순차적 시뮬레이션을 위한 순차적인 Percentile 추정에 관한 연구

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요 약

백분위수는 시뮬레이션 결과의 전체적인 성향을 파악하는데 아주 유용한 측정 기법 중의 하나이다. 그러나, 시뮬레이션으로 수집된 데이터들에 대한 평균이나 표준편차와는 달리 백분위수를 추정하기 위해서는 모든 관측된 데이터들을 저장해야 만 한다. 왜냐하면 백분위수의 추정을 위해서는 관측된 모든 데이터를 분류하여 오름차순으로 정렬하는 등 여러 단계의 처리과정이 필요하기 때문이다. 따라서, 백분위수 추정을 위해서는 관측된 모든 데이터를 저장하기 위한 대용량의 저장장치와 정렬을 위한 계산시간 ($O(n\log_2 n)$)이 요구된다. 이러한 문제점을 해결하기 위한 여러 백분위수 추정 기법들이 제안되었으나 고정된 샘플 크기의 시뮬레이션(fixed sample size simulation)을 수행할 경우에만 적용 가능하다 [11, 12, 21]. 본 논문에서는 3가지 백분위수 추정 기법(*linear* PE, *batching* PE, *spectral* P^2 PE)을 순차적인 안정상태 시뮬레이션(sequential steady-state simulation)에 적용하여 연구하였다. 또한, 3가지의 백분위수 추정 기법들에 대해 coverage 분석을 수행한 결과를 제시하였다.

Sequential Percentile Estimation for Sequential Steady-State Simulation

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ABSTRACT

Percentiles are convenient measures of the entire range of values of simulation outputs. However, unlike means and standard deviations, the observations have to be stored since calculation of percentiles requires several passes through the data. Thus, percentile estimation (PE) requires a large amount of computer storage and computation time. The best possible computation time to sort n observations is ($O(n\log_2 n)$), and memory proportional to n is required to store sorted values in order to find a given order statistic. Several approaches for estimating percentiles in RS (regenerative simulation) and non-RS, which can avoid difficulties of PE, have been proposed in [11, 12, 21]. In this paper, we implemented these three approaches known as : *linear* PE, *batching* PE, and *spectral* P^2 PE in the context of sequential steady-state simulation. Numerical results of coverage analysis of these three PE approaches are presented.

키워드 : 순차적 시뮬레이션(Sequential Simulation), 백분위수 추정(Percentile Estimation), 재생적 데이터 분석 방법(Regenerative Output Data Analysis), 재생적 시뮬레이션(Regenerative Simulation)

1. Introduction

In simulating a stochastic system, such as a queueing or an inventory system, the simulator is frequently more concerned with extreme performance of the system than with long run average. As opposed to averages, percentiles can account for extreme behaviour of the system. Percentiles are convenient measures of the entire range of values of simulation outputs. Analysts find percentiles particularly useful in estimating reasonable capacities for facilities, comparing the overall performance of alternative designs or establishing minimum standards. Therefore, from a practical point of view, the problem of estimating percentiles is quite important.

Let $X_1, \cdots, X_n, \cdots, X_N$ be a sample of independent and identically distributed(i.i.d) random variables from a continuous distribution $F_X(x)$ with a probability density function $f_X(x)$. For $0 < p < 1$, let

$$x_p = \inf\{x : F_X(x) \geq p\} = F_X^{-1}(p),$$

where $F_X^{-1}(p)$ is the inverse of $F_X(x)$ with derivative $1/f_X(x_p)$. The quantity x_p is the $100p^{th}$ percentile of $F_X(x)$.

Let $X_{(1)} \leq \cdots \leq X_{(n)} \leq \cdots \leq X_{(N)}$ be the order statistics corresponding to the sample. The usual non-parametric

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point estimator of x_p is the $100p^{th}$ sample percentile

$$\widehat{x_p} = X_{(\lfloor Np+1 \rfloor)}, \quad 0 < p < 1$$

where $\lfloor z \rfloor$ denotes the integral part of z.

The problem with using $\widehat{x_p}$ as an estimator is that the sample sizes required for adequate precision are prohibitively large. Both sorting times and memory sizes are then unrealistic. A measure of the inflation of sample size over the independence case has been investigated by Blomqvist [1]. Unlike means and standard deviations, the observations have to be stored since calculation of percentiles requires several passes through the data. Thus, percentile estimation (PE) requires a large amount of computer storage and very long runs for securing the credibility of the final results. It makes the amount of computation time very large. The best possible computation time to sort n observations is $O(n \log_2 n)$, and memory proportional to n is required to store sorted values in order to find a given order statistic. For example, extreme percentiles of the M/M/1/∞ queueing system with traffic intensity ρ = 0.9 requires a sample size of roughly 500,000 customers to estimate the 99^{th} percentile of the waiting time distribution to within plus or minus 10% accuracy. For the 99.9^{th} percentile, the required sample size is approximately 23,000,000. Clearly, storing and sorting the entire sequence is impractical in such a case. Actually, to produce an order-statistic point estimate of x_p requires storing the largest $(1-p)/N$ values of the sequence. However, this ordering must be dynamically maintained as the sequence is generated, a computationally expensive operation and an additional storage are also required to estimate the variance. The above results are derived from <Table 8> of Blomqvist [1] by Heidelberger and Lewis [9].

Several approaches for estimating percentiles in RS and non-RS, which can avoid the above difficulties, have been proposed in [11, 12, 21]. These three approaches are originally developed for traditional (non sequential) procedure.

Our motivation is finding the robust estimator of percentiles in sequential steady-state simulation. In this paper we discuss three approaches known as *linear* PE and *batching* PE for RS, and *spectral* P^2 PE for non-RS, in the context of sequential steady-state simulation. In Section 2 detailed sequential PE approaches of RS, which are *linear* PE and *batching* PE, are discussed. In Section 3 detailed sequential PE approach of non-RS, which is *spectral* P^2 PE, is discussed. In Section 4 numerical results of coverage analysis of three PE approaches are presented and conclusions are made in Section 5.

2. Sequential PE Approaches of RS

The regenerative method (RM) of simulation, first suggested by Cox and Smith [2], for analysis of observations collected from a regenerative process $\{X(t) : t \geq 0\}$ has been systematically developed by a number of authors [3-5]. The regenerative approach is motivated by the fact that many stochastic systems have the property of *starting afresh probabilistically* from time to time. The central idea of the RM is to exploit the fact that, when $\{X(t) : t \geq 0\}$ is a regenerative process, random variables between successive regeneration points are i.i.d. thus it can circumvent the autocorrelation problem in estimates. Iglehart [11], Moore [15], Seila [21], and Heidelberger and Lewis [9] have given special methods for processes $\{X_n\}$ with regenerative structure, i.e. processes for which there exist random time points at which the process restarts probabilistically. An example is the waiting time process $\{W_n\}$ in M/M/1/∞ queueing system which regenerates every time a customer arrives to find the queue empty, so that the waiting time of that customer is zero.

Detailed comparisons of Iglehart, Seila and Moore's approaches for PEs in fixed sample size approach are in [22]. Three methods mentioned differ significantly. Each has advantages and disadvantages, and the appropriate method will depend on the specific application. The summary of three methods' comparisons is as following <Table 1>.

<Table 1> Comparisons of Three PE Approaches

Methods	Statistical Precision	Computational Efficiency	Memory Efficiency
Iglehart	Moderate	High	Moderate
Seila	Moderate	High	Moderate
Moore	High	Low	Low

These three PE approaches for RS use fixed sample size analysis method even though sequential analysis of simulation output is generally accepted as the most efficient way for securing representativeness of samples of collected observations. In this paper, we consider two approaches, which are *Iglehart*'s (we will call it *linear*) and *Seila*'s (we will call it *batching*) methods for sequential PE because *Moore*'s approach does not consider memory and computing time ef-

ficiency as we can see the <Table 1>.

Among few possible criteria for stopping the simulation, probably the most commonly used one is based on the relative half width of the confidence interval at a given confidence level $(1-\alpha)$ defined as the ratio

$$\varepsilon = \frac{\Delta_x}{\overline{X}(n)} \quad 0 < \varepsilon < 1; \tag{1}$$

where $\overline{X}(n)$ is the estimation of mean μ_x of an analyzed process from the sequence of collected observations $x_1, x_2, \cdots, x_n$ and Δ_x is the half width of the confidence interval for the estimator. It is well known that if observations $x_1, x_2, \cdots, x_n$ can be regarded as realizations of independent and normally distributed random variables $X_1, X_2, \cdots, X_n$, then

$$\Delta_x = t_{n-1,1-\alpha/2}\ \hat{\sigma}\left[\overline{X}(n)\right],$$

where

$$\hat{\sigma}\left[\overline{X}(n)\right] = \sum_{i=1}^{n} \frac{\{x_i - \overline{X}(n)\}^2}{n(n-1)}$$

is the (unbiased) estimator of the variance of $\overline{X}(n)$, and $t_{n-1,1-\alpha/2}$ is the $(1-\alpha)$ percentile of the t-distribution with $(n-1)$ degrees of freedom.

The ratio of Equation (1) is called the *relative precision of the confidence interval*. The simulation experiment is stopped at the first checkpoint for which $\varepsilon \le \varepsilon_{\max}$, where $\varepsilon_{\max}$ is the required limit relative precision of the results at the $100(1-\alpha)\%$ confidence level, $0 < \varepsilon_{\max} < 1$.

2.1 Sequential PE using Linear Approach

The *linear* approach has originally been developed for the fixed sample size simulation by Iglehart [11]. In this paper, we modified the *linear* approach for sequential PE. First of all, the *linear* approach for sequential PE requires to first specify a grid of $h+1$ points $g_0 < g_1 < g_2 < \cdots < g_h$ so that all observations lie between g_0 and g_h. We used 21 grid points spaced 0.2 units which is reasonable for PE (but not for extreme PE) because all theoretical PE values of M/M/1/∞ queueing system are in grids. Next, this method estimates the cumulative distribution function only at grid points. Then, it interpolates linearly between these estimates to find the percentile estimate until the steady-state parameter has been estimated with the required relative precision.

Let us consider how the percentile Q_p would be estimated in the course of a simulation experiment by collecting observations in h bins, where an observation is put into bin i if the observation is between grid point g_{i-1} and g_i. Then, having simulated n RCs, we would accumulate the number of observations in each bin. If $w_n(i)$, $i=1,\cdots,h$, is the total number of observations in bin i during n RCs, then the empirical cumulative distribution function of the random variable X, $F_n(\cdot)$, estimated after n RCs would jump by $w_n(i)/\beta_n$ at grid point g_i, where β_n is the total number of observations collected during n RCs. Then, a new distribution function $\widehat{F_n}(g_i)$ at grid point g_i, is estimated by linear interpolation between $F_n(g_i)$ and $F_n(g_{i+1})$. Next, the sample percentile $\widehat{Q_p}(n)$ after n RCs would be estimated by taking

$$\widehat{Q_p}(n) = \widehat{F_n^{-1}}(g_i).$$

The variance of this estimator is estimated as

$$\widehat{\sigma^2}\left(\widehat{Q_p}(n)\right) = \widehat{\sigma^2}(y_{ij}(n)) - 2F_n(g_i)\,cov(y_{ij}(n), a_{ij}(n)) + F_n^2(g_i)\,\widehat{\sigma^2}(a_{ij}(n)),$$

where $y_{ij}(n)$ and $a_{ij}(n)$ are the sum and the number of observations collected for bin i in the j^{th} RC over n RCs, respectively [11]. Here, $\widehat{\sigma^2}(\cdot)$ and $cov(\cdot,\cdot)$ are estimates of the variance and covariance, and $F_n(g_i)$ is the empirical cumulative distribution function of the random variable X after n RCs at grid point g_i.

A $100(1-\alpha)\%$ CI for the percentile Q_p can be obtained by dividing $\hat{r}(n) \pm \{s(n)\,t_{n-1,1-\alpha/2}\}/\{\overline{a}(n)\sqrt{n}\}$, where $t_{n-1,1-\alpha/2}$, for $0<\alpha<1$, is the upper $(1-\alpha/2)$ critical point from the Student t-distribution with degrees of freedom $n-1$, with the slope of $F_n(\widehat{Q_p}(n))$ [11]. Then, a $100(1-\alpha)\%$ CI for the percentile Q_p is given by

$$\widehat{Q_p}(n) \pm \frac{t_{df,1-\alpha/2}\,\widehat{\sigma}(\widehat{Q_p}(n))}{\overline{a}F'(\widehat{Q_p}(n))\sqrt{n}},$$

where $t_{df,1-\alpha/2}$ is the $(1-\alpha/2)$ percentile of the t-distribution with $df=n-1$ degrees of freedom, $F'(\widehat{Q_p}(n))$ is estimated by $w_n(\widehat{Q_p}(n)+1)/\beta_m$, which is the slope of $F_n(\widehat{Q_p}(n))$, and $\overline{a} = 1/n\ \left\{\sum_{j=1}^{n}\left(\sum_{i=1}^{h} a_{ij}(n)\right)\right\}$.

2.2 Sequential PE using Batching Approach

Batching approach also has been originally developed for fixed sample size simulation by Seila [21]. In this paper, we modified the *batching* approach for sequential PE. First, the *batching* approach for sequential PE groups data from the RCs into batches and computes sample percentiles by regarding as the batches as a set of independent, identically distributed observations. Before applying this method, the analysts must specify the batch size (the number of RCs in a batch) and in this paper we initially considered the batch size of b = 50 to test the feasibility of this approach.

Batching method groups m cycles[1] in each batch. Within each batch, a percentile estimate is computed using the jackknifed sample percentile until the steady-state parameter has been estimated with the required relative precision.

The *batching* method groups each batch of b RCs, and the three sample percentile estimates are computed from each batch to incorporate a two-fold jackknife procedure in order to reduce bias of the percentile estimators. One sample percentile estimate is computed from all observations collected during a batch, and the other two sample percentile estimates are computed from observations of the first and second half RCs of a batch. Assume that b is even, and let $\widehat{Q_p}(b,i)$, $\widehat{Q_p}(b/2,i_1)$, and $\widehat{Q_p}(b/2,i_2)$ be the estimates of Q_p computed from the b RCs in the i^{th} batch, and the first and second $b/2$ RCs in the i^{th} batch using the ordinary percentile estimator[2], respectively. Then, the jackknifed batch p percentile is

$$J(\widehat{Q_p}(b,i)) = 2\,\widehat{Q_p}(b,i) - \frac{1}{2}(\widehat{Q_p}(b/2,i_1) + \widehat{Q_p}(b/2,i_2)).$$

The sequence $\{J(\widehat{Q_p}(b,i)),\ i=1,2,\cdots,r\}$ over r batches consists of r i.i.d. random variables. Let $J(\overline{Q_p}(b,r))$ and $\widehat{\sigma^2}(J(\overline{Q_p}(b,r)))$ denote the mean and variance of such jackknifed percentile estimators, i.e.,

$$J(\overline{Q_p}(b,r)) = \frac{1}{r}\sum_{i=1}^{r} J(\widehat{Q_p}(b,i));$$

and

$$\widehat{\sigma^2}(J(\overline{Q_p}(b,r))) = \frac{1}{r-1}\sum_{i=1}^{r}(J(\widehat{Q_p}(b,i)) - J(\overline{Q_p}(b,r)))^2.$$

Then, a $100(1-\alpha)\%$ CI for the percentile Q_p is given by

$$J(\overline{Q_p}(b,r)) \pm \frac{t_{df,1-\alpha/2}\,\widehat{\sigma}(J(\overline{Q_p}(b,r)))}{\sqrt{r}},$$

where $t_{df,1-\alpha/2}$ is the $(1-\alpha/2)$ percentile of the t-distribution with $df = r-1$ degrees of freedom.

3. Sequential PE Approaches of Non-RS

A PE approach to overcome the typical difficulties of PE in non-RS has been proposed by Jain & Chlamtac [12]. This PE approach is based on P^2 (*Piecewise-Parabolic*) formula. The detailed discussion of P^2 formula can be found in [12]. PE using P^2 algorithm originally developed for fixed sample size procedure and non-RS.

The P^2 algorithm consists of maintaining five markers : the minimum, the $100p/2^{th}$, $100p^{th}$, and $100(1+p)/2^{th}$ percentiles, and the maximum. The markers are numbered 1 to 5. Markers 2 and 4 are also called middle markers because they are midway between the $100p^{th}$ percentile (marker 3) and the extremes. The y value (height) of each marker is equal to the corresponding percentile value, and its x value is equal to the number of observations that are less than or equal to the marker. The marker heights are the current estimates of the percentiles, and these estimates are updated after every observation. As a new observation comes in, it is compared with the markers, and all markers higher than the observation are moved one position to the right. If a maker is off to the left or right of its ideal location by more than one, then the y and x values are adjusted using a P^2 formula. Pseudocodes of P^2 algorithm and an example calculation using P^2 algorithm are in [12].

PE using P^2 algorithm solves the storage problem by calculating percentiles using a piecewise-parabolic formula dynamically as the observations are generated. The observations are not stored, instead, a few statistical counters are maintained which help refine the estimate. Therefore, PE using P^2 algorithm has a very small storage requirement regardless of the number of observations and a small computing time because no sorting is required.

Extended P^2 approach for PE, which is extended version of Jain and Chlamtac's approach, has been proposed by Raa-

1) A regenerative cycle is the portion of the process between two successive regenerative points, at which the process starts over probabilistically.

2) The sample percentile is obtained from the order statistic.

tikainen [18] and this *extended* P^2 approach simultaneously estimates several percentiles without storing and sorting the observations in fixed sample size procedure. Sequential procedure for simultaneous estimation of several percentiles in non-RS has also been proposed in [19]. This sequential approach uses the *extended* P^2 algorithm to estimate the percentiles and the variances of the percentile estimates are estimated using SA/HW method (Spectral Analysis in its version proposed by Heidelberger and Welch [10]). This approach uses the random length of uniform distribution over the interval [1025, 2048] for deciding the length of the initial transient period.

3.1 Sequential PE using Spectral P2 Approach

In this paper, we consider sequential procedure for a single PE in non-RS and we will call it s*pectral* P^2 approach. P^2 algorithm proposed by Jain & Chlamtac was used for PE and the variances of the percentile estimates are estimated using the SA/HW method. However, the method for detecting the initial transient period used here are based on those described in [16]. The sequential procedure detects the initial transient period by the stationarity test proposed by Schruben et al. [23]. It tests the hypothesis that sufficient numbers of initial transient data have been discarded.

Many heuristic rules are discussed in [16], and one of heuristic rules proposed by Fishman [7], which is *the initial transient period is over after* n_0 *observations if the time series* $x_1, x_2, \cdots, x_{n_0}$ *crosses the mean* $\overline{X}(n_0)$ k *times*, is used for detecting the initial transient period. In Gafarian et al. [8], k = 25 was recommended for M/M/1/∞ queueing systems so we assume k = 25 for simulation experiments. Detailed pseudocodes of the sequential procedure for detecting the initial transient period using Fishman's heuristic rule are in [16]. Detailed methods and pseudocodes of the sequential procedure for testing the required precision of results using the SA/HW method are also in [16].

The $100p^{th}$ percentile which is calculated by the P^2 algorithm (detailed procedure of this method in [12]) would be defined by Q_p. The P^2 formula for the height, which is a percentile value, is actually an approximation of the inverse of the empirical cumulative distribution function,

$$\widehat{F^{-1}}(y) = ay^2 + by + c.$$

The variance of the percentile estimate $\widehat{Q_p}(n)$ is estimated by using the formula given in [19]. As the number of observations becomes large, the variance of $\widehat{Q_p}(n)$ can be approximated by

$$\sigma^2(\widehat{Q_p}(n)) = \frac{S(0;\widehat{Q_p}(n))}{n\widehat{f^2}(\widehat{Q_p}(n))},$$

where $S(0;\widehat{Q_p}(n))$ is the spectral density at frequency 0, estimated using the SA/HW method proposed by Heidelberger and Welch [10], and $\widehat{f}(\widehat{Q_p}(n))$ is the empirical density function, approximated by

$$\widehat{f}(\widehat{Q_p}(n)) = (b + 2a\widehat{F}(\widehat{Q_p}(n)))^{-1}$$

since $\widehat{Q_p}(n)$ is an approximation of the inverse of the empirical cumulative distribution function, $\widehat{F^{-1}}(n) = an^2 + bn + c$ [19].

A $100(1-\alpha)\%$ CI for the percentile Q_p is given by

$$\widehat{Q_p}(n) \pm \frac{t_{df,1-\alpha/2}\,\hat{\sigma}(\widehat{Q_p}(n))}{\sqrt{n}},$$

where $t_{df,1-\alpha/2}$ is the $(1-\alpha/2)$ percentile of the t-distribution with degrees of freedom df = 7.

4. Numerical Results

Implementations of the *batching* PE approach, the *linear* PE approach in RS and s*pectral* P^2 approach in non-RS for analysing sequential PE of output data have been discussed in the previous section.

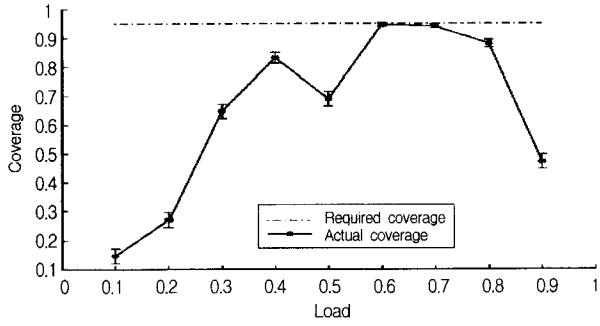
The robustness of any method can be usually measured by the coverage of confidence intervals, defined as the proportion $\hat{p}$ with which the number of the final confidence interval $(\hat{p}-\Delta, \hat{p}+\Delta)$ contains the true value p. An estimator of variance $\widehat{\sigma^2}$ used for determining the confidence interval of the point estimate is considered as valid, i.e. producing valid $100(1-\alpha)\%$ confidence intervals of the point estimate, if the upper bound of the confidence interval of the point estimate $\hat{p}$ equals at least $(1-\alpha)$ [20]. Coverage analysis, however, is to analytically intractable systems, since the theoretical value of the interesting parameter has to be known. Because of this reason, in this paper we estimated percentiles of response times of M/M/1/∞ queueing system.

All numerical results in this paper were obtained by stopping simulation experiments when the final steady-state

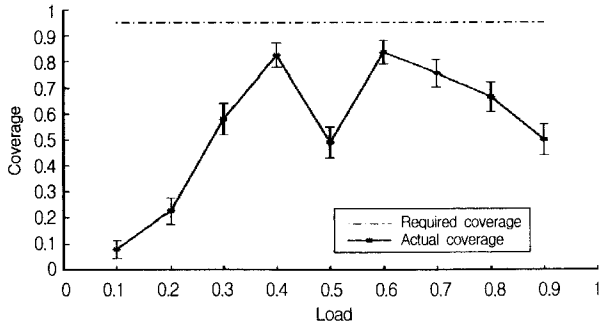
results reached a required precision of 5% or less, at the 0.95 confidence level and 200 or more bad confidence intervals (to secure representativeness in the analysed data) had been collected. All results were also filtered of strangely short simulation runs to secure the statistical properties of interval estimators after 200 bad confidence intervals are collected. The filtering of short simulation runs has improved estimates of coverage for sequential PE in RS except a case. The results of traditional fixed sample size approach with 200 replications are shown in (Figure 2), (Figure 4), and (Figure 6).

Sequential coverage analysis (using F approximation [13, 14], which gives much narrower confidence intervals,) for sequential PE approaches in RS and non-RS on a single processor under MRIP (Multiple Replications In Parallel) scenario of AKAROA [6, 17] are experimented at the 90^{th} percentile.

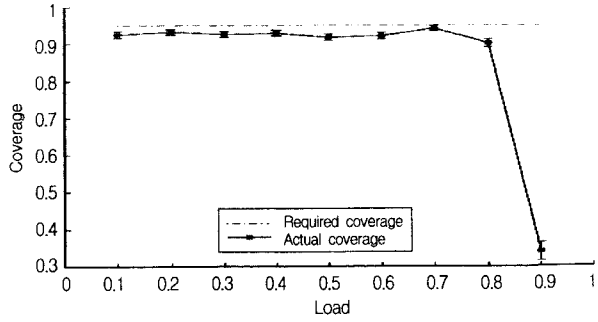
(Figure 1) and (Figure 3) depict the results obtained from *linear* PE and *batching* PE approaches in RS, respectively and (Figure 5) is from *spectral* P^2 PE approach in non-RS. As we can see, *batching* PE approach in RS gives much stable coverage except traffic intensity 0.9.



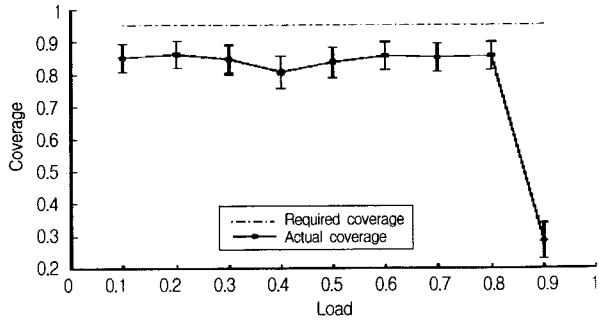
(Figure 1) Coverage Analysis of *Linear* PE in RS Using F Approximation in M/M/1/ ∞ Queueing System (P = 1 & Sequential Analysis)



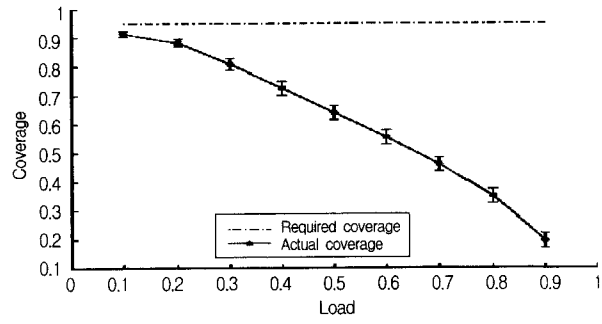
(Figure 2) Coverage Analysis of *Linear* PE in RS Using F Approximation in M/M/1/ ∞ Queueing System (P = 1 & Fixed Sample Size of 200 Replications)



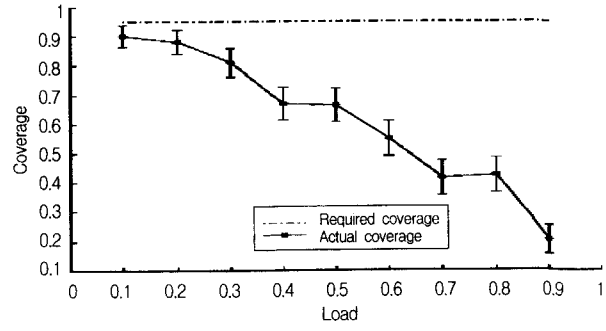
(Figure 3) Coverage Analysis of *Batching* PE in RS Using F Approximation in M/M/1/ ∞ Queueing System (P = 1 & Sequential Analysis)



(Figure 4) Coverage Analysis of *Batching* PE in RS Using F Approximation in M/M/1/ ∞ Queueing System (P = 1 & Fixed Sample Size of 200 Replications)



(Figure 5) Coverage Analysis of *Spectral* P^2 PE in Non-RS Using F Approximation in M/M/1/ ∞ Queueing System (P = 1 & Sequential Analysis)



(Figure 6) Coverage Analysis of *Spectral* P^2 PE in Non-RS Using F Approximation in M/M/1/ ∞ Queueing System (P = 1 & Fixed Sample Size of 200 Replications)

<Table 2> shows the comparison of three PE approaches with the ordinary PE approach regarding the computational time and the required storage. The computational time and the required storage in the ordinary method clearly depend on the number of observations n. However, the other methods do not depend on the number of observations. They only depend on the pre-defined number of bin, h, and the batch size, b. As we can see the computational time and the required storage are significantly reduced.

<Table 2> Comparison of Three PE Approaches

	Ordinary PE	*Linear* PE	*Batching* PE	*Spectral* P^2
Time	$O(n\log_2 n)$	$O(c)$	$O(c)$	$O(c)$
Storage	n	$h+1$	b	5

5. Conclusions

For large number of observations, PE becomes impractical to store and sort the entire sequences. Physical memory limitations of computers used for PE make large numbers of replications impossible, and in others, the shuffling of virtual memory pages slow down the simulation considerably. *Linear* and *batching* PE approaches for RS and P^2 PE approach for non-RS can resolve the problems related with PE but these are not originally developed for sequential simulation. Run length control of simulation is very important as the most efficient way for securing simulation results statistically. Sequential stopping rules, which control the relative width of an estimated confidence interval, can be used in conjunction with RS and non-RS. Therefore, we proposed three PE approaches for RS and non-RS in sequential steady-state simulation: *linear* PE, *batching* PE, and *spectral* P^2 PE. Many other aspects for sequential PE in RS and non-RS will have to be carefully studied and tested with a number of different simulation models before these procedures can be safely used in simulation practice.

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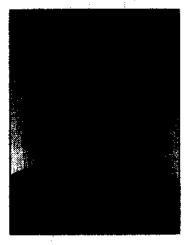

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