , \tag{16}$$

with $\boldsymbol{R}_{\boldsymbol{rr}}(n) = \sum_{i=0}^n \lambda^{n-i}\boldsymbol{r}(i)\boldsymbol{r}^H(i)$. A recursive procedure for updating the solution (16) is thus given by:

$$\begin{aligned} \boldsymbol{k}(n) &= \frac{\boldsymbol{R}_{\boldsymbol{rr}}^{-1}(n-1)\boldsymbol{r}(n)}{\lambda + \boldsymbol{r}^H(n)\boldsymbol{R}_{\boldsymbol{rr}}^{-1}(n-1)\boldsymbol{r}(n)} \ , \\ \boldsymbol{R}_{\boldsymbol{rr}}^{-1}(n) &= \frac{1}{\lambda}\left(\boldsymbol{R}_{\boldsymbol{rr}}^{-1}(n-1) - \boldsymbol{k}(n)\boldsymbol{r}^H(n)\boldsymbol{R}_{\boldsymbol{rr}}^{-1}(n-1) \right) \ , \\ \boldsymbol{D}_b(n) &= \boldsymbol{R}_{\boldsymbol{rr}}^{-1}(n)\boldsymbol{S}_G \\ &= \frac{1}{\lambda}\left(\boldsymbol{D}_b(n-1) - \boldsymbol{k}(n)\boldsymbol{r}^H(n)\boldsymbol{D}_b(n-1) \right) \ . \end{aligned} \tag{17}$$

Notice that, as already anticipated, the above procedure can be implemented with no knowledge on the external interference, nor does it require the usage of known training sequences. Additionally, it permits avoiding on-line inversion of the matrix $\boldsymbol{R}_{\boldsymbol{rr}}(n)$ (which is a square matrix of order NM). Summing up, it may be concluded that the computational complexity of algorithm (17) is $\mathcal{O}\left((NM)^2\right) + \mathcal{O}\left(GNM\right)$.

Even though the RLS algorithms is capable of tracking the true solution (12) with satisfactory performance, there are some

[2] We recall that, as already anticipated, the performance improvement for increasing M is strictly tied to the use of rectangular (infinite-bandwidth) chip-pulses.

[3] Notice that this is exactly the same information required by a bank of G conventional matched-filter detectors.

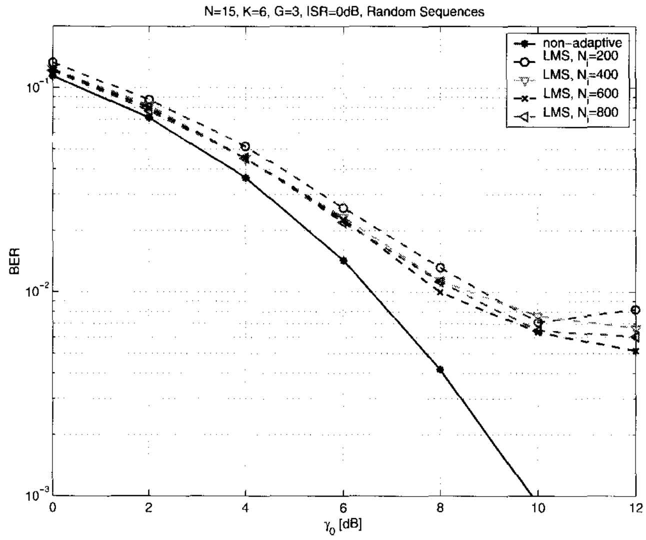

Fig. 6. System BER for the LMS-based adaptive group detector.

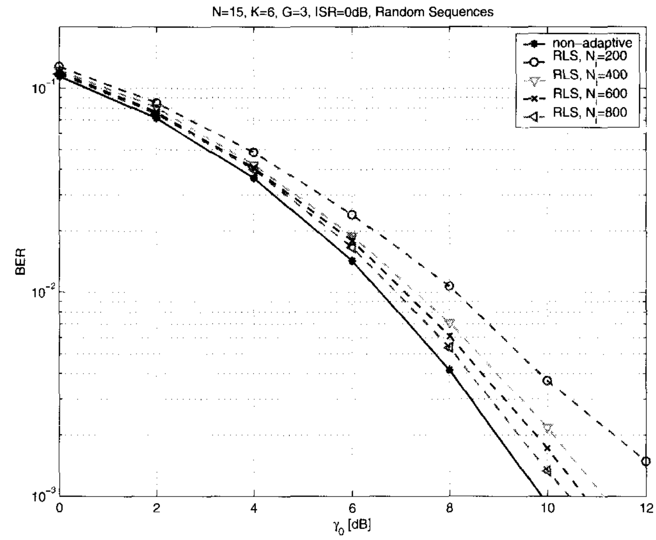

Fig. 7. System BER for the RLS-based adaptive group detector.

reasons that lead to develop alternative procedures. First of all, the computational complexity is quadratic in the product NM; additionally, as pointed out in [8, Part II], its steady-state performance in terms of Signal-to-Interference Ratio (SIR) is poorer than that of a non-blind decision-directed RLS procedure. Finally, both the LMS and the RLS algorithms are very sensitive to the signature waveform mismatch problem: for example, as shown in [6], if, due to imperfect estimates of the delays, the assumed signature waveforms differ from the actual ones, the constrained minimum MOE criterion (15) may lead to complete cancellation of the useful signals at the output of the receiver first-stage. In [7] it has been shown that these drawbacks may be circumvented by resorting to a subspace-based formulation of both the MMSE and ZF linear receivers and applying the PASTd algorithm for their blind adaptive implementation.

In order to extend such an approach to the case at hand, we begin by noticing that the covariance matrix of the observables may be decomposed as

$$\begin{aligned} \boldsymbol{R_{rr}} &= [\boldsymbol{U}_s \ \boldsymbol{U}_n] \begin{bmatrix} \boldsymbol{\Lambda}_s & \\ & \boldsymbol{\Lambda}_n \end{bmatrix} \begin{bmatrix} \boldsymbol{U}_s^H \\ \boldsymbol{U}_n^H \end{bmatrix} \\ &= \boldsymbol{U}_s \boldsymbol{\Lambda}_s \boldsymbol{U}_s^H + \boldsymbol{U}_n \boldsymbol{\Lambda}_n \boldsymbol{U}_n^H . \end{aligned} \tag{18}$$

In the above (18), $\boldsymbol{\Lambda}_s$ is a diagonal matrix containing the r largest eigenvalues of the matrix $\boldsymbol{R_{rr}}$, $\boldsymbol{U}_s$ is an $NM \times r$-dimensional matrix containing the corresponding eigenvectors, r is the signal subspace dimensionality[4], i.e., the rank of the matrix $\boldsymbol{R}$ defined in Secton III-A, $\boldsymbol{\Lambda}_n = 2\mathcal{N}_0 \boldsymbol{I}_{NM-r}$ contains the remaining $NM - r$ eigenvalues of the matrix $\boldsymbol{R_{rr}}$, and, finally, $\boldsymbol{U}_n$ contains the $NM - r$ orthonormal eigenvectors corresponding to the eigenvalue $2\mathcal{N}_0$. It can be easily shown that the column span of $\boldsymbol{U}_s$ coincides with the span of the signatures $\left\{ \{s_k^i\}_{i \in \{-1,0\}} \right\}_{k=G}^{K-1} \bigcup \{s_k^0\}_{k=0}^{G-1}$, whence it is called the *signal subspace*; likewise, the column span of $\boldsymbol{U}_n$ is its orthogonal complement, and is called the *noise subspace*. Since processing the components of the received signal lying in the noise subspace does not add any useful information, it is easily understood that the first-stage linear filter $\boldsymbol{D}$ depends on the signal subspace parameters only: Otherwise stated, it is possible to express the filter $\boldsymbol{D}$ as a linear combination of the columns of $\boldsymbol{U}_s$, i.e.,

$$\boldsymbol{D} = \boldsymbol{U}_s \boldsymbol{C} ,$$

with $\boldsymbol{C}$ denoting an $r \times G$-dimensional coefficient matrix. Substituting the above expression into problems (6) and (10) and solving with respect to $\boldsymbol{C}$ yields the solutions

$$\boldsymbol{D} = \boldsymbol{U}_s \left(\boldsymbol{\Lambda}_s - 2\mathcal{N}_0 \boldsymbol{I}_r \right)^{-1} \boldsymbol{U}_s^H \boldsymbol{S}_G , \tag{19}$$

and

$$\boldsymbol{D} = \boldsymbol{U}_s \boldsymbol{\Lambda}_s^{-1} \boldsymbol{U}_s^H \boldsymbol{S}_G , \tag{20}$$

respectively. (19) and (20) are alternative formulations (in terms of the signal subspace parameters) of expressions (8) and (12), respectively. It is thus seen that the first stage of the proposed receiver is obtained once the signal subspace components are identified. As a consequence, the problem of blind adaptive receiver implementation is now reduced to the problem of blind tracking of the eigenvectors and eigenvalues of the signal subspace. Subspace tracking algorithms can be thus applied so as to come up with several blind adaptive implementations of the filter $\boldsymbol{D}$. Many subspace tracking algorithms have been so far developed (see [23], [24] and references therein). In keeping with [7], in this work we consider the recently proposed PASTd algorithm [22]; the most relevant advantages of such an algorithm lie in almost sure global convergence to the signal eigenvectors and eigenvalues, low computational complexity (which, for the case at hand, is $\mathcal{O}(NMr) = \mathcal{O}(NM(2K - G))$), and rank tracking capability [25]. A thorough exposition of such an algorithm would not add much conceptual value to this paper, since the procedure outlined in [7], [22], and [25] can be directly applied, with no modifications, to the case at hand, so as to track the signal subspace components $\boldsymbol{U}_s$ and $\boldsymbol{\Lambda}_s$ and to obtain an estimate of the filter $\boldsymbol{D}$ through (19) and (20).

[4]If we assume that all of the signatures are linearly independent, it is easily shown that $r = 2K - G$.

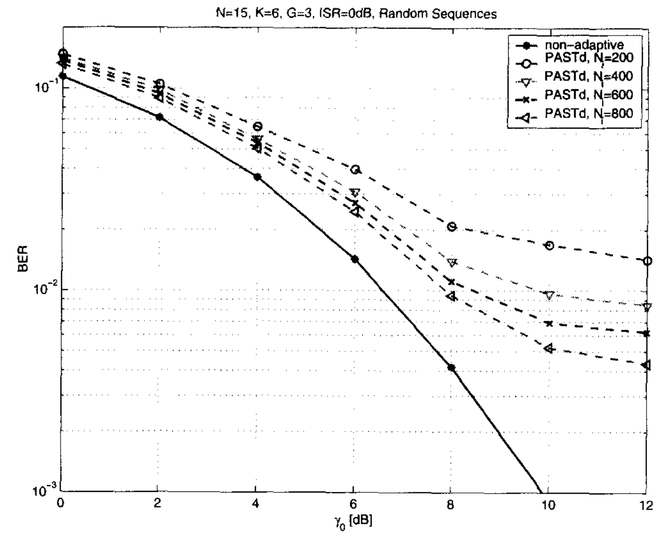

Fig. 8. System BER for the PASTd-based adaptive group detector.

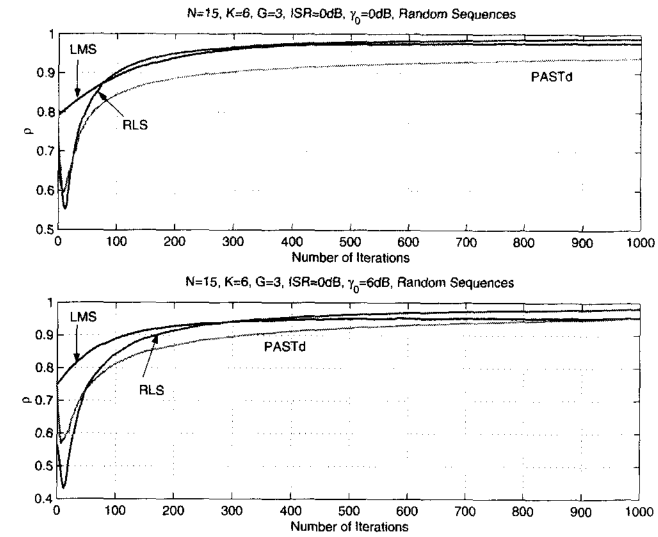

Fig. 9. Convergence dynamics of the adaptive algorithms for $\gamma_0 = 0, 6$dB.

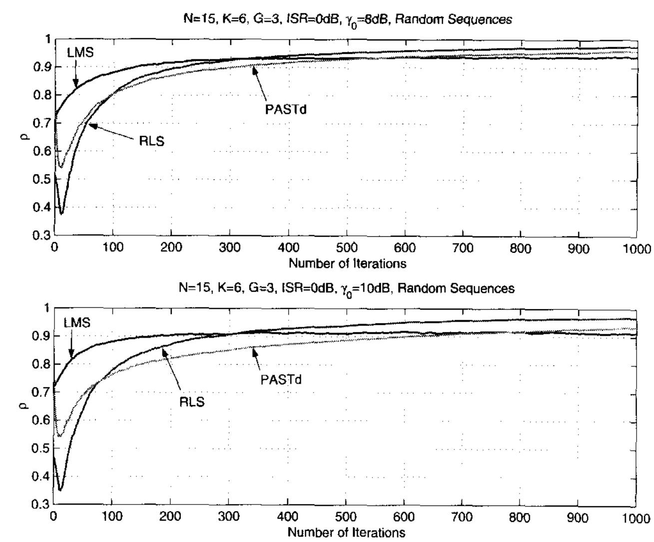

Fig. 10. Convergence dynamics of the adaptive algorithms for $\gamma_0 = 8, 10$dB.

A. Simulation Example

In order to test the effectiveness of the adaptive implementations of the MMSE-based detector, we have considered as performance measure the system BER and the following correlation coefficient:

$$\rho(n) = \frac{\operatorname{tr}\left(\boldsymbol{D}_b^H(n)\boldsymbol{D}\right)}{\sqrt{\operatorname{tr}\left(\boldsymbol{D}_b^H(n)\boldsymbol{D}_b(n)\right)\operatorname{tr}\left(\boldsymbol{D}^H\boldsymbol{D}\right)}}, \quad (21)$$

wherein $\boldsymbol{D}$ is the true solution given by (11) (or, equivalently, by (20)), while $\boldsymbol{D}_b(n)$ is the matrix estimated by the adaptive procedures at the n-th iteration. In the following, we have considered an asynchronous DS/CDMA system employing random spreading sequences with processing gain $N = 15$. The users number K has been set equal to 6, while $G = 3$. The oversampling factor is $M = 1$. In Figs. 6, 7, and 8, we have reported the system BER for the blind adaptive group detector based on the LMS, the RLS and the PASTd algorithm, respectively, versus γ_0. More precisely, in each plot we report the system BER for a number of iterations N_i equal to 200, 400, 600, and 800. For comparison purposes, we also report the BER of the non-adaptive MMSE group detector. The plots shown are the result of an average over 50 independent realizations of the spreading codes and delays for all the users. Since the convergence dynamics of the PASTd algorithm were found to be very sensitive to the initial estimate of the signal space parameters, the eigencomponents of the signal space at the 0-th iteration were obtained by applying a Singular-Value-Decomposition (SVD) to the sample covariance matrix based upon N data vectors. It is seen that the adaptive algorithms exhibit a satisfactory BER performance, especially the RLS, whose performance loss with respect to the non-adaptive system is less than 1 dB. Conversely, the loss incurred by the LMS and the PASTd algorithm (which are simpler to implement than the RLS) is slightly larger. However, in any case the system BER falls below 10^{-2} and, since the effect of coding and interleaving has not been considered here, this is an acceptable value. In Figs. 9 and 10, instead we report the parameter ρ versus the number of iterations for different values of γ_0. Also in this case results confirm that the proposed algorithms are able to converge to the non-adaptive filter $\boldsymbol{D}$ with good accuracy.

VI. CONCLUSIONS

In this work, a group detector for DS/CDMA systems has been presented. It resorts to a two stage receiving structure; the first stage is a linear block aimed at suppressing multiuser interference, while the second stage is a non-linear device implementing a maximum-likelihood decision strategy with respect to the set of the users of interest. Remarkably, the new structure encompasses, as a special case, the well-known Varanasi's group detector, and it outperforms previously derived group detection structures.

Also, the problem of blind adaptive receiver implementation has been tackled, and indeed recursive procedures have been proposed so as to blindly implement the first stage of the receiver, and numerical results have shown that the proposed adaptive algorithms are effective in tracking the true solution.

It is worth recalling that the proposed receiving structures are suited for adoption in a base station so as to suppress the extra-cell interference, as well as in mobile receivers in a multirate CDMA system implemented with a multicode access technique. To this end, a mandatory outgrow of the results presented here is the consideration of a more realistic communication channel, i.e., the generalization of the proposed structures to the case, of primary interest in mobile communications applications, of fading dispersive channels. Preliminary results on this issue have been reported in [26].

APPENDIX

Here we show that, for a synchronous CDMA system with linearly independent signatures the newly proposed receiver based upon the decorrelating approach (and with no oversampling, i.e., $M = 1$) coincides with Varanasi's group detector derived in [10]. First of all, we re-write the matrix $\boldsymbol{D}$ in (9) as

$$\boldsymbol{D} = \boldsymbol{U}\boldsymbol{\Lambda}\boldsymbol{V}^H , \tag{22}$$

wherein $\boldsymbol{U}$ is an $N \times G$-dimensional matrix whose columns are an orthonormal basis for the subspace of $\mathcal{C}^N$, say $\mathcal{R}(\boldsymbol{D})$, spanned by the columns of $\boldsymbol{D}$, $\boldsymbol{\Lambda}$ is a G-dimensional diagonal matrix and $\boldsymbol{V}$ is an unitary $G \times G$-dimensional matrix. Notice also that, due to expression (9), $\mathcal{R}(\boldsymbol{D})$ is the intersection of $\mathcal{R}(\boldsymbol{S}_G)$ and the orthogonal complement to $\boldsymbol{s}_G^0, \cdots, \boldsymbol{s}_{K-1}^0$, i.e., it is a G-dimensional subspace of $\mathcal{C}^N$ orthogonal to the $(K-G)$ MAI signatures and to the $(N-K)$-dimensional noise subspace too. Now, since $\left(\boldsymbol{D}^H\boldsymbol{D}\right)^{-1} = \boldsymbol{V}\boldsymbol{\Lambda}^{-2}\boldsymbol{V}^H$, we have equation (23) at the bottom of this page with $\boldsymbol{r}_D(p) = \boldsymbol{U}\boldsymbol{U}^H\boldsymbol{r}(p)$ the component of $\boldsymbol{r}(p)$ lying in the subspace $\mathcal{R}(\boldsymbol{D}) = \mathcal{R}(\boldsymbol{U})$. Given (23), the decision rule of the newly proposed detection structure is re-written as

$$\widehat{\boldsymbol{b}}_G(p) = \arg \min_{\boldsymbol{x}_G \in \{+1,-1\}^G} \left\| \boldsymbol{U}^H \left(\boldsymbol{r}_D(p) - \sum_{k=0}^{G-1} A_k e^{j\phi_k} x_k \boldsymbol{s}_k^0 \right) \right\|^2 \tag{24}$$

Let us now consider Varanasi's group detector. If we denote by $\boldsymbol{S}_{\overline{G}}$ the $N \times (K-G)$-dimensional matrix containing on its columns the MAI signatures $\left[\boldsymbol{s}_G^0 \cdots \boldsymbol{s}_{K-1}^0\right]$, and by $\boldsymbol{P}$ the projector onto the orthogonal complement to the subspace $\mathcal{R}\left(\boldsymbol{S}_{\overline{G}}\right)$, Varanasi's detection rule, which is based on the application of the GLRT strategy, is written as

$$\widehat{\boldsymbol{b}}_G(p) = \arg \min_{\boldsymbol{x}_G \in \{+1,-1\}^G} \left\| \boldsymbol{P} \left(\boldsymbol{r}(p) - \sum_{k=0}^{G-1} A_k e^{j\phi_k} x_k \boldsymbol{s}_k^0 \right) \right\|^2 . \tag{25}$$

Now, notice that the matrix $\boldsymbol{P}$ performs a projection onto an $N-(K-G)$-dimensional subspace. Such a subspace, in turn, can be decomposed as the direct sum of an $(N-K)$-dimensional subspace, say $\mathcal{S}_n$, defining the noise subspace, and of a G-dimensional subspace. Since the columns of the rank G matrix $\boldsymbol{D}$ in (9) are orthogonal to $\mathcal{R}\left(\boldsymbol{S}_{\overline{G}}\right)$, the latter G-dimensional subspace necessarily coincides with $\mathcal{R}(\boldsymbol{D}) = \mathcal{R}(\boldsymbol{U})$. Otherwise stated, the projector $\boldsymbol{P}$ can be written as

$$\boldsymbol{P} = \boldsymbol{U}\boldsymbol{U}^H + \boldsymbol{U}_n\boldsymbol{U}_n^H , \tag{26}$$

wherein $\boldsymbol{U}_n$ is an $N \times (N-K)$-dimensional matrix containing on its columns a basis for the noise subspace $\mathcal{S}_n$, and $\boldsymbol{U}$ is the matrix defined in (22). Substituting expression (26) into the decision rule (25) we have equation (27) at the bottom of this page wherein the second equality follows from the fact that the desired signals' signatures are orthogonal to $\mathcal{R}(\boldsymbol{U}_n)$ and the third

$$\begin{aligned} &\left(\boldsymbol{r}(p) - \sum_{k=0}^{G-1} A_k e^{j\phi_k} x_k \boldsymbol{s}_k^0 \right)^H \boldsymbol{D}(\boldsymbol{D}^H\boldsymbol{D})^{-1}\boldsymbol{D}^H \left(\boldsymbol{r}(p) - \sum_{k=0}^{G-1} A_k e^{j\phi_k} x_k \boldsymbol{s}_k^0 \right) = \\ &\left\| \boldsymbol{U}^H \left(\boldsymbol{r}(p) - \sum_{k=0}^{G-1} A_k e^{j\phi_k} x_k \boldsymbol{s}_k^0 \right) \right\|^2 = \left\| \boldsymbol{U}^H \left(\boldsymbol{r}_D(p) - \sum_{k=0}^{G-1} A_k e^{j\phi_k} x_k \boldsymbol{s}_k^0 \right) \right\|^2 , \end{aligned} \tag{23}$$

$$\begin{aligned} \widehat{\boldsymbol{b}}_G(p) &= \arg \min_{\boldsymbol{x}_G \in \{+1,-1\}^G} \left\| \left(\boldsymbol{U}\boldsymbol{U}^H + \boldsymbol{U}_n\boldsymbol{U}_n^H \right) \left(\boldsymbol{r}(p) - \sum_{k=0}^{G-1} A_k e^{j\phi_k} x_k \boldsymbol{s}_k^0 \right) \right\|^2 \\ &= \arg \min_{\boldsymbol{x}_G \in \{+1,-1\}^G} \left[\left\| \boldsymbol{U}\boldsymbol{U}^H \left(\boldsymbol{r}(p) - \sum_{k=0}^{G-1} A_k e^{j\phi_k} x_k \boldsymbol{s}_k^0 \right) \right\|^2 + \left\| \boldsymbol{U}_n\boldsymbol{U}_n^H \boldsymbol{r}(p) \right\|^2 \right] \\ &= \arg \min_{\boldsymbol{x}_G \in \{+1,-1\}^G} \left\| \boldsymbol{U}^H \left(\boldsymbol{r}_D(p) - \sum_{k=0}^{G-1} A_k e^{j\phi_k} x_k \boldsymbol{s}_k^0 \right) \right\|^2 , \end{aligned} \tag{27}$$

equality from the fact that $U^H r(p) = U^H r_D(p)$. Since the decision rules (27) and (24) are coincident, the thesis is proven.

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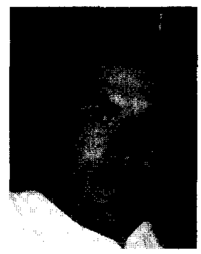

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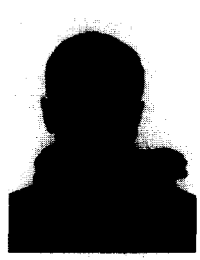

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