Korean Journal of Remote Sensing, Vol.19, No.5, 2003, pp.363~370

An Efficient Rectification Algorithm for Spaceborne SAR Imagery Using Polynomial Model

Man-Jo Kim

Agency for Defence Development

Abstract : This paper describes a rectification procedure that relies on a polynomial model derived from the imaging geometry without loss of accuracy. By using polynomial model, one can effectively eliminate the iterative process to find an image pixel corresponding to each output grid point.

With the imaging geometry and ephemeris data, a geo-location polynomial can be constructed from grid points that are produced by solving three equations simultaneously. And, in order to correct the local distortions induced by the geometry and terrain height, a distortion model has been incorporated in the procedure, which is a function of incidence angle and height at each pixel position.

With this function, it is straightforward to calculate the pixel displacement due to distortions and then pixels are assigned to the output grid by re-sampling the displaced pixels. Most of the necessary information for the construction of polynomial model is available in the leader file and some can be derived from others.

For validation, sample images of ERS-1 PRI and Radarsat-1 SGF have been processed by the proposed method and evaluated against ground truth acquired from 1:25,000 topography maps.

Key Words : Imaging Geometry, Geocoding, Rectification, Zero Doppler Time, Range-doppler Equation.

1. Introduction

SAR geocoding is the process of distorting the original slant-range image intensity or complex radar return values such that the final image format corresponds to a map grid oriented to grid North. The main constraint of the process is to preserve the radiometric content of the warping process, despite the fact that the warping may destroy almost all of the useful radiometric content (e.g. point spreading function, area and point statistics, radar cross-section).

Usually, the only radiometric values preserved are the absolute values of the radar intensities which must be derived by resampling the image using an interpolation function which may be variously defined (e.g. sinc function, spline), but an optimised resampling algorithm will require a regular array of oversampled points.

This resampling must take place on the slant-range image that means that the required pixel positions on the map must be point-to-point transformed onto the slant-range image where the image pixels can be resampled to the desired position. It is clear that this geocoding process is different from the geolocation process, which facilitates the image-to-object approach.

Received 7 August 2003; Accepted 25 September 2003.

Thus, a geocodng algorithm consists, initially, of a mapping function, relating points on the map to points in the image, and then a resampling step that will allow us to derive, for each map point, an intensity value.

The resampling process is highly subjective and a scheme must be chosen such that those radiometric properties required are preserved. There have, as yet, only been a few studies relating to the resampling aspect of SAR geocoding relating to the final image quality but as yet no optimal solution has been located.

The mapping function, however, consists of the transformation of points on the ground to points on the slant-range image. The first stage is to map each map projection coordinate (x, y) to a point in global Cartesian coordinate (X, Y, Z) via local geodetic and Cartesian coordinates using values of the terrain height. The final stage is the evaluation of the image coordinates (R, t) of the point using the range-doppler equation and the platform orbit model (see Fig. 1)(Curlander, 1982).

If

$$\vec{R}_S \equiv \vec{R}_S(t)$$

and

$$\vec{V}_S \equiv \vec{V}_S(t)$$

are the position and velocity vectors of the sensor platform, respectively, and

$$\vec{R}_T \equiv \vec{R}_T(t)$$

and

$$\vec{V}_T \equiv \vec{V}_T(t)$$

are the respective position and velocity vectors of the target on the Earth surface, then we may solve for the azimuth time, t, from doppler equation

$$f_D = \frac{2}{\lambda |\vec{R}_S - \vec{R}_T|} (\vec{R}_S - \vec{R}_T) \cdot (\vec{V}_S - \vec{V}_T) \qquad (1)$$

where, f_D is doppler frequency. Once, t has been determined, the range R is calculated from

$$R = |\vec{R}_S - \vec{R}_T| \qquad (2)$$

If all of the above vectors are given in the ECI, then $\vec{V}_T$ has the form

$$\vec{V}_T = \vec{\omega} \times \vec{R}_T,$$

where,

$$\vec{\omega} = (0, 0, \omega)$$

and ω is the rate of rotation of the Earth.

The above scheme for SAR geocoding is simplified when the input image is processed on zero doppler plane; i.e. where the positions of targets in the slant-range image is such that their position corresponds to that anticipated if the image had been processed to a Doppler centroid of zero. Thus, the value of f_D throughout the image is zero, which allows a consistent geocoding algorithm and ensures a geometrically coherent image.

During the geocoding process, with height information, we can correct distortions due to terrain.

As SAR forms an image in slant range that is directly related to the distance between sensor and targets, a target with height over the reference ellipsoid in slant range is displaced toward the sensor.

The relationship between the target displacement (D) and target height (h) is

$$D = \frac{h}{\tan(i)} \qquad (3)$$

where i is the incidence angle over the reference

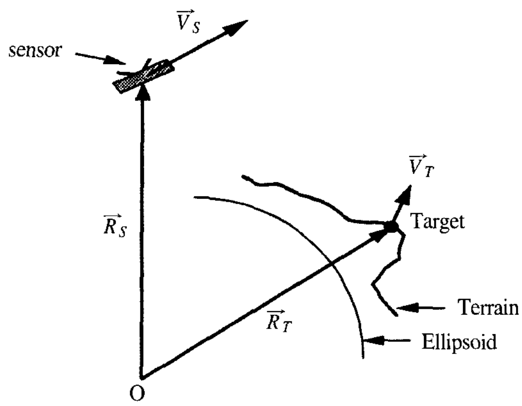


Fig. 1. SAR imaging geometry.

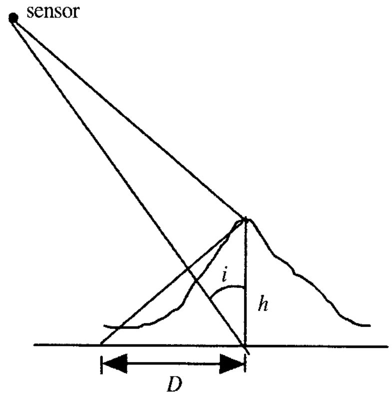


Fig. 2. Target displacement due to terrain height.

ellipsoid at the target position (see Fig. 2)(Kwok *et al.*, 1987).

2. Geocoding Algorithm

1) Overview

In this chapter, a fast geocoding algorithm for SAR imagery will be introduced. This method uses information in the header, does not require orbit interpolation, does not need to find Zero Doppler Time (ZDT) and performs geocoding on the fly.

Basically, the most common difficulties in accurate geocoding is to calculate the state vectors at any time just using given limited numbers of state vectors and other information. In general, it is accepted that the information in the image header is sufficiently accurate and imaging geometry of sensor is quite stable over imaging time.

The main ideas behind the method are twofold. Firstly, it relies on the information given in header and secondly, it constructs a polynomial which relates map coordinates to image coordinates using a least-square fitting method.

We have developed two different procedures specific to sensors, one for ERS SAR and the other for Radarsat, and this procedure can easily accommodate any other SAR sensors such as ENVISAT.

In order to solve the SAR geocoding problem, we need state vectors at any time with sufficient accuracy and must solve the range-doppler equation in space to locate image pixels to map locations.

In spaceborne SAR images, the following parameters are generally available in the header that we can utilise in the geocoding process.

- State vectors (positions, velocities)
- Time of first state vector
- Time interval between state vectors
- Range times (e.g., first, mid, last range pixel positions)
- Azimuth times (e.g., first, mid, last line positions)
- Pixel spacing, line spacing
- Reference ellipsoid parameters
- Conversion polynomial from ground to slant range

2) Method

Just using the parameter identified as above, it is possible to construct a polynomial that performs the geocoding on the fly.

With the given parameters, we solve the range-doppler equation at the azimuth positions corresponding to given state vectors and for every range pixel position. Then we calculate the geodetic positions on the reference ellipsoid. During this process we also calculate the incidence angle at each location.

Then, we relate the image coordinates, azimuth line and range pixel, with geodetic coordinates of that position. With this relationship, it is possible to construct a polynomial using least-square fitting.

Fig. 3 shows the processing flow for the proposed geocoding of spaceborne SAR images.

Each stage in the process for ERS case is described in detail.

(1) Extract Parameters

In the ERS SAR header, there are several parameters

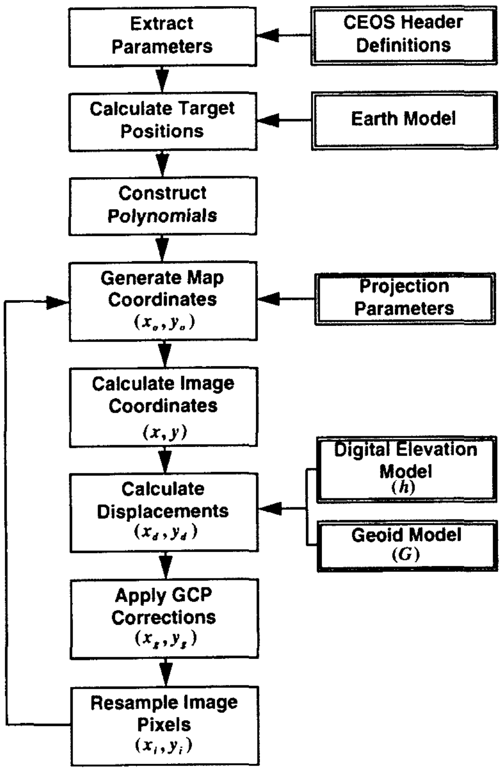


Fig. 3. Functional flow of fast geocoding algorithm.

that we can utilise for geocoding purpose.

During this process, parameters are extracted from the header and calculated for later use.

(2) Calculate Target Positions

This process is the heart of this proposed method. During this process, with given image coordinates, we solve the range-doppler equation and calculate geodetic coordinates of targets on the reference ellipsoid.

Eq. (4) is the one we have to solve and Fig. 4 translates the geolocation geometry.

$$
\begin{aligned}
&|\vec{R}_s - \vec{R}_t| = R \\
&\vec{V}_s \cdot (\vec{R}_s - \vec{R}_t) = 0 \\
&\frac{x_t^2 + y_t^2}{R_e^2} + \frac{z_t^2}{R_p^2} = 1
\end{aligned}
\tag{4}
$$

where

R is slant range to target position,

$\vec{R}_s$ is vector to satellite position,

$\vec{V}_s$ is vector of satellite velocity,

$\vec{R}_t = (x_t, y_t, z_t)$ is target position on reference ellipsoid (i.e. height is zero) and

R_e, R_p is semi-major axis and semi-minor axis of the reference ellipsoid respectively.

Actually, this step involves the solution of three non-linear equations simultaneously and we will refer to these equations as the geolocation equation.

Also, we need to calculate the slant range to each range pixel position.

$$
\begin{aligned}
&T_i = T_0 + \frac{C_0 + C_1 G_i + C_2 G_i^2 + C_3 G_i^3}{F_s} \\
&R_i = \frac{c}{2}(T_i - T_0)
\end{aligned}
\tag{5}
$$

where,

T_0, T_i are range time of first and i^th range pixel positions, respectively,

C_i is coefficients of ground range-to-range time polynomial,

G_i is ground range to i^th range pixel positions,

F_s is range sampling frequency of sensor,

R_i are slant range to i^th range pixel positions,

c is speed of light.

(3) Construct Polynomial

After solving the equations, we have a large number of data samples that consist of image coordinates and

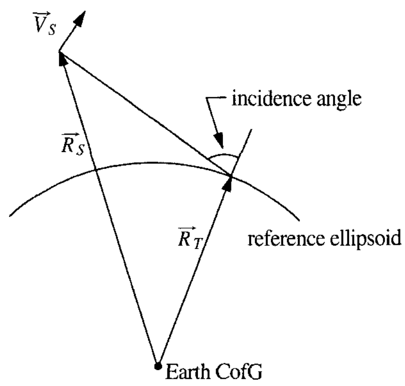


Fig. 4. Geometry of geolocation process.

geodetic coordinates. The maximum number of data samples is number of state vectors times number of range pixels. With these, it is quite straightforward to construct a polynomial using least-square fitting method.

We have used polynomial order of two, bi-quadratic, to fit the two coordinates and have constructed two different set of polynomial, one for image-to-map transformation (or forward transformation) and the other for map-to-image transformation (or backward transformation).

Eqs. (6) and (7) show polynomials and we will refer to as the geocoding polynomials.

$$\begin{aligned} x_0 &= a_0 + a_1x + a_2y + a_3xy + a_4x^2 + a_5y^2 \\ y_0 &= b_0 + b_1x + b_2y + b_3xy + b_4x^2 + b_5y^2 \end{aligned} \tag{6}$$

$$\begin{aligned} x &= c_0 + c_1x_0 + c_2y_0 + c_3x_0y_0 + c_4x_0^2 + c_5y_0^2 \\ y &= d_0 + d_1x_0 + d_2y_0 + d_3x_0y_0 + d_4x_0^2 + d_5y_0^2 \end{aligned} \tag{7}$$

where,

x, y are image coordinates,

x_0, y_0 are map coordinates,

a_i, b_i, c_i, d_i, are the coefficients of geocoding polynomials.

It should not be missed that with geocoding polynomials, we can relate image pixel to map position and vice versa. This is one of the big advantages of this proposed method.

(4) Calculate Target Displacement

The geocoding polynomial gives only image coordinates or map coordinates without displacement. In order to locate the displaced pixel position, we need to calculate the displacement using height from external DEM (refer Fig. 2).

As Eq. (3) requires ellipsoidal height, we have to know the geoidal height of target position.

$$x_d = \frac{(h+G)}{\tan(i)} \tag{8}$$

where

x_d is target displacement,

i is incidence angle,

h, G are elevation and geoidal height of target position, respectively.

As SAR forms an image in slant range, the pixel is displaced toward sensor.

(5) Apply GCP Correction

The header information used is sufficiently accurate. Nevertheless, there still could be error.

So, in order to achieve high degree of accuracy, we need Ground Control Points (GCPs). As imaging geometry of a SAR sensor is quite stable over imaging time, we adapted bi-linear polynomial for most general cases.

Eq. (9) shows the polynomial and we will refer to as the GCP polynomial.

$$\begin{aligned} x_g &= a_0 + a_1x_0 + a_2y_0 \\ y_g &= b_0 + b_1x_0 + b_2y_0 \end{aligned} \tag{9}$$

where,

x_0, y_0 are map coordinates,

x_g, y_g are GCP corrections,

a_i, b_i are coefficients of GCP polynomial.

The GCP polynomial gives the corrections in terms of pixel units and in turn, this will be applied to the evaluated coordinates from the geocoding polynomials.

(6) Calculate Pixel Positions

The actual, final pixel position is affected by three factors as follows.

- Evaluated coordinates from the geocoding polynomials
- Corrections from GCP polynomial
- Displacement due to terrain height

Therefore, the final pixel position is expressed as

$$\begin{aligned} x_i &= x + x_d + x_g \\ y_i &= y + y_g \end{aligned} \tag{10}$$

(7) Calculate Incidence Angles

In order to remove distortions due to terrain height, we need to calculate displacement by evaluating Eq. (11).

$$|\vec{R}_s|^2 = |\vec{R}_t|^2 + |\vec{R}_s - \vec{R}_t|^2 + 2|\vec{R}_t||\vec{R}_s - \vec{R}_t|\cos i \quad (11)$$

In ERS SAR case, these incidence angles directly come from the solution of the geolocation equation.

Eq. (8) requires the incidence angle of target positions over reference ellipsoid.

3) Example

We have processed SAR images of ERS and Radarsat for validation purpose and used DTED level-1 for external height information.

(1) Input Image

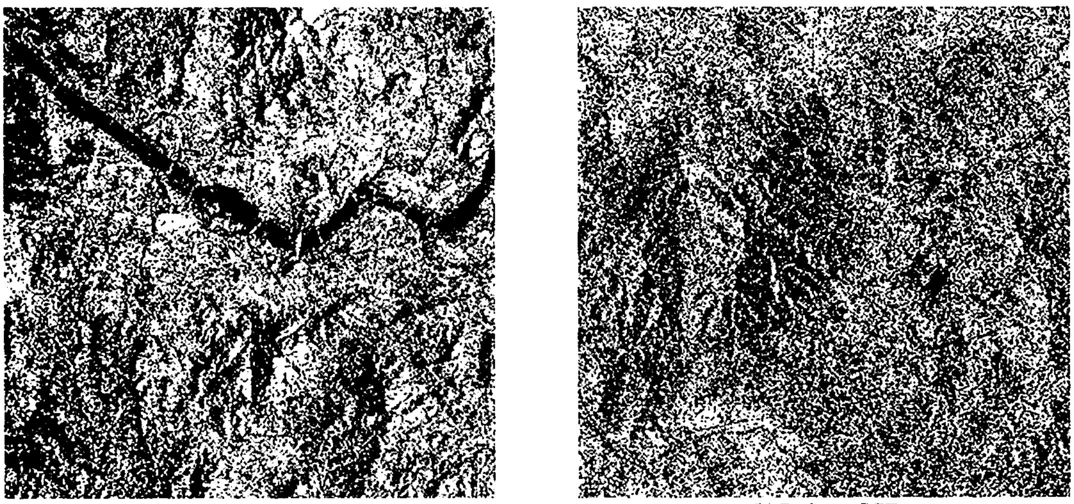
Two images, one ERS PRI and one Radarsat SGF, are selected for validation purpose. Detail information on test images can be found Table 1.

(2) Output Image

Fig. 5 shows a subset of the output image from geocoding process, which has the same orientation as the map projection and also the terrain-distortion has been removed. This sort of image is called an Ortho-Rectified Image (ORI) and Fig. 6 shows the effect of

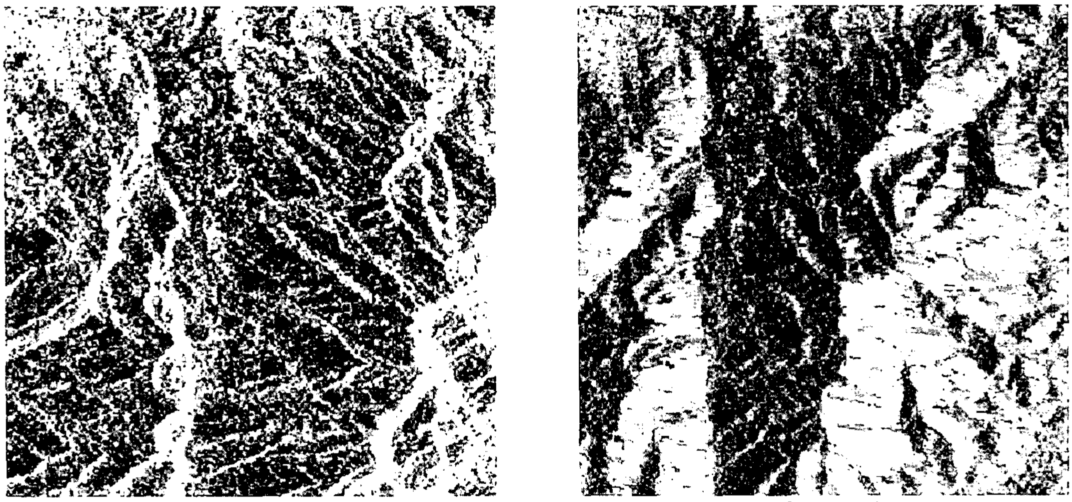
Table 1. Image Details.

	Description	
Product	ERS-1 SAR PRI	Radarsat SGF (Fine 5)
Orbit/Frame	5802 / 2853 Descending	10931 Ascending
Acquisition date	1992/08/25 02:16:21.000	1997/12/08 09:39:11.807
Scene centre	37.3964 N, 126.5909 E	37.4760 N, 126.8431 E
Incidence angle	23.3124 degrees	46.357 degrees
Scene projection	Ground Range	Ground Range
Pixel spacing	20.0 meters	6.25 meters
Line spacing	20.0 meters	6.25 meters
No. of lines	5248 lines	8095 lines
No. of pixels	5120 pixels	6117 pixels



(a) ERS PRI (b) Radarsat SGF

Fig. 5. Ortho-rectified image.



(a) before correction (b) after correction

Fig. 6. Effect of terrain correction.

terrain correction clearly.

With the ORI, we can register it directly onto map and can fuse with other information sources.

4) Validation

In order to validate the method and to get preliminary result, we have manually picked several Ground Control Points (GCPs) from 1:25,000 topography maps and in this specific case, we have been used one GCP (GCP #1) for correction of the geocoding polynomials since we didn't have sufficient number of GCPs.

Table 2 summarises validation result using these GCPs.

3. Conclusions

As you understand SAR imaging geometry, the geometry is actually quite stable and Doppler estimation by SAR processor is quite accurate. From this understanding, if we know the geodetic positions of a sufficient number of pixel positions well spread across the image, then we can construct a fitting polynomial that gives geodetic positions of any pixel positions of the image.

In addition, target displacement due to the terrain height can be analytically determined, which basically requires an iterative process to find zero Doppler time.

Table 2. Validation Results.

(a) ERS PRI

No.	Easting (m)	Northing (m)	Residual	
			Easting	Northing
1	181150.0	461888.0	0.000	0.000
2	203463.0	440150.0	30.896	0.001
3	202975.0	452875.0	-19.273	-4.049
4	200525.0	437062.5	38.973	-0.314
5	181550.0	462100.0	-27.118	19.493

(b) Radarsat SGF

No.	Easting (m)	Northing (m)	Residual	
			Easting	Northing
1	192650.0	456550.0	0.000	0.000
2	170487.5	432350.0	-9.003	-2.951
3	195770.0	442125.0	5.478	0.045
4	180530.0	453812.5	-13.768	12.456
5	176400.0	439150.0	3.947	13.748

This is worth to recall that if a SAR sensor operates in small squint angle, this effectively confines target displacement onto the range direction.

This rationale is the key idea behind this proposed method. Fortunately, there does exist header information viable to enable the calculation of geodetic positions of sample points.

The number of sample points for each case is sufficiently enough. Then, we can construct a fitting polynomial and with this, the transformation between image coordinates and map coordinates is easy.

The proposed method has many advantages as following compared to conventional method that incorporates orbit interpolation and solution of Zero Doppler Time (ZDT). This method

- Does not require orbit interpolation and solution of zero-dopper time.
- Constructs a least squares fitting polynomial that relates image coordinates and map coordinates.
- Enable you to directly transform existing information from such as SPOT image or GIS into slant range image, so in some case, you don't need to generate geocoded image.
- Produces good accuracy with much less number of GCPs compared to conventional methods. In most cases, one or two GCPs will be affordable.

An efficient solution for SAR geocoding has been developed, implemented and validated and this method has dramatically improved processing time and simplified the geocoding process itself.

If this method could be combined with a SAR processor, it could be possible to develop a near real time orthorectification system for SAR imagery.

References

Curlander, J. C., 1982. Location of spaceborne SAR imagery, *IEEE Trans. Geosci and Remote Sensing*, GE-20: 359-364.

Curlander, J. C., 1984. Utilization of SAR data for mapping, *IEEE Trans. Geosci and Remote Sensing*, GE-22: 106-112.

Curlander, J. C., R. Kwok, and S. S Pang, 1987. A post-processing system for automated rectification and registration of spaceborne SAR imagery, *Int. J. Remote Sensing*, 8: 621-638.

Kropatsch, W. and D. Strobl, 1990. The generation of SAR layover and shadow maps from digital elevation models, *IEEE Trans. Geosci and Remote Sensing*, GE-28: 98-107.

Kwok, R., J. Curlander, and S. S. Pang, 1987. Rectification of terrain induced distortion in radar imagery, *Photogram. Eng. and Rem.* Sen., 5: 507-513.

Naraghi, M., W. Stromberg, and M. Daily, 1983. Geometric rectification of radar imagery using digital elevation models, *Photogram. Eng. and Rem. Sen.*, 49: 195-199.