International Journal of Reliability and Applications
Vol. 4, No. 4, pp. 171-181, 2003

Tests for Exponentiality Against Harmonic New Better Than Used in Expectation Property of Life Distributions

A. S. Al-Ruzaiza
Department of Statistics and O. R., College of Science
P. O. Box 2455, King Saud University, Riyadh 11451, Saudi Arabia.

Abstract. This paper proposes a U-test statistic for the problem of testing that a life distribution is exponential against the alternative that it is harmonic new better (worse) than used in expectation upper tail HNBUET (HNWUET), but not exponential on complete data. Selected critical values are tabulated for sample sizes n =5(1)60. The asymptotic normality of the statistic is proved and a comparison is made of the asymptotic efficiency between the statistic and other statistics. The power of the test is studied by simulation. A test for HNBUET in the case of randomly right-censored data is also considered. An application of the proposed test statistic in medical sciences is given.

Key Words : *test exponentiality, convex ordering, life distributions classes, randomly right censored data, asymptotic normality, asymptotic efficiency, product limit estimator.*

1. INTRODUCTION AND DEFINITIONS

Eversince the works of Barlow *et al.* (1963) and Bryson and Siddiqui (1969), various classes of life distributions have been introduced in reliability. Currently the applications of these classes of life distributions can be seen in engineering, maintenance and biometrics. Therefore, statisticians and reliability analysts have shown a growing interest in modeling survival data using classification of life distributions (i.e. distribution function F with $F(0-) = 0$, survival function $\overline{F} = 1 - F$ and finite mean $\mu = \int_0^\infty \overline{F}(u)du$) based on some aspects of aging. Among these aspects are

(i) IFR (increasing failure rate)

Corresponding author.
E-mail address: aruzaiza@ksu.edu.sa

(ii) IFRA (increasing failure rate average)

(iii) NBU (new better than used)

(iv) NBUE (new better than used in expectation)

(v) HNBUE (harmonic new better than used in expectation)

the classes with duals. For definitions and further details see, for example, Haines (1973), Barlow and Proschan (1981) and Zacks (1992). Deshpande *et al.* (1986) introduced a new class of life distribution named HNBUE(3) (HNWUE(3)) (harmonic new better (worse) than used in expectation of third order) which is larger than HNBUE (HNWUE) class. Note that Abouammoh and Ahmed (1989) renamed the class HNBUE(3) by HNBUET (harmonic new better than used in expectation upper tail).

The implications among the above classes of life distributions are

$$IFR \Longrightarrow IFRA \Longrightarrow NBU \Longrightarrow NBUE \Longrightarrow HNBUE \Longrightarrow HNBUET.$$

Similar implications hold for the corresponding dual classes

$$DFR \Longrightarrow DFRA \Longrightarrow NBU \Longrightarrow NBUE \Longrightarrow HNBUE \Longrightarrow HNBUET.$$

Many test statistics have been developed for testing exponentiality against various aging alternatives. Testing exponentiality against the classes of life distributions has received a good deal of attention. For testing against new better than used (NBU), we refer to Hollander and Proschan (1972), Koul (1977) and Ahmad (1994), among others. For new better than used in expectation (NBUE), we refer to Koul (1977) among others. For harmonic new better than used in expectation (HNBUE), we refer to Klefsjo (1982) and Hendi et al. (1998) among others.

The classes HNBUE and HNBUET may be defined on the basis of a variability definitions due to Stoyan (1983), which is the following.

Definition 1. Let X and Y be two random variables with marginal distributions F and G, respectively. We say that X is less variable than Y (or X is smaller than Y in convex ordering) and write $X \leq_{VR} Y$ if $E[h(X)] \leq E[h(Y)]$ for all increasing convex functions h. Clearly $X \leq_{VR} Y$ if and only if $\int_x^\infty \overline{F}(u)du \leq \int_x^\infty \overline{G}(u)du$ for all $x \geq 0$ where $\overline{F}(x) = 1 - F(x)$ and $\mu = \int_0^\infty \overline{F}(u)du$.

It is not difficult to see that X is HNBUE if and only if $X \leq_{VR} X_0$, cf. Ahmad (1995), where X_0 is a random variable with the exponential distribution with mean μ. Then from Definition 1 we have the following:

(i) $F \epsilon$ HNBUE iff

$$\int_x^\infty \overline{F}(u)du \leq \mu \exp\left(\frac{-x}{\mu}\right); \qquad x \geq 0\ ,\ \mu > 0. \tag{1.1}$$

Integrating both sides of Equation (1.1) with respect to x, from t to ∞, we obtain

$$\int_t^\infty \int_x^\infty \overline{F}(u)dudx \leq \mu^2 \exp\left(\frac{-t}{\mu}\right); \qquad x,t \geq 0 \ , \ \mu > 0. \tag{1.2}$$

Equation (1.2) is the definition of the class "harmonic new better than used in expectation upper tail (HNBUET)".

Example 1. Consider the survival function $\overline{F}(x)$ given by

$$\overline{F}(x) = \begin{cases} e^{-x} & \forall\ 0 \leq x \leq 1 \\ 0.7e^{-1} & \forall\ 1 \leq x < 1.5 \\ 0 & \forall\ x > 1.5 \end{cases}$$

It is easy to prove that $\overline{F}(x) \epsilon$ HNBUET.

Al-Ruzaiza et al (2003) are derived a moment inequality for HNBUET, based on this inequality they introduced testing procedure for exponentiality against HNBUET.

The main theme of this paper is organized as follows:
In Section 2, we propose a test statistic, based on the U-statistic, for testing $H_0 : F$ is exponential (μ) against $H_1 : F$ is HNBUET and not exponential. We then present Monte Carlo null distribution critical points for sample sizes n =5(1)60. In Section 3, we calculate the efficiency of the test statistic for some common alternatives and compared them to other procedures. In Section 4, we give simulated values of the power estimates of the test and in Section 5, we consider the problem of dealing with randomly right censored data. Finally, in Section 6, we consider an application in medical sciences, based on a set of data from Susarla and Vanryzin (1978).

2. TESTING THE HNBUET CLASS

2.1 The U-statistics test procedure

The test here depends on a random sample $X_1, \ldots, X_n$ from a population with distribution F. We wish to test the null hypothesis $H_0 : \overline{F}$ is exponential against the alternative hypothesis $H_1 : \overline{F}$ is HNBUET and not exponential, that is,

$$\int_t^\infty \int_x^\infty \overline{F}(u)dudt \leq \mu^2 \exp\left(\frac{-x}{\mu}\right); \qquad x,t \geq 0 \ , \ \mu > 0.$$

In order to test H_0 against H_1 we define the invariant measure of departure from H_0 as

$$\Delta_F = \frac{1}{\mu^2} E\left[\mu^2 \exp(-X/\mu) - \int_X^\infty \int_t^\infty \overline{F}(u)dudt\right] \tag{2.1}$$

i.e.
$$\Delta_F = \frac{1}{\mu^2}\int_0^\infty \{\mu^2 \exp(-x/\mu) - \int_x^\infty \int_t^\infty \overline{F}(u)dudt\}dF(x). \tag{2.2}$$
Integrating (2.2) by parts gives
$$\Delta_F = \frac{1}{\mu^2}\int_0^\infty \{\mu^2 e^{-x/\mu} - \frac{x^2}{2} + x^2\overline{F}(x) + xv(x)\}dF(x), \tag{2.3}$$
where $v(x) = \int_x^\infty \overline{F}(u)du$.

Note that under $H_0, \Delta_F = 0$, while under $H_1, \Delta_F > 0$. We estimate Δ_F by $\hat{\Delta}_{F_n}$ by using a random sample $X_1, \ldots, X_n$ from F. The survival distribution $\overline{F}(x)$ is estimated by the empirical survival distribution $\overline{F}_n(x) = \frac{1}{n}\sum_{j=1}^n I(X_j > x)$, dF is estimated by $dF_n(x) = \frac{1}{n}, v(x)$ is estimated by $\hat{v}_n(x) = \frac{1}{n}\sum_{j=1}^n (X_j - x)I(X_j > x)$ and μ is estimated by the sample mean $\overline{X}$. Then, by using Equation (2.3), $\hat{\Delta}_{F_n}$ can be obtained as:
$$\hat{\Delta}_{F_n} = \frac{1}{\overline{X}^2}\int_0^\infty \{\overline{X}^2 \exp(\frac{-x}{\overline{X}}) - \frac{x^2}{2} + x^2\overline{F}_n(x) + x\hat{v}_n(x)\}dF_n(x) \tag{2.4}$$
i.e.
$$\hat{\Delta}_F = \frac{1}{\overline{X}^2}\int_0^\infty \{(\frac{\sum_{j=1}^n X_j}{n})^2 \exp(\frac{-x}{\overline{X}}) - \frac{x^2}{2} + \frac{x^2}{n}\sum_{j=1}^n I(X_j > x)$$
$$+\frac{x}{n}\sum_{j=1}^n (X_j - x)I(X_j > x)\}dF_n(x) \tag{2.5}$$
i.e.
$$\hat{\Delta}_{F_n} = \frac{1}{n^3\overline{X}^2}\sum_{i=1}^n\sum_{j=1}^n\sum_{k=1}^n \{X_jX_ke^{-X_i/\overline{X}} - \frac{X_i^2}{2} + X_iX_jI(X_j > X_i)\} \tag{2.6}$$
i.e.
$$\hat{\Delta}_{F_n} = \frac{1}{n^3\overline{X}^2}\sum_{i=1}^n\sum_{j=1}^n\sum_{k=1}^n \phi(X_i, X_j, X_k). \tag{2.7}$$
Setting
$$\phi(X_1, X_2, X_3) = X_2X_3e^{-X_1/\mu} - \frac{X_1^2}{2} + X_1X_2I(X_2 > X_1) \tag{2.8}$$
and defining the symmetric kernel $\psi(X_1, X_2, X_3) = \frac{1}{3!}\sum_R \phi(X_{i1}, X_{i2}, X_{i3})$, where the sum is over all arrangements of X_1, X_2 and X_3, it is seen that $\hat{\Delta}_{F_n}$ is equivalent to U-statistics
$$U_n = \frac{1}{\binom{n}{3}}\sum_{i<j<k} \psi(X_i, X_j, X_k). \tag{2.9}$$

The following theorem summarizes the large sample properties of $\hat{\Delta}_F$ as U_n.

Theorem 1. (i) As $n \to \infty$, $\sqrt{n}(U_n - \Delta_F)$ is asymptotically normal with mean 0 and variance σ^2 given by

$$\begin{aligned}\sigma^2 &= Var\{\mu^2 e^{-x/\mu} - \frac{1}{2}X_1^2 + X_1 \int_{X_1}^{\infty} y dF(y) + 2X_1\mu \int_0^{\infty} e^{-y/\mu} dF(y) \\ &\quad - \int_0^{\infty} y^2 dF(y) + X_1 \int_0^{X_1} y dF(y) + \int_0^{\infty} u \int_u^{\infty} y dF(y) dF(u)\}\end{aligned}$$

(ii) Under H_0, the variance reduces to

$$\sigma_0^2 = E(-\frac{3}{2} + 2X_1 - \frac{1}{2}X_1^2 + e^{-X_1})^2 = 0.8333$$

(iii) If F is continuous HNBUET, then the test is consistent.

Proof. The proofs of (i) and (ii) follow from the standard theory of U-statistics (see, Lee (1990)) and direct calculation, respectively.

To prove (iii) let $D(x) = \mu^2 e^{-x/\mu} - \frac{x^2}{2} + x^2\overline{F}(x) + xv(x)$ in (2.3).

Since F is HNBUET, not exponential and continuous, then $D(x) > 0$ for at least one (x), call it x_0.

Set $(x_1) = \inf\{(x)/x \le x_0, \overline{F}(x) = \overline{F}(x_0)\}$. Thus

$$\begin{aligned}D(x_1) &= \mu e^{-x_1/\mu} - \frac{x_1^2}{2} + x_1^2\overline{F}(x_1) + x_1 v(x_1) \\ &\ge \mu e^{-x_1/\mu} + x_1^2\overline{F}(x_1) + x_1 v(x_1) - \frac{x_0^2}{2} \\ &= \mu e^{-x_0/\mu} + x_0^2\overline{F}(x_0) + x_0 v(x_1) - \frac{x_0^2}{2} = D(x_0)\end{aligned}$$

and $F(x_1 + \delta) - F(x_1) > 0$ and since x_1 is point of increase of F, thus $\Delta_F > 0$.

2.2 Monte Carlo Null Distribution Critical Points

We have simulated the upper percentile points for 90%, 95%, 98% and 99%. Table 1 gives these percentile points of the statistic $\hat{\Delta}_{F_n}$ in (2.6). The calculations are based on 5000 simulated samples of sizes n =5(1)60. It is clear from Table 1 that percentile values change slowly as n increases.

Table 1. Critical values of the $\hat{\Delta}_{F_n}$ statistics

n	90%	95%	98%	99%	n	90%	95%	98%	99%
5	0.1803	0.2058	0.2279	0.2389	33	0.1373	0.1618	0.1872	0.1996
6	0.1806	0.2057	0.2285	0.2436	34	0.1401	0.1652	0.1863	0.2014
7	0.1827	0.2090	0.2316	0.2439	35	0.1360	0.1575	0.1810	0.1922
8	0.1816	0.2069	0.2306	0.2440	36	0.1351	0.1563	0.1803	0.1958
9	0.1801	0.2027	0.2292	0.2442	37	0.1329	0.1547	0.1768	0.1897
10	0.1801	0.2073	0.2288	0.2442	38	0.1346	0.1589	0.1792	0.1913
11	0.1786	0.2030	0.2278	0.2423	39	0.1288	0.1547	0.1775	0.1922
12	0.1719	0.2007	0.2250	0.2388	40	0.1299	0.1536	0.1780	0.1948
13	0.1711	0.1978	0.2220	0.2347	41	0.1287	0.1520	0.1751	0.1908
14	0.1724	0.1964	0.2228	0.2362	42	0.1279	0.1497	0.1758	0.1909
15	0.1699	0.1930	0.2178	0.2336	43	0.1263	0.1492	0.1743	0.1881
16	0.1665	0.1919	0.2176	0.2321	44	0.1264	0.1487	0.1699	0.1856
17	0.1637	0.1906	0.2176	0.2300	45	0.1250	0.1464	0.1700	0.1847
18	0.1629	0.1887	0.2111	0.2226	46	0.1260	0.1486	0.1681	0.1839
19	0.1575	0.1818	0.2033	0.2210	47	0.1240	0.1460	0.1672	0.1788
20	0.1561	0.1824	0.2056	0.2193	48	0.1231	0.1461	0.1681	0.1818
21	0.1588	0.1843	0.2098	0.2234	49	0.1190	0.1418	0.1656	0.1808
22	0.1565	0.1817	0.2080	0.2186	50	0.1217	0.1434	0.1662	0.1782
23	0.1540	0.1791	0.2042	0.2174	51	0.1220	0.1420	0.1622	0.1746
24	0.1479	0.1751	0.1965	0.2109	52	0.1176	0.1399	0.1665	0.1796
25	0.1482	0.1757	0.1954	0.2084	53	0.1160	0.1363	0.1586	0.1719
26	0.1470	0.1704	0.1959	0.2105	54	0.1185	0.1389	0.1611	0.1755
27	0.1468	0.1699	0.1945	0.2064	55	0.1153	0.1385	0.1623	0.1748
28	0.1411	0.1652	0.1893	0.2045	56	0.1155	0.1372	0.1585	0.1710
29	0.1449	0.1687	0.1909	0.2066	57	0.1140	0.1358	0.1552	0.1697
30	0.1413	0.1653	0.1892	0.2019	58	0.1148	0.1369	0.1627	0.1743
31	0.1377	0.1621	0.1862	0.1990	59	0.1127	0.1334	0.1538	0.1675
32	0.1389	0.1625	0.1859	0.2038	60	0.1111	0.1318	0.1530	0.1648

3. PITMAN ASYMPTOTIC RELATIVE EFFICIENCY (PARE)

In this section, we compare the statistic $\hat{\Delta}_{F_n}$, given in Equation (2.6), with the statistic $\hat{\Delta}_3^{(1)}$, proposed by Al-Ruzaiza *et al.* (2003). The construction of the statistic $\hat{\Delta}_3^{(1)}$ is based on the moments inequalities for harmonic new better than used in expectation property. The comparisons are achieved by using the Pitman asymptotic relative efficiency (PARE), which is defined as follows:

Let T_{1n} and T_{2n} be two test statistics for testing $H_0 : F_\theta \epsilon \{F_{\theta_n}\}$, $\theta_n = \theta + Cn^{-1/2}$ where C is an arbitrary constant. Then the PARE of T_{1n} relative to T_{2n} is defined by

$$e(T_{1n}, T_{2n}) = \{\mu_1'(\theta_0/\sigma_1(\theta_0)\}/\{\mu_2'(\theta_0)/\sigma_2(\theta_0)\}$$

where $\mu_i'(\theta_0) = \lim_{n\to\infty}\{\frac{\partial}{\partial\theta}E(T_{in})\}_{\theta\to\theta_0}$ and $\sigma_i^2(\theta_0) = \lim_{n\to\infty} Var(T_{in})$, $i = 1, 2$ is the null variance.

We consider the following two alternative distributions:

(i) Linear failure rate family : $\bar{F}_1(x) = \exp(-x - \frac{\theta}{2}x^2)$, $\theta > 0, x \geq 0$
(ii) Makeham family : $\bar{F}_2(x) = \exp[-x - \theta(x + e^{-x} - 1)]$, $\theta > 0, x \geq 0$

By direct calculations, the asymptotic efficiencies of the tests $\hat{\Delta}_{F_n}$ are 0.9583 and 0.228 for F_1 and F_2 respectivly. The asymptotic efficiencies of the tests $\hat{\Delta}_3^{(1)}$ for the same class are 0.900 and 0.200 for F_1 and F_2 respectivly. This shows that our U-statistic $\hat{\Delta}_{F_n}$ performs better than the statistic $\hat{\Delta}_3^{(1)}$ for all the alternative distributions considered.

The PARE of $\hat{\Delta}_{F_n}$ and $\hat{\Delta}_3^{(1)}$ can be obtained using their PAE by taking the ratio of $\hat{\Delta}_{F_n}$ to the corresponding $\hat{\Delta}_3^{(1)}$. Results are presented in Table 2 below which clearly show that the test statistics $\hat{\Delta}_{F_n}$ performs better than $\hat{\Delta}_3^{(1)}$.

Table 2. PARE of $\hat{\Delta}_{F_n}$ with respect to $\hat{\Delta}_3^{(1)}$.

Distribution	PARE $e(\hat{\Delta}_{F_n}, \hat{\Delta}_3^{(1)})$
Linear failure rate	1.0653
Makeham	1.7

4. THE POWER ESTIMATES

We calculate the estimate of the statistic $\hat{\Delta}_{F_n}$ defined in (2.6) at level 95% upper percentile and for following alternative distributions:

(i) Linear failure rate $:f_1(\theta) = (1+\theta)e^{-x-\frac{1}{2}x^2}$, $x \geq 0, \theta \geq 0.$
(ii) Makeham $:f_2(\theta) = (1+\theta(1-e^{-x}))e^{-x-\theta(x+e^{-x}-1)}$, $x \geq> 0, t \geq 0.$
(iii) Weibull $:f_3(\theta) = \theta x^{\theta-1}e^{-x^\theta}$, $x \geq 0, \theta \geq 0.$

All these distributions are IFR (for an appropriate restriction on θ), hence they all belong to a wider class. Moreover, all these distributions reduce to exponential distribution for (i) and (ii) when the value $\theta = 0$ and for (iii) when the value of $\theta = 1$. Table 3 contains the power estimate for the $\hat{\Delta}_{F_n}$ test statistic with respect to these distributions. The estimates are based on 5000 simulated samples of size $n = 10$, 20, 30 and 40 at level 95% upper percentile.

Table 3. Power estimate for $\hat{\Delta}_{F_n}$-statistic

Distribution	Parameter	Sample size			
	θ	n=10	n=20	n=30	n=40
$f_1\theta$	2	0.2222	0.4348	0.5978	0.7198
(Linear failure	3	0.2606	0.5180	0.7114	0.8256
Rate)	4	0.3092	0.5828	0.7686	0.8808
$f_2\theta$	2	0.1502	0.2752	0.3724	0.4472
Makeham	3	0.1964	0.3596	0.4882	0.5954
	4	0.2336	0.4316	0.5750	0.6786
$f_3\theta$	2	0.7124	0.9686	0.9970	1.0000
Weibull	3	0.9888	1.0000	1.0000	1.0000
	4	1.0000	1.0000	1.0000	1.0000

The power estimate in Table 3 shows clearly the departure from exponentiality towards (HNBUET) properties as θ increases.

5. TEST FOR HNBUET IN THE CASE OF RANDOMLY RIGHT-CENSORED MODEL

In this section, a test statistic is proposed to test H_0 against H_1 with randomly right censored data. Such a censored data is usually the only information available in a life-testing model or in a clinical study where patients may be lost (censored) before the completion of a study. This experimental situation can formally be modeled as follows:

Suppose n objects are put on a test, and $X_1, \ldots, X_n$ are their true life times. Assume that $X_1, \ldots, X_n$ are independent, identically distributed (i.i.d.) according to a continuous life distribution F. Let $Y_1, \ldots, Y_n$ be i.i.d according to a continuous censoring distribution G. Also assume that the X's are independent of the Y's. In the randomly right-censored model, we observe the pair (Z_j, δ_j), $j = 1, \ldots, n$ where $Z_j = \min(X_j, Y_j)$ and,

$$\delta_j = \begin{cases} 1 & \text{if } Z_j = X_j \ (j^{th} \text{ observation is uncensored}) \\ 0 & \text{if } Z_j = Y_j \ (j^{th} \text{ observation is censored}) \end{cases}$$

Let $Z_{(0)} = 0 < Z_{(1)} < Z_{(2)} < \ldots < Z_{(n)}$ denote the ordered Z's and δ_j is the δ corresponding to Z_j, respectively.

Using the censored data $(Z_{(1)}, \delta_j)$, $j = 1, \ldots, n$ Kaplan and Meier (1958) propose the product limit estimator

$$\overline{F}_n(x) = 1 - F(x) = \prod_{[j:Z_{(j)} \leq 1} \{(n-j)/(n-j+1)\}^{\delta_{(j)}}, \qquad x \epsilon [0, Z_{(n)}]. \tag{5.1}$$

Now, for testing $H_0 : \Delta_F = 0$ against $H_1 : \Delta_F > 0$, using the randomly right censored data, we propose the following test statistic

$$\hat{\Delta}^c_{F_n} = \frac{1}{\hat{\mu}^{c^2}} \int_0^\infty \{\hat{\mu}^{c^2} e^{-x/\hat{\mu}^c} - \frac{x^2}{2} + x^2 \overline{F}_n(x) + x \hat{v}^c_n(x)\} dF_n(X) \tag{5.2}$$

where $\overline{F}_n$ is the product limit estimator given in (5.1), above. For computational purposes, $\hat{\mu}^c, \hat{\Delta}^c_{F_n}$ and $\hat{v}^c_n(x)$ can be written as

$$\hat{\mu}^c = \sum_{j=1}^{n} \{\prod_{k=1}^{j-1} C_k^{\delta(k)}\} (Z_{(j)} - Z_{(j-1)}), \tag{5.3}$$

$$\begin{aligned} \hat{\Delta}^c_{F_n} \quad = \quad & \frac{1}{\hat{\mu}^{c^2}} \sum_{j=1}^{n} \{\hat{\mu}^{c^2} e^{-\frac{Z_j}{\hat{\mu}^c}} - \frac{Z_j^2}{2} + Z_j^2 \prod_{k=1}^{j-1} C_k^{\delta(k)} \\ & + Z_j \left[\sum_{i=j}^{n} (\prod_{k=1}^{i-1} C_k^{\delta(k)}) (Z_{(i)} - Z_{(i-1)}) \right] \} \\ & \{\prod_{k=1}^{j-2} C_k^{\delta(k)} - \prod_{k=1}^{j-1} C_k^{\delta(k)}\} \end{aligned} \tag{5.4}$$

where

$$\hat{v}_n^c(x) = \sum_{i=j}^{n} \prod_{k=1}^{i-1} C_k^{\delta(k)} (Z_{(i)} - Z_{(i-1)}) \tag{5.5}$$

and

$$dF_n(x) = \overline{F}_n(Z_{j-1}) - \overline{F}_n(Z_j) = \prod_{k=1}^{j-2} C_k^{\delta(k)} - \prod_{k=1}^{j-1} C_k^{\delta(k)} \tag{5.6}$$

with $C_k = [n-k][n-k+1]^{-1}$

Table 4 gives the critical values percentiles of $\hat{\Delta}_{F_n}^c$ test for sample sizes 5(1)60(5)80,81.

Table 4. The critical values percentiles of $\hat{\Delta}_{F_n}^c$.

n	0.01	0.02	0.05	0.10	0.90	0.95	0.98	0.99
5	0.0000	0.0000	0.0000	0.2021	0.5115	0.5620	0.6014	0.6245
6	0.0000	0.0000	0.1676	0.2072	0.5099	0.5630	0.6028	0.6326
7	0.0000	0.1428	0.1759	0.2589	0.5303	0.5682	0.6023	0.6355
8	0.0648	0.1397	0.2021	0.2706	0.5317	0.5694	0.6127	0.6339
9	0.1121	0.1570	0.2362	0.2790	0.5410	0.5790	0.6151	0.6370
10	0.1250	0.1907	0.2431	0.2812	0.5476	0.5816	0.6197	0.6452
11	0.1250	0.1848	0.2455	0.2877	0.5519	0.5874	0.6254	0.6462
12	0.1432	0.1869	0.2464	0.2824	0.5559	0.5928	0.6266	0.6506
13	0.1511	0.1904	0.2467	0.2838	0.5606	0.5952	0.6276	0.6490
14	0.1546	0.1931	0.2384	0.2771	0.5600	0.5915	0.6263	0.6483
15	0.1501	0.1878	0.2363	0.2769	0.5542	0.5945	0.6265	0.6509
16	0.1278	0.1844	0.2364	0.2752	0.5667	0.5949	0.6263	0.6492
17	0.1294	0.1803	0.2370	0.2730	0.5702	0.6017	0.6311	0.6589
18	0.1426	0.1811	0.2327	0.2657	0.5744	0.6067	0.6428	0.6634
19	0.1416	0.1815	0.2341	0.2708	0.5784	0.6077	0.6415	0.6604
20	0.1145	0.1666	0.2256	0.2666	0.5800	0.6084	0.6428	0.6605
21	0.1396	0.1743	0.2250	0.2615	0.5787	0.6089	0.6392	0.6602
22	0.1354	0.1719	0.2245	0.2617	0.5777	0.6097	0.6417	0.6588
23	0.1281	0.1670	0.2218	0.2609	0.5772	0.6096	0.6458	0.6669
24	0.1325	0.1732	0.2208	0.2575	0.5825	0.6130	0.6447	0.6687
25	0.1296	0.1678	0.2187	0.2586	0.5832	0.6146	0.6466	0.6675
26	0.1314	0.1651	0.2183	0.2535	0.5839	0.6147	0.6436	0.6646
27	0.1005	0.1562	0.2127	0.2491	0.5867	0.6162	0.6516	0.6723
28	0.1292	0.1695	0.2142	0.2521	0.5879	0.6162	0.6456	0.6522
29	0.1125	0.1556	0.2088	0.2494	0.5831	0.6149	0.6480	0.6649
30	0.1216	0.1585	0.2089	0.2478	0.5899	0.6186	0.6511	0.6672
31	0.1218	0.1575	0.2079	0.2435	0.5910	0.6217	0.6539	0.6715
32	0.1026	0.1499	0.2062	0.2416	0.5909	0.6191	0.6479	0.6681
33	0.1225	0.1569	0.2080	0.2446	0.5905	0.6167	0.6469	0.6651
34	0.1173	0.1530	0.1995	0.2392	0.5873	0.6210	0.6497	0.6703
35	0.1068	0.1477	0.2019	0.2397	0.5954	0.6222	0.6539	0.6716
36	0.1092	0.1506	0.2003	0.2378	0.5942	0.6232	0.6513	0.6708
37	0.1050	0.1523	0.1963	0.2340	0.5937	0.6217	0.6518	0.6720
38	0.1104	0.1503	0.1989	0.2371	0.5941	0.6255	0.6543	0.6730
39	0.0949	0.1426	0.1990	0.2361	0.5934	0.6268	0.6517	0.6678
40	0.1006	0.1434	0.2005	0.2340	0.5925	0.6192	0.6525	0.6677
41	0.1089	0.1470	0.1972	0.2328	0.5937	0.6225	0.6518	0.6704
42	0.1020	0.1408	0.1924	0.2309	0.5958	0.6254	0.6561	0.6727
43	0.0990	0.1391	0.1898	0.2290	0.5974	0.6258	0.6561	0.6774
44	0.1039	0.1394	0.1887	0.2292	0.5945	0.6233	0.6515	0.6704
45	0.1164	0.1496	0.1967	0.2308	0.5968	0.6260	0.6574	0.6712
46	0.1059	0.1453	0.1920	0.2271	0.5974	0.6273	0.6579	0.6724
47	0.1049	0.1404	0.1925	0.2293	0.5939	0.6215	0.6494	0.6698
48	0.0931	0.1370	0.1885	0.2240	0.5972	0.6265	0.6537	0.6671
49	0.0925	0.1287	0.1865	0.2258	0.5974	0.6254	0.6556	0.6753
50	0.0973	0.1426	0.1882	0.2223	0.5948	0.6253	0.6524	0.6680
51	0.0916	0.1310	0.1821	0.2256	0.5950	0.6233	0.6509	0.6710
52	0.1003	0.1349	0.1860	0.2271	0.5963	0.6276	0.6567	0.6715
53	0.0955	0.1406	0.1905	0.2256	0.5968	0.6266	0.6571	0.6714
54	0.1012	0.1379	0.1893	0.2245	0.5934	0.6255	0.6541	0.6725
55	0.1041	0.1420	0.1877	0.2236	0.5954	0.6243	0.6472	0.6703
56	0.1099	0.1425	0.1893	0.2236	0.5983	0.6289	0.6558	0.6769
57	0.1001	0.1331	0.1815	0.2190	0.5970	0.6258	0.6549	0.6774
58	0.1031	0.1355	0.1832	0.2213	0.5989	0.6273	0.6531	0.6746
59	0.1030	0.1362	0.1800	0.2178	0.5949	0.6264	0.6581	0.6742
60	0.1022	0.1363	0.1823	0.2197	0.5944	0.6233	0.6489	0.6716
65	0.0843	0.1270	0.1748	0.2160	0.5976	0.6263	0.6539	0.6723

Table 4. The critical values percentiles of $\hat{\Delta}^c_{F_n}$ (Continued).

70	0.0930	0.1245	0.1718	0.2098	0.5912	0.6235	0.6552	0.6698
75	0.0921	0.1295	0.1745	0.2134	0.5897	0.6244	0.6548	0.6744
80	0.1008	0.1304	0.1764	0.2100	0.5926	0.6261	0.6568	0.6761
81	0.1049	0.1312	0.1753	0.2082	0.5937	0.6251	0.6571	0.6760

6. APPLICATION

Consider the data in Susarla and Vanryzin (1978). These represent 81 survival times of patients of melanoma. Out of these 46 represent non-censored data, and the ordered values are: 13, 14, 19, 19, 20, 21, 23, 23, 25, 26, 26, 27, 27, 31, 32, 34, 34, 37, 38, 38, 40, 46, 50, 53, 54, 57, 58, 59, 60, 65, 65, 66, 70, 85, 90, 98, 102, 103, 110, 118, 124, 130, 136, 138, 141, 234.

The ordered censored observations are: 16, 21, 44, 50, 55, 67, 73, 76, 80, 81, 86, 93, 100, 108, 114, 120, 124, 125, 129, 130, 132, 134, 140, 147, 148, 151, 152, 152, 158, 181, 190, 193, 194, 213, 215.

Now, ignoring the censored data, one can apply the methodology of section 2 to test the hypothesis H_0: the survival times are exponential versus H_1: the survival times follow HNBUET and not exponential.

Computing the statistic $\hat{\Delta}_{F_n}$ from (2.6) we get $\hat{\Delta}_{F_n} = 0.166$ which is significant at $\alpha = 0.05$. But, taking into account the whole set of survival data (both censored and uncensored), and computing the statistic $\hat{\Delta}^c_{F_n}$ from (5.4) we get $\hat{\Delta}^c_{F_n} = 0.5594$ which is not significant at $\alpha = 0.05$.

ACKNOWLEDGEMENT

The author is grateful to Professor M. Hendi for his valuable discussions when preparing the paper.

REFERENCES

Abouammoh, A. M. and Ahmed, A. N. (1989). Two notations of positive ageing based on stochastic dominance, *Statische Hefte*, **30**, 213-219.

Ahmad, I. A. (1994). A class of Statistics useful in testing increasing failure rate average and new better than used life distribution, *J. Statist. Plant.Inf.*, **41**, 141-149.

Ahmad, I. A. (1995). Nonparametric testing of classes of life distributions derived from a variability ordering, *Parisankhyan Samikkha.*, **2**, 13-18.

Al-Ruzaiza, A. S.,Hendi, M. I. and Abu-Youssuf, S. E. (2003). A note on moment inequalities for harmonic new better than used in expectation property with hypotheses testing applications, *J. Nonparametric Statist.*, **15**, 267-272.

Barlow, R. E. and Proschan, F. (1981). *Statistical Theory of Reliability and Life Testing*, To Being, with Silver Spring, MD.

Barlow, R. E., Marshall, A. W. and Proschan, F. (1963). Properties of probability distributions with monotone hazard rate, *Ann. Math. Statist.*, **34**, 375-389.

Bryson, M. C. and Siddiqui, M. M. (1969). Some criteria for aging, *J. Amer. Statist. Assoc.*, **64**, 1472-1483.

Deshpande, J. V., Kochar, S. C. and Singh, H. (1986). Aspects of positive ageing, *J. Applied Prob.*, **23**, 748-758.

Haines, A. (1973). *Some contributions to the theory of restricted of distributions with applications to reliability*, Ph.D. dissertation. The George Washington University.

Hendi, M. I., Al-Nachawati, H., Montasser, M. and Alwasel, I. A. (1998). An exact test for HNBUE class of life distributions, *J. Statist. Computation and Simulation*, **60**, 261-275.

Hollander, M. and Prochan, F. (1972). Testing whether new better than used, *Ann. Math. Statist.*, **43**, 1136-1146.

Kaplan, E. L. and Meeier, P. (1958). Nonparametric estimation from incomolete observation, *J. Amer. Statist. Assooc.*, **53**, 457-481.

Koul, H. I. (1977). A new test for new better than used, *Commu. Statist. Thear. Meth.*, **6**, 563-573.

Klefsjo, B. (1982). The HNBUE and HNWUE class of life distribution, *Noval Res. Logistics Quarterly*, **21**, 331-344.

Lee, A. J. (1990). *U-statistics*, New York: Marcel Dekker.

Stoyan, D. (1983). *Comparisons methods for Queues and their stochastic models*, New York: John Wiley.

Susarla, V. and Vanryzin, J. (1978). Empirical bayes estimation of survival functionright cencored observation, *Ann. Sttist.*, **6**, 710-755.

Zacks, S. (1992). Introduction to Reliability Analysis, *Springer-Verlage, New York.*