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P-Δ Effects on the Reliability of Offshore Platforms

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ABSTRACT

A typical marine platform in the Bay of Campeche is studied from the standpoint of structural reliability, and several characteristics of its deck such as slenderness and diameter/thickness ratios of the legs and actual degree of correlation between some variables are taken into account. The global and local buckling capacities of the deck legs are compared and the correlation coefficient between the critical axial load and the critical moment is assessed in order to validate its influence on the structural reliability. In addition, the influence of the vertical load, and its uncertainty, on the variance of the decks capacity, and latter on, on the platform's failure probability is assessed.

The results presented may be used in future studies to further extend and upgrade the first version of the Reference Norm (PEMEX, 2000) and, in the longer term, to improve the current practice in the Design and Requalification of Offshore Marine Platforms in the Bay of Campeche.

Keywords: P-Δ effect, structural reliability, platform deck, slenderness ratio, axial capacity, lateral capacity

1. Introduction

Design and assessment of steel structures for offshore platforms in the Bay of Campeche have been under considerable improvement since the 1960s. After the Roxanne hurricane, the IMP and PEMEX started extensive studies to develop risk based design and requalification criteria for those structures. A result of such efforts came out in 1997 in the form of the first edition of the Transitory Criteria for Design and Requalification of Offshore Platforms in the Bay of Campeche (Instituto Mexicano del Petróleo, 1997), which later became the PEMEX Reference Norm (PEMEX, 2000).

Whenever an offshore platform operates with heavy vertical load (due to the weight of equipment used for exploration and exploitation of oil and gas) and its deck legs are slender, the P-Δ effect of the deck may have a significant influence on the safety level of the platform (De León, 1999, 2001). It is not unrealistic for the deck legs to have high slenderness ratios because of the common requirement for the deck elevation to be higher than the wave height that occurs during strong storms.

It is well known that the axial capacity of slender elements may be governed by either global or local buckling. For this purpose, the deck legs of typical offshore platforms in the Bay of Campeche, Mexico were analyzed and their global and local buckling capacities compared to identify the instability mode that governs their axial capacities.

Because the wave loading and the structural capacity are uncertain variables, structural reliability techniques are used to explicitly include the uncertainties underlying the safety assessment process (Václavek *et al.*, 2001).

The structural safety of marine platforms under lateral loads is commonly assessed (Mortazavi *et al.*, 1996) by assuming that the platform is a series system composed of: the deck (superstructure), two or more bays of jackets, and the pile foundation.

Several assumptions regarding the coefficients of variation and the correlation between some of the main variables are examined from the standpoint of structural reliability. The impact of the P-Δ effect on the deck reliability is also addressed in this paper.

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2. Capacity of the Deck Legs

2.1 Structural Model

The failure mode commonly assumed for estimating the deck capacity considers that the ends of all legs reach their plastic capacity simultaneously. From the virtual work principle, and taking into account the P-Δ effect, the lateral capacity of the deck may be estimated. See Fig 1 for the model of the deck.

2.2 Lateral capacity of the Deck.

The wave height and the steel yield strength are considered as random variables. Deck failure probability is estimated by assuming that the wave loading and the deck lateral capacity are lognormal random variables and the platform failure probability is calculated from the above mentioned model of a series system. Data obtained from typical platforms in Mexico are used to illustrate the formulation and quantify its numerical significance.

If the contribution of the jacket stiffness under the deck is considered through a rotational spring and if the ultimate lateral displacement is Δ_u the lateral capacity of the deck, including the P-Δ effect is:

$$P_u = (2nM_u - Q\Delta)/H_d \quad (1)$$

where:

$$M_u = M_{cr}cos\alpha \quad (2)$$

$$\alpha = (\pi/2)[(Q/n)/P_{cr}] \quad (3)$$

Finally,

$$P_u = M_{cr}cos\alpha\{(2n/H_d) - Q[H_d/(6EI_d) + 1/C_r]\} \quad (4)$$

and

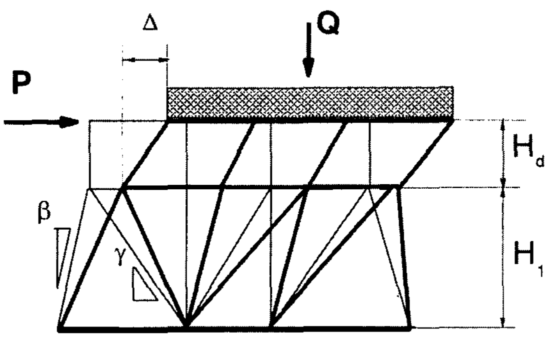

Fig. 1. Structural model of the platform deck

$$\Delta_u = M_u H_d[H_d/(6EI_d) + 1/C_r] \quad (5$$

In these expressions: Q = total vertical (dead + live load, n = number of platform legs, H_d = decks height, E = elastic modulus of the deck legs, I_d = moment o inertia of the deck legs, C_r = rotational stiffness o the jacket.

If the P-Δ effect is not taken into account:

$$P_u = 2nM_u/H_d \quad (6$$

Cox developed the following expressions (Cox, 1987 for the mean value of M_{cr} and P_{cr} (local buckling) fron regression analysis:

$$M_{cr} = M_p\{1.113exp[-1.638f_yD/(Et)]\} \quad (7$$

$$P_{cr} = P_y[1.79 - 0.25(D/t)^{0.25}] \quad (8$$

where: $M_p = Zf_y$, $P_y = Af_y$, f_y = yielding stress of tubula leg section, D = leg diameter, t = leg thickness Z = plasti modulus of tubular leg section, A = cross section area, an E = steel elastic modulus.

The bias and coefficient of variation are, for Eq. (3) 1.29 and 0.106 respectively, whereas for Eq. (4) are 1.2 and 0.117, according to Cox.

However, if global buckling governs the axial capacit of the legs (Cox, 1987):

$$P_{cr} = P_y[1.03 - 0.24(kH_d/r)^2 f_y/E] \quad (9$$

where:

kH_d/r = slenderness ratio

And the corresponding bias and coefficient of variation for several values of $\lambda = 1/\pi\ (kH_d/r)\ (f_y/E)^{1/2}$ are shown i Table 1:

2.3 Effect of the correlation between P_{cr} and M_{cr}

By recognizing that M_{cr} and P_{cr} are functions of the common random variable f_y, which is a common source o uncertainty, the correlation coefficient between the tw random variables M_{cr} and P_{cr} can be evaluated as

Table 1. Bias and coefficient of variation for global bucklin capacity

λ	0.4	0.6	0.8	1	1.2	1.4
Bias	1.196	1.200	1.208	1.220	1.239	1.273
CV	0.099	0.100	0.106	0.119	0.150	0.212

$$\rho_{M_{cr},P_{cr}} = [E(M_{cr}P_{cr}) - E(M_{cr})E(P_{cr})]/(\sigma_{M_{cr}}\sigma_{P_{cr}}) \quad (10)$$

where: $E()$ = expected value, M_{cr} = bending moment capacity, P_{cr} = local or global axial buckling capacity and σ = standard deviation, This correlation is usually assumed to be perfect. In order to validate this assumption, second order approximations and Monte Carlo simulation techniques (Cornell, 1969; Ang *et al.*, 1984) were performed to estimate the correlation coefficient between critical axial buckling load and buckling moment. For this purpose, it is necessary to know which buckling mode governs the axial capacity.

Typical marine platforms, all octapods (n = 8), in Mexico are analyzed. See Fig. 2 for the elevation views of one of them.

Local and global capacities are assessed through Eqs. (8) and (9) for typical slenderness and D/t ratios and for A-36 steel for actual offshore platforms in Mexico. See Table 2 for some of the geometrical properties. Fig. 3 shows a comparison between the buckling capacities for a range of slenderness and D/t ratios.

From Fig. 3, it may be observed that, for all the slenderness and D/t ratios shown, the global buckling mode governs the axial capacity.

The global and local buckling capacities of the deck legs of five actual offshore platforms, whose geometry data were shown in Table 2, were calculated. A comparison is shown in Fig. 4. As was found before, it may be observed that for these 5 cases, the global buckling mode also governs the axial strength.

Next, the correlation coefficient between Pcr and Mcr is calculated by using the Eqs. (7), (9) and (10) and the second order second moments approximation. By developing the approximations, it is shown (see Appendix A) that the coefficient of correlation is, approximately, 1. This is because of the almost linear relationship of P_{cr} and M_{cr} with respect to f_y. It is known that for two linear functions of a single variable, the correlation coefficient is 1.0.

Also, Monte Carlo simulations were performed to corroborate the value of the correlation coefficient. The steel

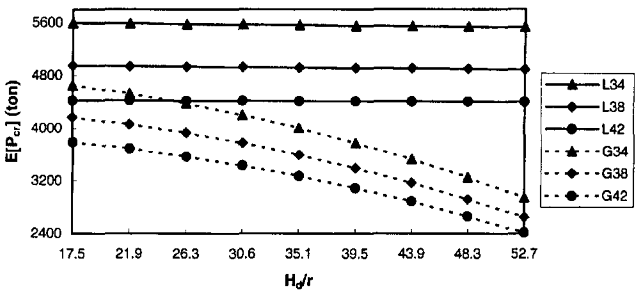

Fig. 3. Comparison between critical local (L) and global (G) buckling loads for several D/t and slenderness ratios

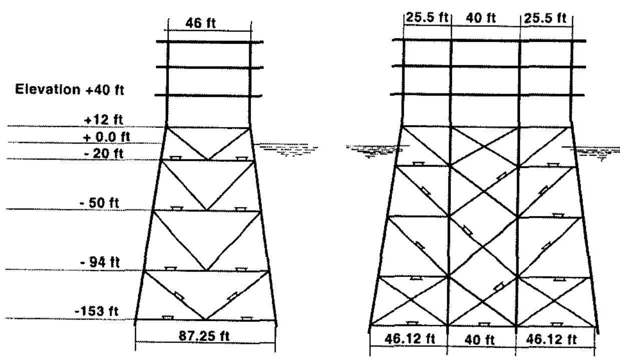

Fig. 2. Elevations of vertical frames

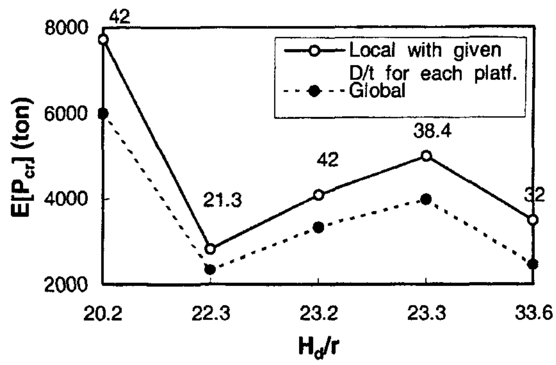

Fig. 4. Axial capacity of deck legs for 5 platforms on the Bay of Campeche

Table 2. Geometrical properties of 5 typical offshore platforms in Mexico

	Deck bays	Jacket bays	Lower deck elev. (ft)	Water depth (ft)	Weight of decks (ton)	D (in)	t (in)	D/t	H_D (in)	r (in) (in)	H_D/r
A	2	3	50.43	157	7907	48	1.50	32.00	384	16.71	22.98
B	3	4	39.99	153	5415	48	1.25	38.40	384	16.75	22.92
C	2	3	47.24	123	5315	48	2.25	21.33	326.8	16.58	19.71
D	2	3	47.01	111	2501	42	1.00	42.00	324	14.67	22.08
E	3	3	47.28	118	2675	36	1.38	26.18	411.4	12.49	32.95

yielding strength was assumed to have a lognormal distribution. The number of simulations performed in the Monte Carlo process were 5,000. The results, for several slenderness and D/t ratios are shown in Fig. 5.

In addition, the constant values usually recommended for the coefficient of variation of the critical axial load and buckling moment were reviewed (De León and Campos, 1999). In order to assess the numerical significance of these recommendations, analytical expressions for these coefficients of variation are developed, by using second moments also, and applied to a platform in the Bay of Campeche (See Appendix A). Estimations were made for A-36 steel, whose nominal yield level f_y = 36 ksi (2530 kg/cm^2), which corresponds to the grade of steel used for typical production platforms in Mexico. The value of CV_{fy} (coefficient of variation of f_y) is taken from previously reported results (Bruneau *et al.*, 1988; SSPC, 1994).

The coefficient of variation of the critical moment CV_{Mcr} is plotted, for A36 steel, against D/t and compared with the recommended (assumed) constant value. The comparison is shown in Fig. 6.

Fig. 7 shows the comparison between the actual coefficient of variation of the critical axial load and the recommended (suggested) value for several slenderness ratios. These values compare reasonably well with those in Table 1.

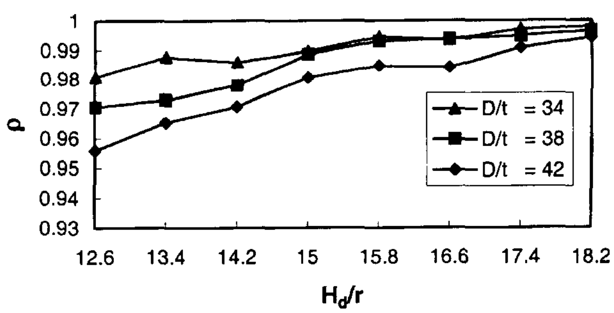

Fig. 5. Correlation coefficient between critical moment and global axial load from Monte Carlo simulations

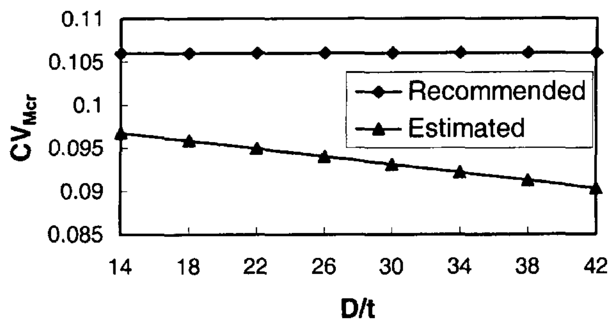

Fig. 6. Recommended and estimated coefficient of variation of the critical moment for several D/t ratios

2.3 Mean and coefficient of variation of the deck lateral capacity including the P-Δ effect

The variance of the vertical load is neglected in the existing formulation. In order to include it, and examine its numerical significance on the structural reliability, P_u is treated as a function of the random variables M_{cr}, P_{cr} and Q. It is assumed that correlation exists only between M_{cr} and P_{cr}. If second order moments are estimated in Eq. (1), it may be shown that (see Appendix B):

$$CV_{P_u}^2 = CV_{M_{cr}}^2 + E^2(a)CV_{P_{cr}}^2 + 2\rho_{M_{cr},P_{cr}} + E(a)CV_{M_{cr}}CV_{P_{cr}} + E^2(b)CV_Q^2 \tag{11}$$

where:

$$a = \alpha tan\alpha \tag{12}$$

and:

$$b = 1 - a - 2nM_{cr}cos\alpha/(P_uH_d) \tag{13}$$

However for Pu obtained without considering the P-Δ effect (from Eq. 5):

$$b' = -a \tag{14}$$

And the coefficient of variation of the lateral deck capacity without P-Δ effect is:

$$CV_{P_u}^2 = CV_{M_{cr}}^2 + E^2(a)CV_{P_{cr}}^2 + 2\rho_{M_{cr},P_{cr}} + E(a)CV_{M_{cr}}CV_{P_{cr}} + E^2(a)CV_Q^2 \tag{15}$$

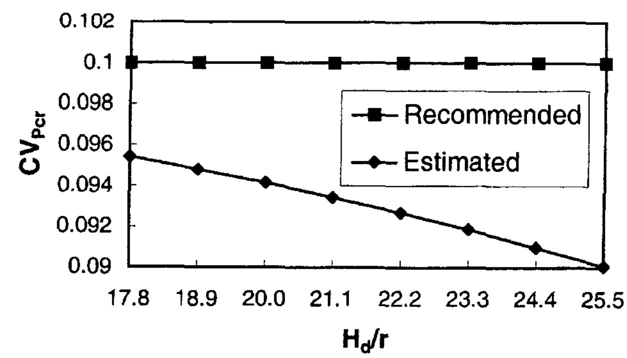

Fig. 7. Recommended and estimated coefficient of variation of the critical global axial load for several slenderness ratios

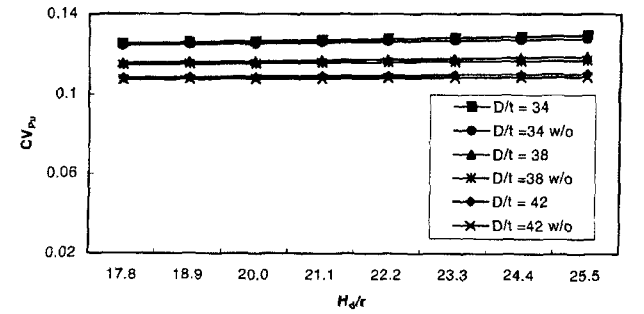

Fig. 8. Coefficient of variation of the decks lateral capacity for several slenderness and D/t ratios, E[Q]= 7,000 ton and for CV_Q = 0.2 with and without (w/o) P-Δ effect.

The influence of the slenderness and D/t ratios on the coefficient of variation of the deck lateral capacity, for E[Q] = 7,000 ton and CV_Q = 0.2, is shown in Fig. 8.

The effect of the slenderness and D/t ratios on the expected lateral deck capacity, for A-36 steel, is shown in Fig. 9. For each point in this figure, the deck height H_d is shown in meters. It is observed that, as the ratio D/t increases, the P-Δ effect also increases.

3. Reliability under Wave Loading

As an example of the calculation of the deck and platform reliability against wave loading, the mean and coefficient of variation of the lateral capacity of the deck, for the case B of the 5 actual platforms (see Table 2) were estimated. The estimations were performed for both conditions: with and without taking into account the P-Δ effect and for CV_Q = 0.2. The calculation of the deck reliability index is performed through:

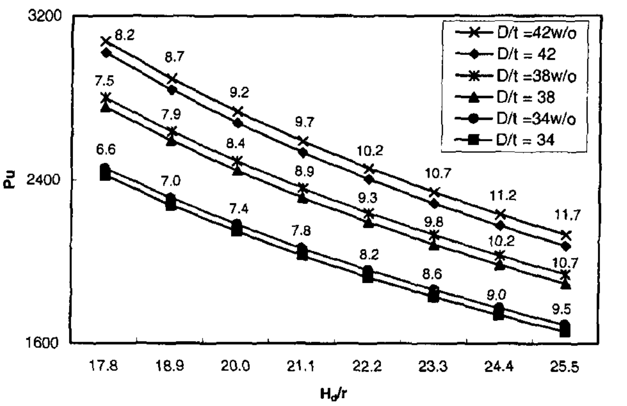

Fig. 9. Expected value of the decks lateral capacity for several slenderness and D/t ratios, E[Q]= 7,000 ton and for CV_Q = 0.2. For each point, H_d in m.

$$\beta = (\lambda_R - \lambda_w)/\sqrt{\zeta_R^2 + \zeta_w^2} \tag{16}$$

where:

$$\lambda = In\mu - 1/2\zeta^2 \tag{17}$$

for the capacity R and the wave loading W, and:

$$\zeta = \sqrt{In(1 + CV^2)} \tag{18}$$

Assuming that the failure probability of the other components does not change with the P-Δ effect of the deck, the effect over the global reliability may be calculated. The deck reliability is estimated through the calculation of the mean load, at the deck level, produced by the maximum wave height of 55 ft (De León, 2001), which is typical for extreme wave loading on the Bay of Campeche (De León *et al.*, 2003). Industry-wide software is used for that purpose (Stear *et al.*, 1997) A lognormal distribution is assumed for the wave load and the lateral deck capacity. The resulting deck reliability indices, with and without the P-Δ effects are 2.43 and 2.45, respectively.

The global reliability is estimated from the assumption that the global probability of failure is the geometrical mean between the first order bounds of the global failure probability. These bounds are obtained from the corresponding component reliabilities. The global reliabilities are 3.73.

Therefore, the reduction on deck reliability, for typical data of offshore platforms in Mexico, is not significant. This is because the actual design practice is conservative enough to avoid the P-Δ effect. Also, it is observed that the P-Δ effect does not change the global reliability index. This is because the slight difference on the deck is absorbed by the other component reliabilities which are larger that those of the deck.

4. Discussion of Results

From Fig. 3, the global buckling mode was found to be the one that governs the axial capacity of deck legs for the ranges of slenderness ratios and other geometrical characteristics that are typical of offshore platforms in Mexico. And also, from Fig. 4, it is clear that, for the 5 actual platforms that were analyzed, the global buckling governs the axial capacity of the deck legs.

The results from Fig. 5 verify the one obtained from Appendix A. It is observed that the Monte Carlo simulation process approximates the result of perfect correlation and, for higher slenderness ratios, the approximation improves.

The recommended constant values for the coefficient of variation of the critical axial load and moment are reasonable, as shown in Figs. 6 and 7, and the conservative margin increases slightly for higher steel grade, D/t ratios and slenderness.

From Figs. 8 and 9, the following observations may be made:

a) For practical values of slenderness and D/t ratios and vertical load, there is no significant change in the coefficient of variation of the deck lateral capacity.

b) Also, for a total vertical load of 7,000 tons, the deck lateral capacity reduces slightly as the slenderness and D/t ratios increase.

Finally, the deck reliability, for the typical values of slenderness and D/t ratios of actual offshore platforms in Mexico, showed a slight reduction when the P-Δ effect is included. Typical extreme oceanographical and meteorological conditions were considered for the wave loading.

5. Conclusions and Recommendations

The practice of assuming a perfect correlation between axial and moment critical buckling load is reasonable; also the respective assumed coefficients of variation are reasonable and may be used without introducing significant errors.

As the deck's failure mode is not a dominant in the platform's failure, for the particular platform used in the illustration, the global effect of the P-Δ effect, as well as the variance of the vertical load, is not significant. However, for local design and damage assessment of existing platforms, their effects are important and should not be neglected. In any case, it is important to assess the impact of other damage modes, such as corrosion and fatigue, before this conclusion may be generalized for the Bay of Campeche.

The mean and dispersion in the vertical load increase the failure probability and, consequently this load should be carefully estimated at the design stage, especially when the slenderness effects are significant. The practice of increasing the vertical load should be carefully examined because of its effect on the safety level, particularly in the case of platforms that tend to induce P-Δ effects which may produce an increment on the deck failure probability, as was shown here.

It is suggested to extend the research on the structural safety of facilities with slenderness problems, particularly when they are under strong vertical load and significant lateral loading.

Also, the effect of the non-uniform distribution of vertical load on the deck area is an aspect that remains to be assessed.

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APPENDIX A

Considering Eqs. (7), (9) and (10), $E(M_{cr}\ P_{cr})$ may be estimated by:

$$E(M_{cr}P_{cr}) = (M_{cr}P_{cr})_{E(fy)} + 1/2[\partial^2(M_{cr}P_{cr})/\partial f_y^2]_{E(f_y)}\sigma_{f_y}^2 \quad (19)$$

But, σ^2_{Mcr} may be estimated from Eq. (15) as:

$$\sigma_{M_{cr}}^2 = E^2(M_{cr})\{1+In[E(M_{cr})/(1.13E(M_p))]\}^2 CV_{f_y}^2 \quad (20)$$

That is:

$$CV_{M_{cr}} = \{1+In[E(M_{cr})/(1.13E(M_p))]\}CV_{f_y} \quad (21)$$

On the other hand, given that P_{cr} is a linear function of f_y,

$$CV_{P_{cr}} = CV_{f_y} \quad (22)$$

From Eqs. (18), (19) and with the second order approximations of $E(P_{cr})$ and $E(M_{cr})$:

$$E(P_{cr}) = (P_{cr})_{E(fy)} + 1/2[\partial^2(P_{cr})/\partial f_y^2]_{E(f_y)}\sigma_{f_y}^2 \quad (23)$$

and:

$$E(M_{cr}) = (M_{cr})_{E(fy)} + 1/2[\partial^2(M_{cr})/\partial f_y^2]_{E(f_y)}\sigma_{f_y}^2 \quad (24)$$

It may be shown that:

$$\rho_{M_{cr},P_{cr}} = [-E(\beta)-1]/\{1+In[E(M_{cr})/1.13/E(M_p)]\} \quad (25)$$

where:

$$\beta = 1.638Df_y/(Et)-2 \quad (26)$$

Finally:

$$\rho = \{1-1.638DE[f_y]/(Et)\}/\{1+(-1.638DE[f_y]/(Et))\} = 1 \quad (27)$$

APPENDIX B

If P_u is a function of the random variables M_{cr}, P_{cr} and Q and there is correlation only among M_{cr} and P_{cr}, the variance of P_u may be estimated by second order moments:

$$\sigma_{P_u}^2 = (\partial P_u/\partial M_{cr})^2\sigma_{M_{cr}}^2 + (\partial P_u/\partial P_{cr})^2\sigma_{P_{cr}}^2 + 2\rho_{M_{cr},P_{cr}}(\partial P_u/\partial P_{cr})(\partial P_u/\partial M_{cr})\sigma_{M_{cr}}\sigma_{P_{cr}} + (\partial P_u/\partial Q)^2\sigma_Q^2 \quad (28)$$

$$\partial P_u/\partial M_{cr} = P_u/M_{cr} \quad (29)$$

$$\partial P_u/\partial P_{cr} = -M_{cr}sin\alpha[P_u/(M_{cr}cos\alpha)](-\alpha/P_{cr}) \quad (30)$$

$$\partial P_u/\partial Q = M_{cr}cos\alpha\{-[H_d/(6EI)+1/C_r] + \pi/2/(nP_{cr})[P_u/(M_{cr}cos\alpha)](-M_{cr}sin\alpha) \quad (31)$$

where:

$$a = \alpha tan\alpha \quad (32)$$

And,

$$CV_{P_u}^2 = CV_{M_{cr}}^2 + E^2(a)CV_{P_{cr}}^2 + 2\rho_{M_{cr},P_{cr}}E(a)CV_{M_{cr}}CV_{P_{cr}} + E^2(b)CV_Q^2 \quad (33)$$

where:

$$b = 1-a-2nM_{cr}cos\alpha/(P_uH_d) \quad (34)$$