Block Error Performance of Transmission in Slow Nakagami Fading Channels with Diversity

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Abstract**—**In this paper presents equations which describe an average weighted spectrum of errors and average block error probabilities for noncoherent frequency shift keying (NCFSK) used in D-branch maximal ratio combining (MRC) diversity in independent very slow nonselective Nakagami fading channels. The average is formed over the instantaneous receiver signal to noise ratio (SNR) after combining. the analysis is limited to additive Gaussian noise.

Index Terms**—**Diversity reception, error probability, Nakagami fading channels, block error probabilities

I. INTRODUCTION

This paper presents an analysis of the performance of maximal ratio combining (MRC) diversity reception in a Nakagami fading environment. The theoretical model for deriving the Nakagami distribution assumes that the received signal is the sum of vectors with random modulo and random phases. Nakagami fading any occur in practical situations, especially in mobile communications. Experimental and theoretical work has shown that the Nakagami distribution is the best-fit distribution for data obtained from a model of urban radio multipath channels [1], [8].

This letter presents on one formula the average probability of bit error for MRC employing noncoherent frequency shift keying (NCFSK), differentially coherent phase shift keying(DPSK), coherent frequency shift keying (CFSK), and coherent phase shift keying (CPSK). Formulas for the average weighted spectrum of errors and the average block error probability for NCFSK are also presented. Although the formulas for bit error probability for DPSK and NCFSK are very similar the weighted spectrum of errors and the block error probability of DPSK are difficult to obtain due to bit error correlation. For CPSK and CFSK, the bit errors are independent but the weighted spectrum of errors and block error probability cannot be easily simplified.

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II. AVERAGE PROBABILITY OF BIT ERROR

The bit error probability ("static" bit error probability [7]) for several common binary modulation schemes with optimum detection of nonfading signals in Gaussian noise is given by

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$$P_s(\rho) = \frac{\Gamma(b, a\rho)}{2\Gamma(b)} \quad (1)$$

where

ρ instantaneous signal to noise ratio (SNR);
Γ(x,y) second incomplete gamma function [3];
Γ(a) gamma function [3] ;
a coefficient indicating the modulation type;

$$a = \begin{cases} \frac{1}{2} & \text{for } \textit{frequency shift keying(FSK),} \\ 1 & \text{for } \textit{phase shift keying(PSK)} \end{cases}$$

b coefficient indicating the detection mechanism:

$$b = \begin{cases} \frac{1}{2} & \text{for } \textit{coherent detection,} \\ 1 & \text{for } \textit{noncoherent differentially coherent detection differentiallycoherentdetecton} \end{cases}$$

Assume that the Nakagami fading is very slow, nonselective, and independent. Then the instantaneous SNR remains the same over a block of N bits. MRC is also assumed. The combiner must know each path envelope and phase in order to perform perfect combining. The received signals in MRC are combined to obtain the maximum SNR of the weighted sum at the combiner output. The output SNR is the sum of the instantaneous branch SNR's i.e.

$$\rho = \sum_{k=1}^{D} \rho_k \quad (2)$$

where ρ_k is the instantaneous SNR per branch and D is the number of diversity branches.

For Nakagami fading, ρ_k has gamma probability density function

$$p(\rho_k)=\frac{1}{\Gamma(m_{sk})}\left(\frac{m_{sk}}{\rho_{ok}}\right)^{m_{sk}}\rho_k^{m_{sk}-1}\exp\left(-\frac{m_{sk}}{\rho_{ok}}\rho_k\right),\quad \rho_k\geq 0 \qquad (3)$$

where $m_{sk}\geq 1/2$ is the inverse fading depth parameter and ρ_{ok} is the average SNR per branch.

According to [1] the probability density function for the MRC output SNR is

$$p(\rho)=\frac{1}{\Gamma(m_{sw})}\left(\frac{m_{sw}}{\rho_{ow}}\right)^{m_{sw}}\rho_k^{m_{sw}-1}\exp\left(\frac{m_{sw}}{\rho_{ow}}\rho\right),\quad \rho\geq 0 \qquad (4)$$

where $m_{sw}=\sum_{k=1}^{D}m_{sk}$ and $\rho_{ow}=\sum_{k=1}^{D}\rho_{ok}$

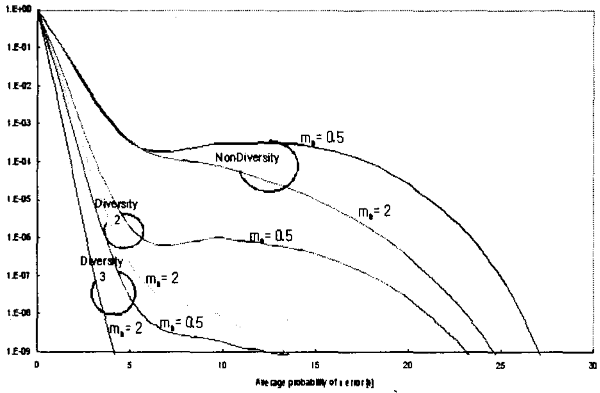

Fig. 1 Average probability of n errors in N bits of MRC diversity and NCFSK when m_s = 2, 0.5 and D = 1, 2, 3, N = 30, and $p_D{}' = 10^{-3}$.

With sufficiently slow flat fading the average probability of bit error ("dynamic" probability of bit error[7]) $p_D(\rho_{ow}, m_{sw})$ equals $p_s(\rho)$ averaged over the distribution of output SNR ρ

$$p_D(\rho_{ow}, m_{sw}) = \int_0^{\infty} P_s(\rho)p(\rho)d\rho = E[P_s(\rho)] \qquad (5)$$

where E(x) denotes the expected value of the argument.

Inserting (1) and (4) in (5) givers the average probability of bit error for MRC as [5]

$$p_D(\rho_{ow}, m_{sw}) = \frac{1}{2B(m_{sw}, b)}B_{\frac{m_{sw}}{m_{sw}+a\rho_{ow}}}(m_{sw}, b) \qquad (6)$$

where $B_g(x,y)$ is the beta function [3] and $B_g(x,y)$ is the incomplete beta function [3] the error rata formula (6) is valid for four principal cases of binary modulation in Nakagami channels with MRC. It should be noted that the special case result of (6), i.e., for coherent detection when b=1/2, is identical to that obtained in [7]. The result over a Rayleigh channel described in [2] can be obtained from our result (6) when $m_{sk}=1$.

Assume that m_{sk} and ρ_k are the same for all diversity branches i.e., $m_{sk}=m_s$ and $E(\rho_k)=\rho_{ok}=\rho_0$. Then (4) simplifies to

$$p(\rho)=\frac{1}{\Gamma(Dm_s)}\left(\frac{m_s}{\rho_0}\right)^{Dm_s}\rho^{Dm_s-1}\exp\left(-\frac{m_s}{\rho_0}\rho\right),\quad \rho\geq 0 \qquad (7)$$

and (6)is simplified to

$$P_D(\rho_0, m_s)=\frac{1}{2B(Dm_s, b)}B_{\frac{m_{sw}}{m_{sw}+a\rho_{ow}}}(Dm_s, b) \qquad (8)$$

III. AVERAGE WEIGHTED SPECTRUM OF ERRORS

In case of digital transmission over a fading channel, the time variation causes a changing bit error probability with the effect of clustering errors in the received signal, fellow by the appearance of a stream of errors. Digital systems employing

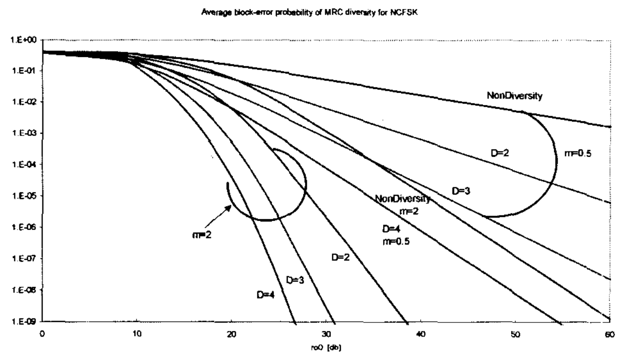

Fig. 2 Average block error probability of MRC diversity for NCFSK when m_s = 2, 0.5 and D = 1, 2, 3, 4

error detection or error correction coding are generally based on the transmission of blocks of N bits. The average probability of bit error specified by (6) neither describes the number of error nor their placement in the stream of errors. In order to describe the stream of error it is possible to use the average weighted spectrum of errors or the average bloc error probability

The weighted spectrum of errors, i.e., the probability of n bit errors s given by the binomial distribution

$$P(L_n=n)=\binom{N}{n}P_s^n(\rho)[1-P_s(\rho)]^{N-n}, n=0,1,2 \cdots, N. \qquad (9)$$

The average probability of n errors in a block of N bits (average weighted spectrum of errors) is denoted as

$$P_D(L_n=n)=\int_0^{\infty}P(L_N=n)p(\rho)d\rho = \binom{N}{n}\sum_{j=0}^{N-n}\binom{N-n}{j}(-1)^j E[P_s^{J+n}(\rho)] \qquad (10)$$

The average in (10) is formed over the instantaneous SNR ρ which has the probability density function p(ρ) it is assumed that p(ρ) is described by (7).

For noncoherent FSK with MRC, the average weighted spectrum of errors is given by an equation in [5]

$$P_D(L_n=n) = (L_n\binom{N}{n}(2m_s)^{Dm_s}\sum_{j=0}^{N-n}\binom{N-n}{j}(-1)^j \bullet (\frac{1}{2})^{j+n}[(n+j)\rho_0+2m_s]^{-Dm_s} \quad (11)$$

Equation (11) has been used to calculate the average probability of n errors in N bits. Fig. 1 shows the $P_D(L_n=n)$ for N=30 and $m_s=0.5$ or $m_s=2; P_D^1(\rho_0, m_s)$ denotes the average probability of bit error with no diversity (D=1).

IV. AVERAGE BLOCK ERROR PROBABILITY

The average probability of more than n errors in a block of N bits, i.e., the average block error probability, is denoted by

$$\begin{aligned} &P_D(L_n>n) \\ &= 1-\sum_{i=0}^{n}\binom{N}{i}\int_0^\infty P_s^i(\rho)[1-P_s(\rho)]^{N-1}p(\rho)d\rho \\ &1-\sum_{i=0}^{n}\binom{N}{i}\sum_{i=0}^{N-i}\binom{N-i}{j}(-1)^j E\left[P_s^{j+i}(\rho)\right] \end{aligned} \quad (12)$$

Thus, from (12), (1), and (7), the average block error probability for NCFSK with MRC is [5]

$$\begin{aligned} &P_D(L_n>n) \\ &= 1-2(m_s)^{Dm_s}\sum_{i=0}^{n}\binom{N}{i}\sum_{i=0}^{N-i}\binom{N-i}{j}(-1)^j(\frac{1}{2})^{j+i}\bullet[(i+j)\rho_0+2m_s]^{-Dm_s} \end{aligned} \quad (13)$$

For Rayleigh channel, we obtain the same result as presented in [6]. In addition, (13) for $m_{s=1}$ and $D=1$ describes the performance in a Rayleigh channel with no diversity, according to the result presented in [2]. Fig. 2 shows the average block error probability $P_D(L_{30}>0)$, i.e., the probability of at least one error in N=30 bits, for NCFSK, $m_s=2$, 0.5, and $D=1, 2, 3, 4$, respectively

V. CONCLUSION

Equations (6), (11), and (13) have considerably simple and unified form. The equations are valid for nonselective, very slow, and independent Nakagami fading with MRC when all branches are identical, i.e., they have equal SNR and the same fading parameter. The performance in a Rayleigh channel can be calculated from these equations for $m_s=1$. For $D=1$ the performance without diversity is obtained.

Presented results can be useful for error-detection or error-correction. Performance of digital systems using this kind of coding depends on the probability of occurrence of various numbers of errors in a block of N bits. In case of error detection, a block os received correctly only if all N bits are received without error. If an error-correction code enabling the correction of up to n errors in each block of N bits os employed, then the quality of system can be described by the probability of more than n errors in a block. Thus, the results presented in this letter provide useful design data and show the necessity for specialized treatment of the block error in fading environment.

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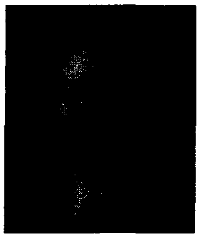

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