몬테칼로법과 생애함수를 이용한 교량의 파괴확률예측

양 승 이
콜로라도 주립대 토목환경공학과
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The Prediction of Failure Probability of Bridges using Monte Carlo Simulation and Lifetime Functions

Seung-Ie Yang
Department of Civil and Environmental Engineering, University of Colorado at Boulder
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Abstract : Monte Carlo method is one of the powerful engineering tools especially to solve the complex non-linear problems. The Monte Carlo method gives approximate solution to a variety of mathematical problems by performing statistical sampling experiments on a computer. One of the methods to predict the time dependent failure probability of one of the bridge components or the bridge system is a lifetime function. In this paper, FORTRAN program is developed to predict the failure probability of bridge components or bridge system by using both system reliability and lifetime function. Monte Carlo method is used to generate the parameters of the lifetime function. As a case study, the program is applied to the concrete-steel bridge to predict the failure probability.

초 록 : 몬테칼로법은 복잡한 특히 비선형문제를 푸는데 강력한 공학도구중의 하나이다. 이 방법은 컴퓨터에서의 통계적인 표본추출방법을 이용하여 각종 공학적인 문제에 근사적인 해를 준다. 교량 하나의 요소나 전체교량의 시간 의존적 파괴확률을 예측하는 방법중의 하나로 생애함수가 있다. 이 논문에서는 교량의 요소나 전체 교량의 파괴확률을 예측하기 위하여, 시스템 신뢰성과 생애함수를 이용하여 포트란 프로그램을 개발하였다. 몬데칼로법은 생애함수의 매개변수를 생성하는데 이용되었다. 적용례로서, 개발된 프로그램은 콘크리트-강 합성 교량에 적용되어, 파괴확률을 예측하는데 이용되었다.
Key Words : bridges, monte carlo method, system reliability, lifetime function, failure probability

1. Introduction

Monte Carlo method is one of the powerful engineering tools especially to solve the complex non-linear problems. The Monte Carlo method gives approximate solution to a variety of mathematical problems by performing statistical sampling experiments on a computer.

One of the methods to predict the time dependent failure probability of one of the bridge components or the bridge system is a lifetime function. The lifetime function describes the evolution of the risk. Although there are several lifetime functions, the survivor function is used to predict the time-dependent failure probability in this paper. The lifetime function have three parameters; failure rate λ, scale factor λ_s, and shape factor x. To give the uncertainty to them, Monte Carlo simulation is used. FORTRAN program is developed to predict the failure probability of bridge components or bridge system by using both system reliability and lifetime function. Monte Carlo method is used to generate the parameters of the lifetime function. As a case study, the program is applied to the concrete-steel bridge to predict the failure probability.

yangsi@colorado.edu

2. Lifetime Distribution and System Reliability

Most engineering systems are designed to be used over some period of time. The lifetime of some systems is short or long. But most systems are intended for use over a much longer period. It is very important to predict the condition of the system in the future. There are several lifetime distributions which describe the evolution of the risk of components. In this section, the lifetime function is used as a tool to forecast the time-dependent failure probability of the system using system reliability concept.

2.1. Condition State of Components and Systems

Structure function and Reliability function[1] are useful tools to describe the state of a system of n components. Structure function defines the system state as a function of the component state. The structure function has two values as follows

$$\phi(x) = \begin{cases} 0 & \text{if the system has failed} \\ 1 & \text{if the system is functioning} \end{cases} \quad (1)$$

where

x = System state vector ; $\{x_1, x_2, ..., x_n\}$

x_i = Component i state; 0 = component has failed, 1= component is functioning

As an example to obtain the structure function, a four-component system shown in Fig. 1 is used.

The first reduction step is a parallel system between components 3 and 4. In the parallel system, the failure of all components causes the system failure. The first subsystem can be expressed as follows

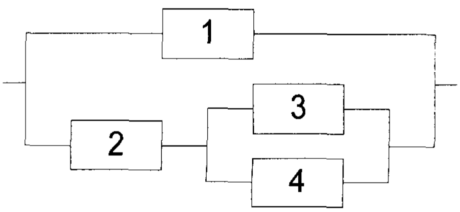

Fig. 1. Four-component system

$$\phi_{s1}(x) = 1-(1-x_3)(1-x_4) \quad (2)$$

The second reduction step is a series system between component 2 and subsystem 1. In the series system, any one component failure in a system causes the system failure. The second reduction can be expressed as follows

$$\phi_{s2}(x) = x_2[1-(1-x_3)(1-x_4)] \quad (3)$$

The last reduction step is a parallel system between component 1 and subsystem 2 produced by Eq. (3) and the last reduction produces the structure function of the system.

$$\phi_s(x) = 1-(1-x_1)\{1-x_2[1-(1-x_3)(1-x_4)]\} \quad (4)$$

The structure function is deterministic. This assumption may be unrealistic for certain types of components or a system. So, reliability function[1], r(p), is necessary to model the structures. x_i was defined to be the deterministic state of component i in the structure function. In the reliability function, x_i is a random variable. The probability that component i is functioning is given by

$$p_i = P[x_i = 1] \quad (5)$$

Where

p_i = Probability that component i is functioning

If there are n components, the reliability vector of a system can be written as follows

$$p = \{p_1, p_2, ..., p_n\} \quad (6)$$

The system reliability is defined

$$r(p) = P[\phi(x)=1] \quad (7)$$

The reliability function gives the probability that the system is functioning. In order to obtain the reliability function for a four-component system shown in Fig. 1, the same procedure is necessary. However, the component reliability function is used in each step instead of component state x.

2.2. Type of Lifetime Functions

There are several lifetime functions to describe the evolution of the probability of failure. In this paper, survivor function is introduced and explained. The survivor function can be applied to both discrete and continuous lifetime.

The survivor function is the generalization of reliability because the survivor function gives the reliability that a component is functioning at one particular time. The survivor function is expressed

$$S(t) = P[T \geq t] \quad t \geq 0 \tag{8}$$

It is assumed that when $t \geq 0$, $S(t)$ is one. The survivor function has to satisfy three conditions. These are

1) $S(0) = 1$
2) $\lim_{t \to \infty} S(t) = 0$
3) $S(t)$ is non-increasing without any maintenance

Several distributions are used as survivor functions. The exponential distribution, Weibull distribution, log-logistic distribution, and exponential power distribution are introduced in this paper. These survivor functions are shown in Table 1.

Table 1. Survivor function

Distribution	Survivor function
Exponential	$\exp(-\lambda t)$
Weibull	$\exp(-(\lambda_s t)^{\kappa})$
Log-logistic	$\frac{1}{1+(\lambda_s t)^{\kappa}}$
Exponential- power	$\exp(1-\exp(\lambda_S t)^{\kappa})$

Where
λ = Failure rate
λ_s = Scale factor
κ = Shape factor
t = Time, $t \geq 0$

For a four-component system shown in Fig. 1, if each component has an exponential survivor function, the system reliability function is shown in Eq. 9.

$$r(p)= 1-(1-e^{-\lambda t}[1-(1-e^{-\lambda t})(1-e^{-\lambda t})] \tag{9}$$

3. Monte Carlo Method

The notion of Monte Carlo method comes from the gaming tables at the casinos[2]. The randomness can be used together with the mathematics of probability theory. The Monte Carlo method gives approximate solution to a variety of mathematical problems by performing statistical sampling experiments on a computer. This method applies to problems with no probabilistic content as well as to those with inherent probabilistic content.

Because both the loads and resistances are time-variant random variables, it is necessary to assess the uncertainty of parameters of lifetime distributions. Monte Carlo method is a useful tool to assess the uncertainty of lifetime parameters and generate the random parameters of the lifetime distributions.

The failure rates and scale factors of components are not certain. It is possible to have precise output with deterministic input data but it is impossible to assess the uncertainty of this input. For real structures, such as highway bridges, both the loads and resistances are time-variant random variables. So, it is necessary to assess the uncertainty in the input data. For complex systems whose components have random failure rates, it is not easy to compute their time-variant reliability. In this case, Monte Carlo simulation is a powerful engineering tool which enables one to perform a statistical analysis of the uncertainty in structural engineering problems. This tool is particularly useful for complex nonlinear problems. The fundamental step in Monte Carlo analysis is a development of a set of uniformly generated random numbers. The cumulative distribution function (CDF) of uniform distribution is used to explain Monte Carlo simulation. If the parameters of uniform distribution are a and b, the CDF is shown in Fig. 2.

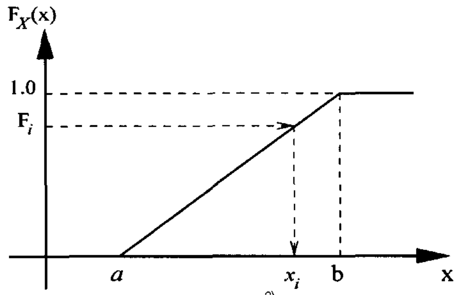

Fig. 2. CDF of uniform distribution[3]

At first, the uniform random numbers, F_i, are generated in the range between zero and one, and they are transferred to the numbers which are in between a and b by Equation (10).

$$x_i = a + (b - a)F_i \tag{10}$$

Where

F_i = Uniformly generated random number

This is the basic idea. Usually, in Monte Carlo method, uniformly generated random numbers are increased to obtain accurate solution. When uniformly generated random numbers are increased to obtain accurate results, the computing time is increased. So, the computing time is a major concern in Monte Carlo simulation. Equation (10) is called Inverse Cumulative Distribution Function (ICDF). Although ICDF is easily derived for uniform distribution, this is not the case for other distributions. In this study, the ICDF for each distribution type(Uniform, Triangular, Log-Normal, Beta, Gamma, and Exponential Distributions) is programmed, and failure rate for random variables are simulated by using Monte Carlo method.

4. Applications

In this section, the results of the program application are shown for correlated parameters, uncorrelated parameters and a hypothetical bridge (concrete-steel girder bridge).

4.1. Uncorrelated Parameters

The probability of failure is predicted for series systems, parallel systems, a three-component system in this section.

Fig. 3 shows the failure probability of series systems.

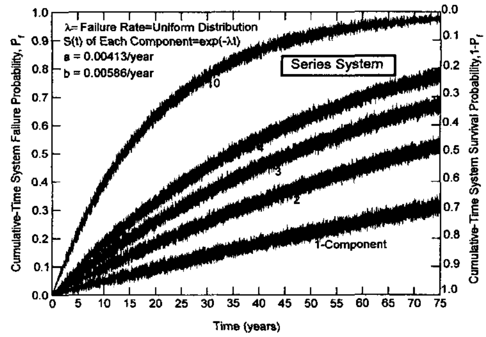

Fig. 3. Effect of number of components on cumulative-time failure probability of a series system

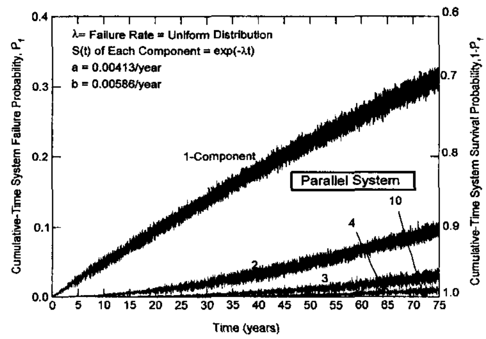

Fig. 4. Effect of number of components on cumulative-time failure probability of a parallel system

The type of lifetime function used in Fig. 3 is exponential. It is assumed that the failure rate has a uniform distribution whose parameters are a = 0.00413/year and b = 0.00586/year. Because failure of any one or more components make the system failure in a series system, a series system is called "Weakest Link System". From Fig. 3, it can be seen that when the number of components increases, the system is more dangerous than any components. Fig. 4 shows the failure probability of parallel systems.

The exponential survivor function is used as a lifetime function of each component in a parallel system. The parameters used in Fig. 4 are the same as that of Fig. 3. In parallel system, every component failure makes a system failure. From Fig. 4, it can be seen that when the number of components increases,

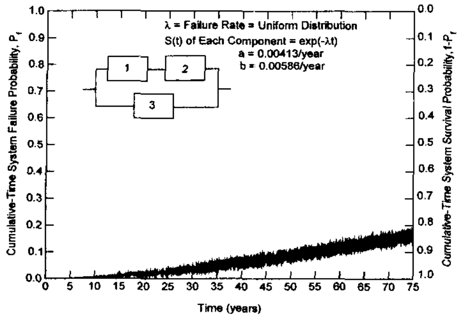

Fig. 5. Cumulative-time failure probability of a three-component system

the system is stronger than any components. The probability of failure is predicted for a three-component system. The result is shown in Fig. 5.

The system reliability function of a three-component system is

$$r(p) = 1-(1-e^{-\lambda t}e^{-\lambda t})(1-e^{-\lambda t}) \tag{11}$$

The parameters of each survivor function are the same as that of a parallel system and a series system.

4.2. Correlated Parameters

There are many cases in which random variables are not independent. FORTRAN program is developed when each random variable is correlated by using CHOLESKY decomposition. If there is a positive correlation between failure rates of lifetime functions, the probability of failure is bigger than that of uncorrelated case or negative case. The example is shown in Fig 6.

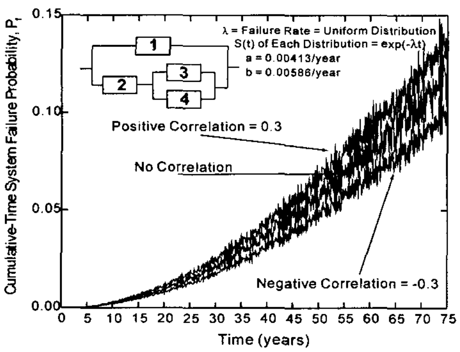

Fig. 6. Cumulative-time failure probability of a four-component system

A four-component system is used. Each component has an exponential survivor function whose parameter (failure rate) has a uniform distribution. Parameters of uniform distribution are a = 0.00413/year and b= 0.00586/year. When each parameter has a positive correlation (0.3), the probability of failure is bigger than that of no correlation case and positive correlation case. Also, it can be seen that a positive correlation case is safer than that of no correlation case.

The series system and the parallel system are shown in Figs. 7 and 8 to show effect of the correlation. Survivor functions and the parameter's distribution types are the same of that of the four-component system.

4.2. Case Study

As a case study, a hypothetical bridge (concrete-steel girder bridge) is used to predict the failure probability. The cross section is shown in Fig. 9.

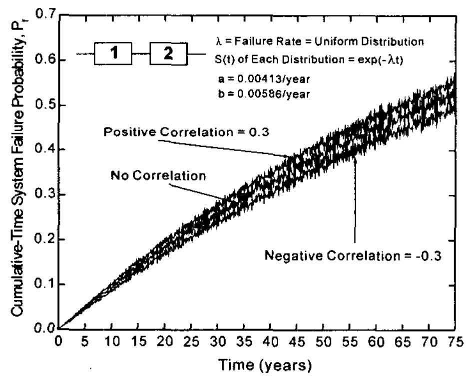

Fig. 7. Effect of correlation on cumulative-time failure probability of a series system

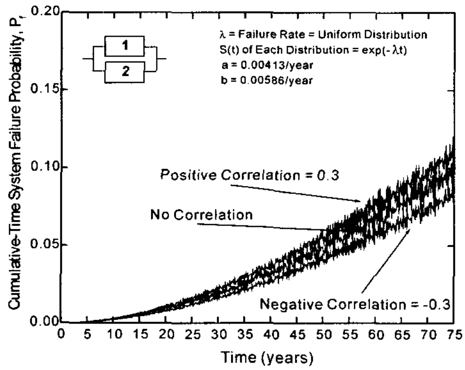

Fig. 8. Effect of correlation on cumulative-time failure probability of a parallel system

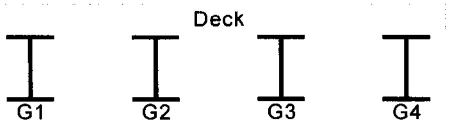

Fig. 9. Cross section of the bridge

Table 2. Parameters of survivor functions

	Scale Factor	Shape Factor
Concrete	a = 0.0084/year b = 0.012/year	2.91
Steel	a = 0.0087/year b = 0.012/year	2.86

In this case study, it is assumed that the concrete and steel have Weibull survivor function whose distribution type of scale factor is uniform. The parameters are summarized in Table 2.

It is important subject to obtain the parameters of the lifetime functions. The report, "Serviceable Life of Highway Structures and their Components[4)]", summarizes the work done by Maunsell Ltd. for the Highways Agency. In this report, the best fit distribution is calculated, and the parameters of lifetime time function are presented. The best fit distribution was decided as Weibull distribution. The parameters of Weibull distribution for each bridge component are shown in this report. Based on this report, the value of Table 2 is obtained.

Two failure modes of the bridge are assumed: The first failure mode - any two adjacent girder failure or

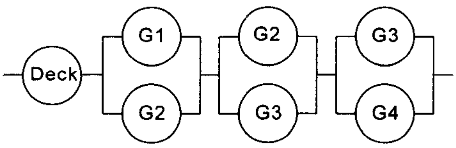

Fig. 10. The first failure mode

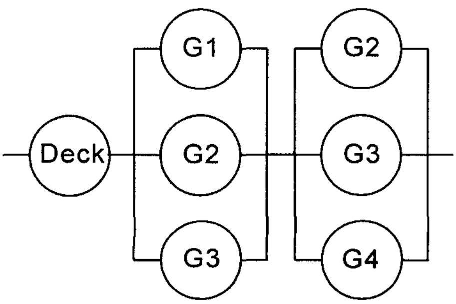

Fig. 11. The second failure mode

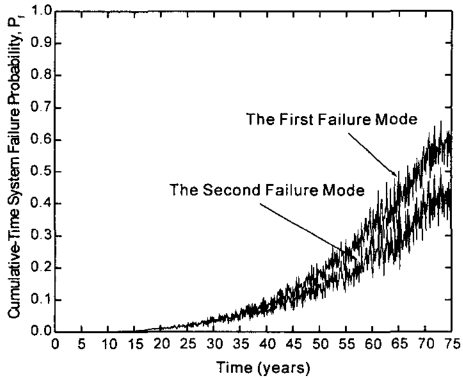

Fig. 12. Failure probability of failure modes

deck failure result in bridge failure; The second failure mode - any three adjacent girder failure or deck failure cause the bridge failure. The first failure mode is shown in Fig. 10.

The second failure mode is in Fig. 11.

The failure probability of failure modes is predicted in Fig. 12.

In this figure, it is assumed that all components are independent.

5. Conclusions

In this paper, Monte Carlo simulation was briefly explained. By using Monte Carlo simulation and lifetime functions, the failure probability was predicted for a component or system. As a case study, a hypothetical bridge (concrete-steel girder bridge) was used to predict the failure probability. The conclusions are followed

1) By using Monte Carlo simulation, it is possible to give the uncertainty into the input data.

2) Lifetime functions are powerful tool to predict the failure probability of a component or a system.

3) The program developed in this paper can be applied to real bridges. In order to apply it to real bridges, it should be performed to do the data analysis for each bridge component.

4) The program developed in this paper can be applied to a bridge network. In this case, a component in a system is assumed as one bridge.

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