

# Computation of Non-Linear Wave Height Distribution in the Seogwipo Harbor Using Finite Element Method

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**ABSTRACT:** In this paper, finite element method is applied for the numerical analysis of wave height distribution. The mild-slope equation is used as the basic equation. The key of this model is to impose the effect of nonlinear waves. Numerical results are presented and agreed well with the results from experimental measurements and other numerical analysis. The present method to determine wave height distribution can be broadly utilized for the analysis of new harbor and port designs in the future.

## 1. Introduction

The computation of wave height distribution in ports and harbors is very important to the design and operation of coastal structures. Most of the waves entering a port and harbor are nonlinear and irregular in term of period, direction and wave height. Thus, the effect of the nonlinear and irregular waves must be considered in computations. In this study, the finite element method was used with semi-circle. A number of numerical analysis and experimental studies have been carried out for nonlinear waves, such as the experimental data by Berkhoff et al.(1982), and numerical results by Demirbilek and Panchang(1998). Furthermore, it is difficult to prepare an appropriate mesh in accordance with the variation of water depth. Kim et al.(2000) have developed an automated mesh generation method in which the element length is matched with the wavelength according to the depth of the entire domain. In this study, the nonlinear wave height distribution of the Seogwipo Harbor is analyzed using the mild-slope equation. The analysis of the calmness of the Jeju Harbor has been presented in Kim and Hur(2002, 2003).

## 2. Governing Equations and Boundary Conditions

The fluid is assumed to be incompressible and inviscid,

and the flow is assumed to be irrotational. As shown in Fig. 1, the mild-slope equation(Berkhoff, 1972; Chen, 1986) is applied for the analysis of harbor oscillation in shallow water of variable depth.

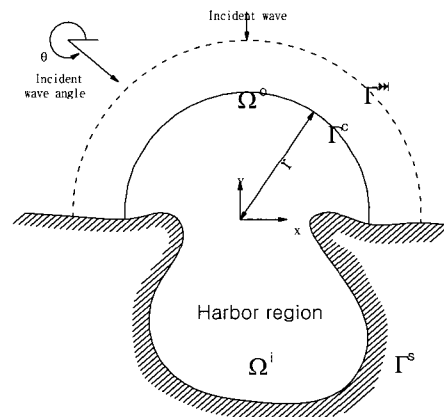


Fig. 1 Definition of variables

$$\frac{\partial}{\partial x} \lambda c c_g \frac{\partial \phi}{\partial x} + \frac{\partial}{\partial y} \lambda c c_g \frac{\partial \phi}{\partial y} + \frac{\omega^2 c_g}{c} \phi = 0 \text{ in } \Omega; \quad (1)$$

$$\lambda = \frac{1}{1 + \frac{i\beta a_0}{h \sinh kh} e^{i\tau}} \quad (2)$$

$$\omega^2 = gk \tanh(kh) \quad (3)$$

where  $h$  is the depth,  $\phi$  is the velocity potential,  $\omega$  is the angular frequency,  $g$  is the gravitational acceleration,

$\Omega$  is the field region,  $\beta$  is the bottom friction coefficient,  $a_0$  is the incident wave amplitude,  $k$  is the wave number,  $i$  is  $\sqrt{-1}$ , and  $\gamma(=\pi/4)$  is the phase difference. Along the solid wall,  $\Gamma_s$  as the absorbing boundary condition similar to the acoustic impedance condition is applied, and is expressed in terms of the reflection coefficient of the boundary,  $K_r$ .

$$\frac{\partial \phi}{\partial n} - \alpha \phi = 0 \quad \text{on } \Gamma_s \quad (4)$$

where  $n$  is the unit normal vector pointing outward from the water region, and  $\alpha = ik \frac{1 - K_r}{1 + K_r}$ . The matching boundary condition on the interface,  $\Gamma_c$ , can be expressed as follows:

$$\left( \lambda h \frac{\partial \phi}{\partial n} \right)_{\Omega_i} = \left( \lambda h \frac{\partial \phi}{\partial n} \right)_{\Omega_o}, \quad \phi_{\Omega_i} = \phi_{\Omega_o} \quad \text{on } \Gamma_c \quad (5)$$

In the outward region  $\Gamma_\infty$ , The radiation condition is applied.

$$\lim_{r \rightarrow \infty} \sqrt{r} \left( \frac{\partial}{\partial r} - ik \right) (\phi - \phi_0) = 0 \quad \text{on } \Gamma_\infty \quad (6)$$

where  $(r, \theta)$  are the polar coordinates.  $\phi_0$  is the velocity potential of incident wave, and it is expressed as follows:

$$\phi_0 = -\frac{iga_0}{w} \{ \exp[ikr \cos(\theta - \theta_0)] + K_r \exp[ikr \cos(\theta + \theta_0)] \} \quad (7)$$

where  $\theta_0$  is the incident wave angle. Applying boundary conditions (4), (5) and (6) to the basic equation (1), numerical simulation of the harbor can be performed.

In addition to the above mechanisms, nonlinear waves may be simulated using the nonlinear dispersion relation (Kirby and Dalrymple, 1986). The following equation can be used in place of equation (3).

$$w^2 = gk [1 + (ka)^2 F_1 \tanh^5 kh] \tanh \{ kh + ka F_2 \} \quad (8)$$

where

$$\left. \begin{aligned} F_1 &= \frac{\cosh(4kh) - 2 \tanh^2(kh)}{8 \sinh^4(kh)} \\ F_2 &= \left( \frac{kh}{\sinh(kh)} \right)^4 \end{aligned} \right\} \quad (9)$$

### 3. Numerical Results and Discussions

#### 3.1 Verification analysis for nonlinear wave by finite element model

Surface water waves propagation over the shoal presented by Berkhoff et al.(1982) are modeled. This test case demonstrates the ability of the model to simulate the effects of complex coastal bathymetric features. The shoal shown in Fig. 2 is oriented such that the major axis of the shoal is parallel to contours of the water depth.

Incident wave period and amplitude is 1 sec and 0.0232m respectively. The boundary conditions are as follows: total reflection about the sidewall and total absorption about the downstream end of the direction of wave propagation. The coefficient of friction is 0. There are two steps in the numerical analysis. In the first step, the linear model is applied. In the second step, the nonlinear model is applied via an iterative algorithm.

The nodes are distributed such that the number of nodes per wavelength remains constant throughout the domain. Grid densities of twelve elements per (local) wavelength were used. The resulting finite element grids is composed of 57,766 nodes and 114,776 elements.

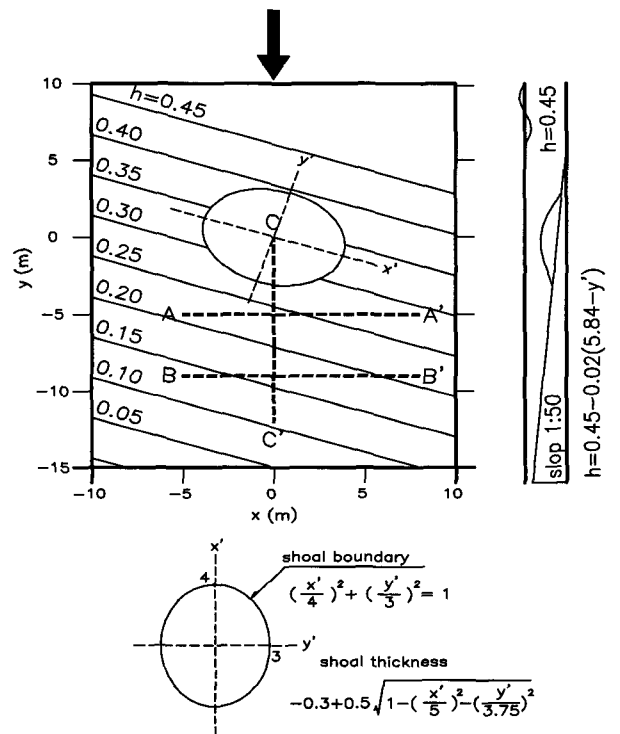


Fig. 2 Computational domain of Berkhoff et al.(1982)

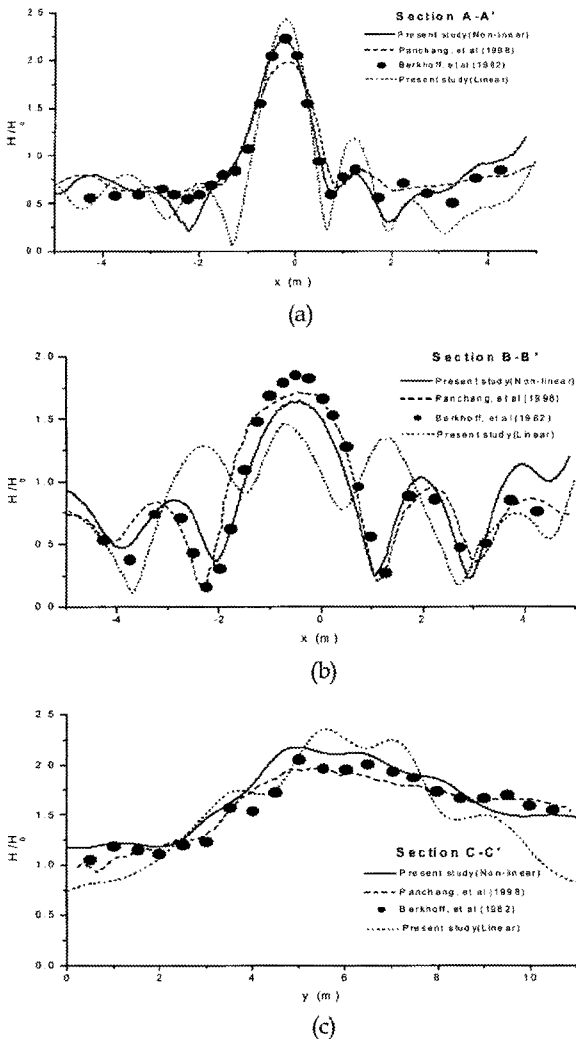


Fig. 3 Comparison of relative wave height distribution in shoal

In Fig. 3, the ordinate is the amplification ratio and the abscissa is the horizontal cross sectional distance within the domain. As shown in Fig. 2, Three comparison sections are Line A-A' and Line B-B' in the horizontal direction, and Line C-C' in the vertical direction. For the section of Line A-A', Line B-B' and Line C-C', the numerical results indicate that the linear model did not agree well with the experiment measurements by Berkhoff et al.(1982) and the numerical predictions by Panchang et al.(1998). However, the results of nonlinear model are in good agreement with experimental measurements and numerical predictions by Panchang et al.(1998). Therefore, it is reasonable to conclude that the nonlinear model is able to capture the nonlinear wave effect better than the linear model.

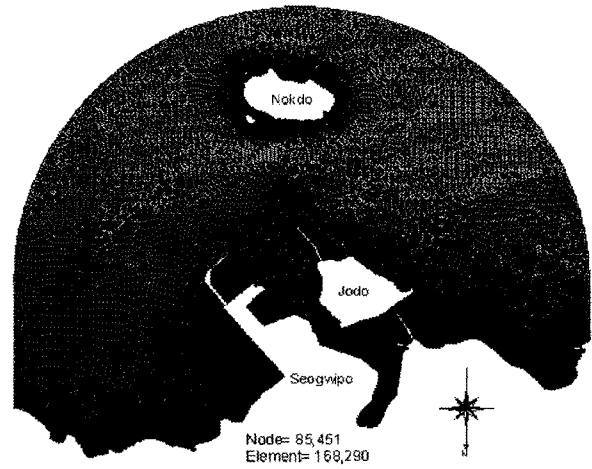


Fig. 4 Finite element mesh diagram of the Seogwipo Harbor

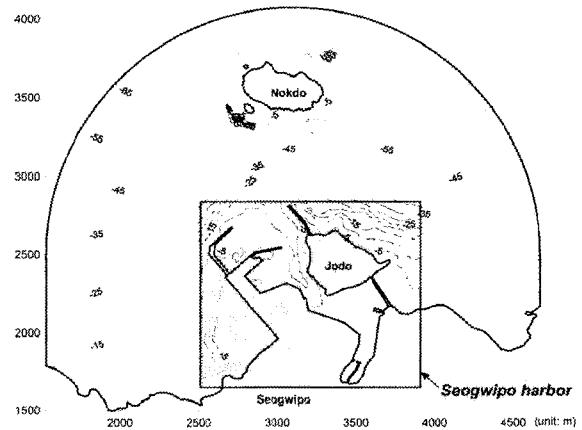


Fig. 5 Water depth contour of the Seogwipo Harbor

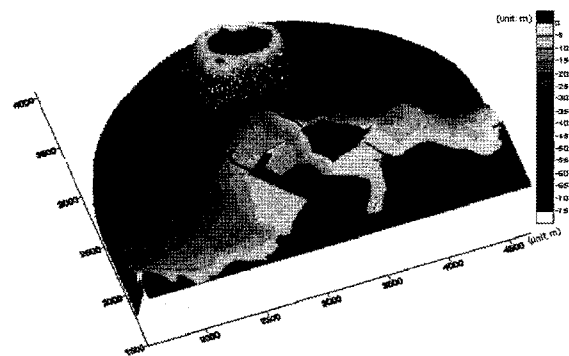


Fig. 6 Topographical map of the Seogwipo Harbor

**3.2 Wave height distribution of the Seogwipo Harbor**

The Seogwipo Harbor is the gateway to Seogwipo and located on the south of Jeju Island. As shown in Fig. 5 and Fig. 6, a notable feature of the Seogwipo Harbor is steep slope topography and fronted by islands. In order to estimate the amplification condition of the Seogwipo Harbor, Fig. 4 shows the finite element idealization mesh obtained by the present automatic mesh generation technique. The total number of finite elements and nodal points are 85,451 and 168,290, respectively. All element sizes satisfy the proposition that the ratio of the wavelength to the element size is greater than 10.

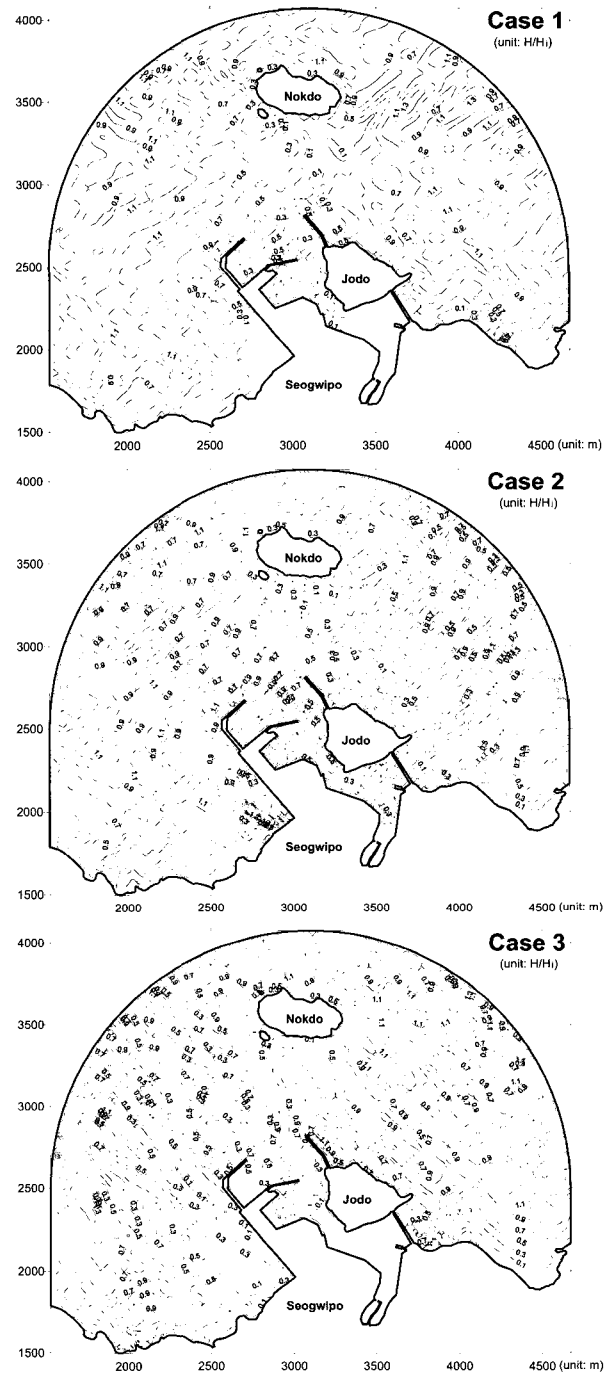
The numerical computations are carried out for three cases as shown in Table 1. we assumed that the deep water waves arrive at the Nokdo. The reflection coefficient on the wave dissipating structure is assumed to be 0.4-0.6, on the perpendicular structure is assumed to be 0.9. The friction coefficient of the bottom is 0.02(Horikawa, 1978). Fig. 7 shows the computed relative wave height distribution using the finite element mesh shown Fig. 4. Fig. 8 shows the computed wave height distribution of the Seogwipo Harbor using three-dimensional graphic techniques. As shown in Fig. 7 and Fig. 8, the wave height distribution is relatively low for Cases 1 and Case 3 because the Nokdo and Jodo prevent the incident waves respectively. However, it is high for Cases 2 because the incident waves directly come into the inner harbor without reflection due to Nokdo. Fig. 9 shows the velocity profile of water particles for the three cases in Seogwipo Harbor. This Figure depict the enlarged area of the Seowipo Harbor in Fig. 5.

**Table 1** Incident wave condition(MOMAF, 1992)

Cases	Wave height (m)	Period (sec)	Direction
1	10.9	15.0	S
2	10.0	13.0	SE
3	10.0	13.0	SW

**4. Conclusions and Remarks**

In this study, the mild-slope equation is employed to determine the wave height distribution. The numerical model accounts for the effect of nonlinear waves via the nonlinear dispersion relation.



**Fig. 7** Computed relative wave height of Seogwipo Harbor

The nonlinear analysis of the shoal in the sloping beach are excellent very good agreement with numerical results presented Panchang et al.(1998) and experiment measurements in Berkhoff et al.(1982).

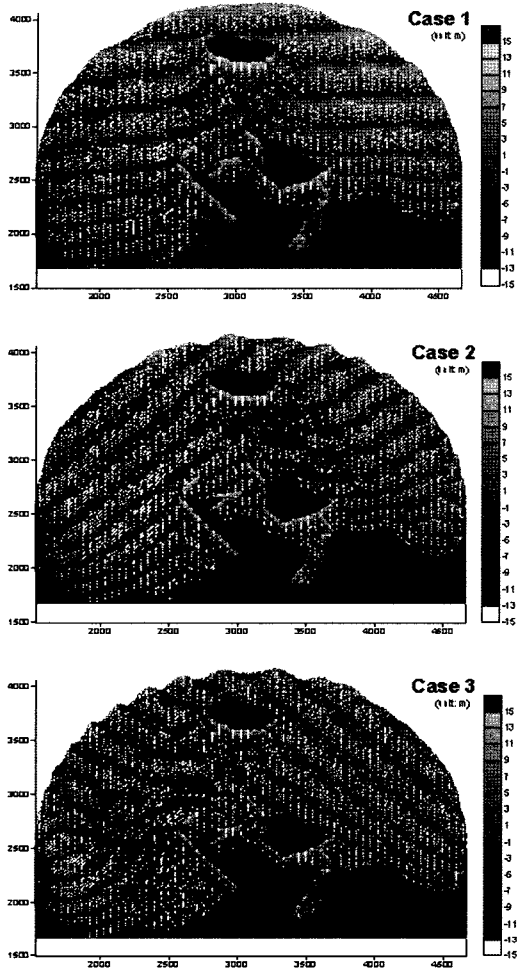


Fig. 8 Computed three-dimensional wave profiles of the Seogwipo Harbor

The predicted relative wave height profile obtained using the nonlinear wave model compared better with experimental measurements than the linear wave model.

The predicted wave height distribution of the Seogwipo Harbor using the nonlinear wave model seem reasonable. In Conclusion, the present numerical method with nonlinear wave effect can be applied broadly for the analysis of wave height distribution of harbors and ports.

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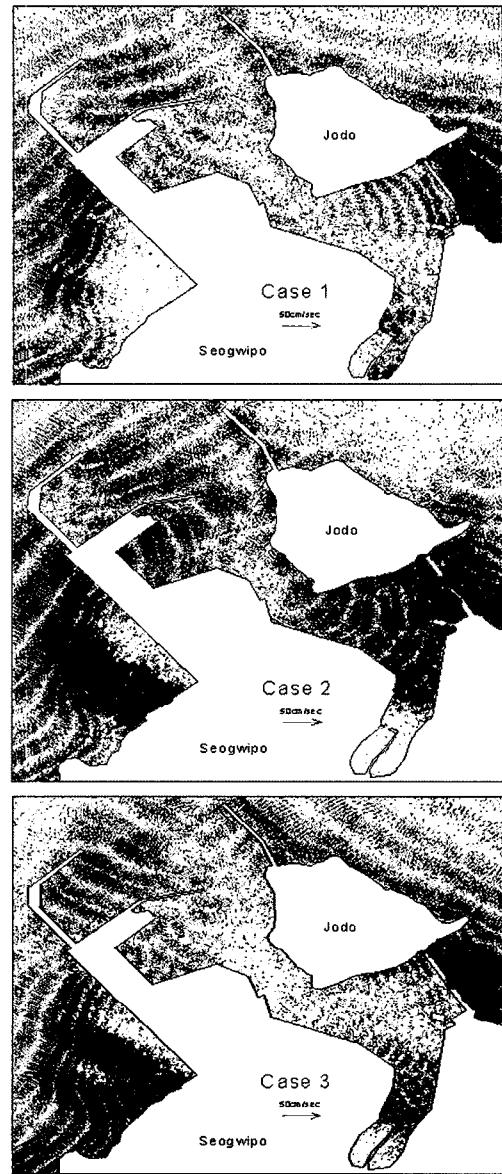


Fig. 9 Water particle velocity profiles of the Seogwipo Harbor

**References**

Berkhoff, J.C.W. (1972). "Computation of Combined Refraction-Diffraction", Proc. 13th International Conference on Coastal Engineering, ASCE, pp 471-490.

Berkhoff, J.C.W., Booij, N. and Radder, R.C. (1982). "Verification of Numerical Wave propagation Models for Simple Harmonic Linear Waves", Coastal Engineering, Vol 6, pp 255-279.

Chen, H.S. (1986). "Effects of Bottom Friction and Boundary Absorption on Water Wave Scattering", Applied Ocean

- Research, Vol. 8, No 2, pp 99-104.
- Demirbilek, Z. and Panchang, V.G. (1998). CGWAVE: A Coastal Surface Water Wave Model of the Mild Slope Equation, Technical Report CHL-98-26, U.S. Army Corps of Engineers.
- Horikawa, K. (1978). Coastal Engineering, University of Tokyo Press, pp 37-39.
- Kim, N. and Hur, Y. (2002). "The Numerical Simulation of Harbor Calmness by Finite Element Method", Journal of Ocean Engineering and Technology, Vol 16, No 1, pp 22-26.
- Kim, N. and Hur, Y. (2003). "Calmness Analysis of Jeju Harbor by Finite Element Technique", Journal of Ocean Engineering and Technology, Vol 17, No 1, pp 16-20.
- Kim, N., Yang, J. and Park, S. (2000). "Automatic Mesh Generation Method in Shallow Water Area Considering Water Depth", Journal of the Korean Institute of Port Research, Vol 14, No 1, pp 97-105.
- Kirby, J.T. and Dalrymple, R.A. (1986). "An Approximate Model for Nonlinear Dispersion in Monochromatic Wave Propagation Models", Coastal Engineering, Vol 9, pp 545-561.
- Ministry of Maritime Affairs & Fisheries (1992). Execution Design Report of Outer Harbor Breakwater in Seogwipo Harbor, Execution Design Report (in Korean).

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