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Statistical Characteristics of Polarization-Sensitive Optical Coherence Tomography for Tissue Imaging

Jung-Taek Oh*

Institute for Medical Engineering, Yonsei Univ., Wonju 220-842, KOREA

Beop-Min Kim

Department of Biomedical Engineering, Yonsei Univ., Wonju 220-710, KOREA

Seung-Woo Kim

Department of Mechanical Engineering, Korea Advanced Institute of Science and Technology, Daejeon 307-101, KOREA

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Statistical characteristics of the backscattered light from turbid tissues obtained by polarization-sensitive optical coherence tomography are investigated. The amplitude of the backscattered light is found to faithfully follow the Rayleigh distribution predicted by the scattering theory of electromagnetic waves in random media. The probability density function of the phase difference between the two orthogonal polarization components of the backscattered light is explicitly derived and then verified in comparison with the experimental data measured from *in-vitro* tissues of porcine ligament.

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Optical Coherent Tomography (OCT) provides highly depth-resolved images of translucent light-scattering objects by localizing the backscattered light by means of low coherence interferometry [1]. Polarization Sensitive Optical Coherence Tomography (PS-OCT) is an evolved form of OCT, which enables to monitor not only the amplitude of the backscattered light but also the change of polarization caused by the internal medium of the specimen [2,3]. A recent advance made in the PS-OCT equipment allows determination of the Stokes parameters in combination of two orthogonal polarization components of the backscattered light, which consequently offers diverse images with enhanced discernment of the internal structure of the specimen [4,5].

The amplitude and phase data actually measured from PS-OCT equipment are found significantly affected by many spurious signals such as the optical speckles generated by randomly scattered light and also by the electric noises with relatively low signal-to-noise ratio due to heavy scattering. Besides, the depolarization of the backscattered light due to complicated scattering phenomena causes many difficulties in performing accurate measurement of polarization change. For better interpretation of the images of the PS-OCT, the measured amplitude and phase data should be evaluated by means of appropriate statistical analysis considering not only the equipment used but also the random nature of the optical scattering within the specimen.

General statistics of the backscattered electric field from random media has been investigated for many years especially for the particular application of remote sensing using polarized electromagnetic waves [6,7]. As a result, the amplitude of the backscattered field is known to be the Rayleigh distributed, while the phase is uniformly distributed. In addition, the probability density function of the phase difference between two orthogonal polarization components of the scattered field has been explicitly derived in terms of the mean effective phase difference and the degree of polarization [8]. In this paper, the general scattering theory of electromagnetic waves in random media is applied to the particular case of PS-OCT for imaging turbid tissues, and its validity is examined in comparison with the experimental data observed from *in-vitro* tissues of porcine ligament.

Scattering behaviors within random media such as turbid tissues are so complex that they need be treated in a statistical way. The backscattered light from translucent light-scattering media undergoes depolarization, whose precise level of polarization loss depends on the type of the particles involved in scattering [8,9]. The backscattered light also experiences turbulent fluctuations in amplitude and phase because the local refractive index of light-scattering media varies microscopically and randomly. Most tissues are found to exhibit intensity fluctuations with a spatial frequency of 0.02~5 μm^{-1} within a continuum length of 4 to 10 μm. When the light source used has a 10 μm coherent length, the backscattered light may be regarded as a sum of individual fields backscattered from as many as 20 scattering cross-sections dispersed within the coherent length [10]. Consequently, one may take a simplification that tissues are made up of multiple stratified laminates whose layer thickness is about the same as the coherent length of the source. In addition, each laminate may well be assumed statistically homogeneous, stationary, and independent of the others.

In PS-OCT, the electric field E of the backscattered light from a single laminate is captured in terms of two orthogonal polarization components such as $E_1 = a_1 \exp(j\delta_1)$ and $E_2 = a_2 \exp(j\delta_2)$. The subscripts 1 and 2 indicate two polarization components, and a and δ represent the amplitude and the phase, respectively. It may be assumed that the electric fields E_1 and E_2 are sums of many independent scattered fields that are Gaussian distributed, then the central limit theorem says that the amplitudes a_1 and a_2 are Rayleigh distributed, and the phases δ_1 and δ_2 are uniformly distributed [11,12]. In particular, for the partially polarized light scattered from tissues, the amplitude and phase of two orthogonal polarizations are random but statistically dependent, with the statistical relationship described in terms of the joint probability function of [6,8]

$$f_{a_1 a_2 \delta}(\delta) = \frac{2a_1a_2}{\pi\sigma_1^2\sigma_2^2(1-p^2)} \exp\left(-\frac{(\sigma_2^2a_1^2 + \sigma_1^2a_2^2 - 2\sigma_1\sigma_2a_1a_2p\cos(\delta-\beta))}{\sigma_1^2\sigma_2^2(1-p^2)}\right) \tag{1}$$

Note that δ is the phase difference, i.e., $\delta = \delta_2 - \delta_1$, p is the degree of polarization, and β is the mean effective phase difference that is defined as $p\exp(j\beta) =< a_1a_2\exp(j\delta) > /\sigma_1\sigma_2$. In addition, $\sigma_1^2 =< a_1^2 >$ and $\sigma_2^2 =< a_2^2 >$, where $<\ >$ denotes the ensemble average. Then, by integrating the above joint probability function with respect to a_1 and a_2, the probability density function for the phase difference δ is explicitly obtained as [7]

$$f_\delta(\delta) = \begin{cases} \frac{1-p^2}{2\pi}\left[1 + \frac{(\pi - \tan^{-1}(\gamma))(1+\gamma^2)}{\gamma^3} + \frac{1}{\gamma^2}\right] & \text{if} \cos(\delta-\beta) > 0 \\ \frac{1-p^2}{2\pi}\left[1 - \frac{\tan^{-1}(\gamma)(1+\gamma^2)}{\gamma^3} + \frac{1}{\gamma^2}\right] & \text{if} \cos(\delta-\beta) \le 0 \end{cases} \tag{2}$$
$$\text{where} \quad \gamma = \sqrt{1-p^2\cos^2(\delta-\beta)}/p\ |\cos(\delta-\beta)|$$

The result indicates that the probability density function of the phase difference δ takes a unique distribution, which is different from the normal distribution and governed by the degree of polarization.

The PS-OCT system configured in this investigation is shown in Fig. 1. A super-luminescent diode emits a source beam of 900 μW through a single-mode fiber. The source beam has a center wavelength of λ_0 = 684.5 nm and its spectral bandwidth is 9.1 nm. The output power of the source beam reduces to 500 μW after passing through the linear polarizer adopted for selecting a single linear polarization component. In the reference arm, a quarter wave Fresnel-Rhomb prism is set to rotate the incident light beam by 22.5° to the horizontal plane, so the returning wave experiences a total rotation of 45°. Consequently, the reference beam is evenly divided into two orthogonal polarization components, which are then individually measured by two separated detectors. In the test arm, the beam is made circularly polarized by use of

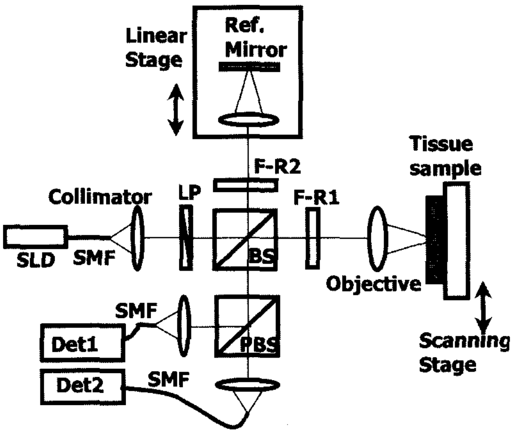

FIG. 1. Schematic of the PS-OCT system. SLD: super luminescent diode, LP: linear polarizer, F-R: Fresnel-Rhomb prism, BS: non-polarizing beam splitter, PBS: polarizing beam splitter, SMF: single-mode fiber, Det: detector. F-R1 and F-R2 rotate by 22.5° and 45°,respectively, from the polarizing direction of LP.

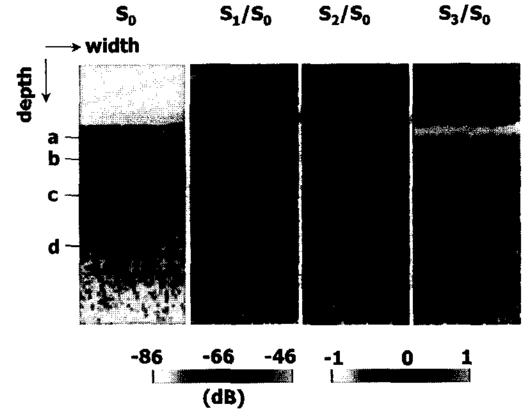

FIG. 2. Two-dimensional images of four Stokes parameters measured for the specimen of porcine ligament. The physical size of images is 1.0 mm in depth by 0.35 mm in width. The degree of polarization at the depths marked **a**, **b**, **c** and **d** in the above images reads 0.86, 0.76, 0.53 and 0.26, respectively.

another Fresnel-Rhomb prism to rotate polarization by 45°. The test and the reference beams are recombined and then divided by a polarizing beam splitter (horizontal and vertical) and coupled to two single-mode fibers operating as spatial filters with a small aperture size. The two orthogonal polarization components of the light returning from the specimen are heterodyned with the reference beam, which provides a 7 kHz Doppler frequency, and sampled by analog-to-digital converters. The sampled signals are band-pass filtered with a bandwidth of 40Hz being centered at the Doppler frequency. The Hilbert transformation pair of both the interference signals is obtained first, and then the envelope and the phase of both the signals are extracted to determine the Stokes parameters as well as the phase difference [5].

An in-vitro tissue of porcine ligament, whose predominant constituent is the type I collagen, was selected as the specimen. Fig. 2 shows the two-dimensional maps of the four Stokes parameters obtained from the specimen. The measured result indicates that there is a strong birefringence layer parallel to the top surface, and that the Stokes parameters vary significantly in the depth direction, while no apparent change is detected to the width direction. The observation justifies well the assumption that the specimen is regarded as a sum of individual stratified laminates.

The assumption that the electric field of the backscattered light from turbid tissues is a sum of randomly scattered individual fields of the Gaussian distribution may be verified by means of checking the distribution of measured intensity data. The statistical theory based on the central limit theorem tells whether the electric field is Gaussian distributed, its amplitude is Rayleigh distributed and its intensity I holds the unique statistical property of $< I^n >=< I >^n n!$ for n=positive integer [12]. Fig. 3 shows a result of correlation between $< I^n >$ and $< I >^n n!$ for the intensity values measured at the four different depths indicated in Fig. 2. The result reveals a good linear agreement, confirming that the assumption of Gaussian distribution may well be justified for tissue imaging.

Fig. 4 presents four cases with different values of degree of polarization (DOP), where the theoretical prediction of the probability density function (PDF) of Eq.(2) is verified. Experimental data of phase differences measured at the four different depths previously selected in Fig. 2 were used to obtain the actual

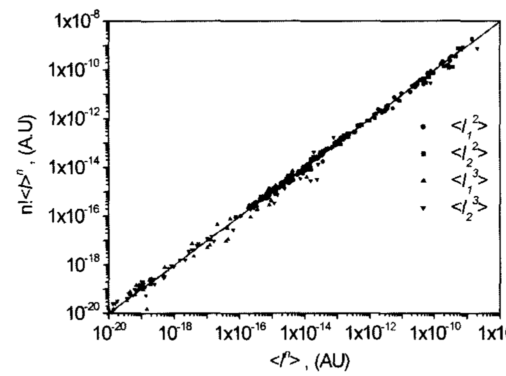

FIG. 3. Test result for verification of the Rayleigh distribution of the amplitude of the intensity signal monitored from the specimen of porcine ligament. The 2^{nd} and 3^{rd} moments of intensity are compared with $< I >^2$ 2! and $< I >^3$ 3! for two orthogonal polarization signals.

TABLE 1. chi-square goodness-of-fit test result

DOP	χ^2 test statistics	α level of significance
0.86	88.1	< 0.005
0.76	40.2	0.005
0.53	38.3	0.01
0.26	36.7	0.01

(Number of observations: $N = 216$, number of bins: $K = 24$; degree of freedom: $n = 21$, cutoff for α level $\chi^2_{n;\alpha}$: $\chi^2_{21;0.025} = 35.5$, $\chi^2_{21;0.01} = 38.9$, $\chi^2_{21;0.005} = 41.4$)

distribution of phase differences. DOP values were determined from the averaged Stokes parameters obtained from the experimental data. The mean effective phase difference β, which indicates the birefringence direction of the tissue specimen, is clearly seen in all the cases. For high DOP values, the measured PDF yields a sharp peak near the mean effective phase difference. On the other hand, for low DOP values, the measured PDF tends to yield a broadened profile as the theoretical PDF predicts, indicating less reliability of the measured phase data. The chi-square goodness-of-fit test was adopted to quantify the correlation between the theoretical and measured PDFs. The result is summarized in Table.1 [13]. The hypothesis of the equivalence between the theoretical and measured PDFs is accepted at the α=0.01 or 0.005 level of significance. For the case of highest DOP in Fig. 4(a), the chi-square test fails because the very small expected value on the tail of the measured distribution induces numerical instability [13].

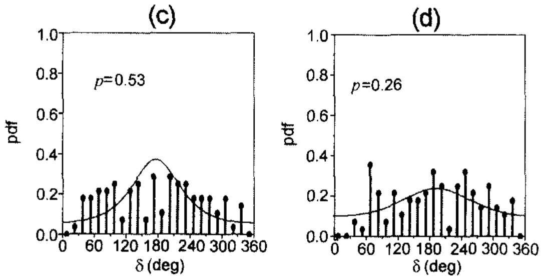

FIG. 4. Comparison of experimental data with theoretical probability density function (PDF); δ : phase difference, p: degree of polarization. Dots denote experiment data and the solid lines are theoretical curves. Experimental data for (a), (b), (c) and (d) respectively correspond to the depths **a**, **b**, **c** and **d** marked in Fig. 2.

Statistical characteristics of polarization-sensitive optical coherence tomography were investigated especially for tissues imaging. Experimental data obtained from *in-vitro* tissues of porcine ligament was analyzed to confirm that the amplitude of the backscattering faithfully follows the Rayleigh distribution predicted by the scattering theory of electromagnetic waves in random media. The theoretical probability density function of the phase difference between the two orthogonal polarization signals was in good agreement especially when the degree of polarization is high. It is also concluded that the measured phase information to estimate Stokes parameters loses reliability when the degree of polarization becomes low.

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*Corresponding author : cliff@dragon.yonsei.ac.kr.

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