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Estimation of Displacements Using the Transformed Response in Time and Frequency Domain

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ABSTRACT: *If the accelerometers are used in measuring the response, the absolute values of the velocity and displacement are not usually obtainable because their initial values are not accounted for in the integration of the acceleration response. A new dynamic response conversion algorithm of both the time domain and the frequency domain is proposed for the problem in estimating the displacement data by defining the transformed responses. In this algorithm, the displacement response can be obtained from the measured acceleration records by integration without requiring the knowledge of the initial velocity and displacement information. The applicability of the technique is tested by an example problem using the real bridge's superstructure under several cases of moving load. In the response conversion procedure of the frequency domain, the identified response according to the frequency can be estimated by changing over the limits of integration. If the reliability of the identified responses is ensured, it is expected that the proposed method for estimating the impact factor can be useful in the bridge's dynamic test. This method can be useful in those practical cases when the direct measurement of the displacement is difficult as in the dynamic studies of huge structure.*

1. Introduction

The displacement response is one of the important parameters to determine the vibration characteristics and the state evaluation of a structure. When studying the structural integrity of a large system it is much easier to measure accelerations than displacements. However, many engineering standards are based on displacements which are proportional to the stresses in an elastic structure. The displacement can be directly used to determine the accumulated damage(Ribeiro et al., 1999). The displacement response is measured by the use of LVDTs and similar direct displacement measuring devices which require a fixed reference to work properly. The use of optical transducers to measure displacements may be expensive. After measuring a signal of acceleration that is relatively convenient for measure and not needing a set of fixed references, on the necessary point of structure, a digital method to double integrate accelerometer data in order to measure displacements is tried to base on a dynamic loading test of structure (Ribeiro et al., 1997). A vibration problem of structure can be approached to the wavelet transform in the frequency domain (Kwon et al., 2000). If the accelerometers are used in measuring the response, the absolute values of the velocity and displacement are not usually obtainable because their initial values are not accounted for in the integration of the acceleration response (Yang, 1998).

In this paper, the theoretical aspects of the dynamic response conversion algorithm that is able to remove an information about the initial condition of velocity and displacement responses are described. The theory is based on reformulation of the structural system matrix that is induced by the process of a time domain modal vibration test technique (Ibrahim and Mikulcik, 1973). Digital integration to measure displacements is accomplished indirectly by defining the change amounts and the transformed responses about three kinds of physical responses in the time and frequency domain respectively. The method presents a solution for the initial value problem under the moving load. In this algorithm, the displacement response can be obtained from the measured acceleration records only. The feasibility for physical application of the proposed technique is tested by the example problems using the real bridge's superstructures under several cases of moving load and the results are compared to the actually measured displacements.

2. Response Conversion Theory

In a finite element formulation, the characteristics of a structure are defined in terms of the stiffness matrix K, damping matrix C, and mass matrix M. The dynamic response conversion technique is based on reformulation of the ordinary differential equation of motion into state variable form,

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$$\begin{bmatrix} \dot{y} \\ \ddot{y} \end{bmatrix} = A\begin{bmatrix} y \\ \dot{y} \end{bmatrix} + \begin{bmatrix} 0 \\ M^{-1}f \end{bmatrix} \tag{1}$$

$$A = \begin{bmatrix} 0 & I \\ -M^{-1}K & -M^{-1}C \end{bmatrix} \tag{2}$$

where y is a vector of displacements under applied force vector f, and A-matrix is composed of the vibration parameters of the system. This first order matrix differential equation is referred to herein as the response conversion model of system. The dynamic characteristics of a structure and the A-matrix are not dependent on the initial conditions of excitation(Ibrahim, 1973), so it may be possible that the response conversion theory can be induced to eliminate the requirement for knowledge of initial velocity and displacement information. The response conversion algorithm is based on assuming that the acceleration response measured at n-stations on a structure is made up of two components,

$$\ddot{y}(t) = \delta\ddot{y}(t) + \ddot{y}(t_1) \tag{3}$$

where $\ddot{y}(t_1)$ is the acceleration response at the first initial time $t = t_1$ which is selected arbitrary. $\delta\ddot{y}(t)$ is the change in acceleration response from the initial time to any time t. The velocity response at any time is calculated from integration of the acceleration response in the time domain.

$$\dot{y}(t) = \delta\dot{y}(t) + \dot{y}(t_1) \tag{4}$$

$$\delta\dot{y}(t) = \int_{t_1}^{t}\ddot{y}(\tau)\,d\tau \tag{5}$$

In order that the velocity response contains the initial component and the change component, the first term of the right side of Eq. (4) is defined as the change of velocity $\delta\dot{y}(t)$. Integration of the velocity results in the displacement response, and the change of displacement $\delta y(t)$ is defined as follows.

$$y(t) = \delta y(t) + y(t_1) \tag{6}$$

$$\delta y(t) = \int_{t_1}^{t}\int_{t_1}^{\xi}\ddot{y}(\tau)\,d\tau\,d\xi + (t - t_1)\dot{y}(t_1) \tag{7}$$

2.1 Definition of transformed response

The state equation must be satisfied for arbitrary time including the initial time. In this paper, two initial times are considered. The first initial time t_1 is the starting time of the measured acceleration response, and the second initial time t_2 is the starting time of the transformed response which is described hereafter. Hence two state equations for the each initial time, $t = t_1$ and $t = t_2(>t_1)$, can be expressed. The responses corresponding to the second equation can be partitioned into the initial value and the altered quantity by substitution of Eq. (3), Eq. (4) and Eq. (6) into the state equation, then the first state equation is substituted for the partitioned second state equation. The result equation is derived as follows,

$$\begin{bmatrix} \delta\dot{y}(t_2) \\ \delta\ddot{y}(t_2) \end{bmatrix} = A\begin{bmatrix} \delta y(t_2) \\ \delta\dot{y}(t_2) \end{bmatrix} + \begin{bmatrix} 0 \\ M^{-1}\delta f(t_2) \end{bmatrix} \tag{8}$$

where $\delta f(t_2)$ is the change of force vector. As the same way, the relationship corresponding to the time $t(\geq t_2)$ can be expressed as the following equation.

$$\begin{bmatrix} \delta\dot{y}(t) \\ \delta\ddot{y}(t) \end{bmatrix} = A\begin{bmatrix} \delta y(t) \\ \delta\dot{y}(t) \end{bmatrix} + \begin{bmatrix} 0 \\ M^{-1}\delta f(t) \end{bmatrix} \tag{9}$$

Multiplying Eq. (8) by t/t_2 and subtracting it from Eq. (9), the transformed response can be produced.

$$\begin{bmatrix} \dot{Y} \\ \ddot{Y} \end{bmatrix} = A\begin{bmatrix} Y \\ \dot{Y} \end{bmatrix} + \begin{bmatrix} 0 \\ M^{-1}F \end{bmatrix} \tag{10}$$

where F is the transformed force and $\ddot{Y}$ is the transformed acceleration response, $\dot{Y}$ and Y are the transformed velocity and displacement respectively. Provided that the initial time of the measured acceleration is zero, the transformed responses are calculated by using the acceleration signal only. These transformed responses take the following new definitions by substitution of Eq. (3), Eq. (5) and Eq. (7).

$$\begin{aligned} \ddot{Y}(t) &= \delta\ddot{y}(t) - \frac{t}{t_2}\delta\ddot{y}(t_2) \\ &= \ddot{y}(t) - \frac{t}{t_2}[\ddot{y}(t_2) - \ddot{y}(t_1)] - \ddot{y}(t_1) \end{aligned} \tag{11}$$

$$\dot{Y}(t) = \int_{t_1}^{t}\ddot{y}(\tau)\,d\tau - \frac{t}{t_2}\int_{t_1}^{t_2}\ddot{y}(\tau)\,d\tau \tag{12}$$

$$\begin{aligned} Y(t) &= \int_{t_1}^{t}\int_{t_1}^{\xi}\ddot{y}(\tau)\,d\tau\,d\xi - \frac{t}{t_2}\int_{t_1}^{t_2}\int_{t_1}^{\xi}\ddot{y}(\tau)\,d\tau\,d\xi \\ &= \int_{t_1}^{t}(t-\tau)\ddot{y}(\tau)\,d\tau - \frac{t}{t_2}\int_{t_1}^{t_2}(t_2-\tau)\ddot{y}(\tau)\,d\tau \end{aligned} \tag{13}$$

It is seen from Eq. (12) and Eq. (13) that the transformed velocity response and the transformed displacement response can be obtained from the experimental acceleration records by integration without requiring the knowledge of the initial velocity $\dot{y}(t_1)$ and displacement $y(t_1)$ information. The transformed response curves are only affected by the initial conditions of the acceleration response $\ddot{y}(t_1)$ and $\ddot{y}(t_2)$. The relationship of Eq. (11) is also applied to the case of displacement responses. Therefore the transformed response of Eq. (13) is related to the real response as follows.

$$Y(t) = y(t) - \frac{t}{t_2}[y(t_2) - y(t_1)] - y(t_1) \tag{14}$$

2.2 Frequency domain integration

When the Euler formula(Newland, 1993) is used, and the linearity of the Fourier transformation is applied, the Fourier transform $FT\{\ddot{y}(t)\}$ of the acceleration function can be represented by the Fourier transform of the displacement function(Bendat et al., 1991).

$$FT\{\ddot{y}(t)\} = -\omega^2 FT\{y(t)\} - i\omega y(0) - \dot{y}(0) \quad (15)$$

where ω is the angular frequency. In case that the initial values of Eq. (15) are presented, the time history of the displacement response can be estimated by the inverse Fourier transformation. In order to apply the proposed response conversion algorithm into the integration scheme of the frequency domain, the Fourier transform of the displacement function has to be separated into the frequency component of the measured acceleration signal and the frequency component of the initial values.

$$\begin{aligned} FT\{y(t)\} &= FT\{y_1(t)\} + FT\{y_2(t)\} \\ &= -\frac{1}{\omega^2} FT\{\ddot{y}(t)\} + \left[\frac{1}{i\omega} y(0) - \frac{1}{\omega^2}\dot{y}(0)\right] \end{aligned} \quad (16)$$

The second term of the right side of Eq. (16) is derived into $y_2(t) = \alpha_1 t + \alpha_2$ for all $t \geq 0$ by the inverse Fourier transformation. Where α_1 and α_2 are the real constants for integration. So, the actual response function is related to the response which is calculated from integrating the acceleration signal only in the frequency domain.

$$y(t) = y_1(t) + \alpha_1 t + \alpha_2 \quad (17)$$

Substituting Eq. (17) into Eq. (14), the transformed displacement response is produced as follows

$$\begin{aligned} Y(t) &= y_1(t) - \frac{t}{t_2}[y_1(t_2) - y_1(t_1)] - y_1(t_1) \\ &+ \left[\alpha_1 t - \frac{t}{t_2}\{\alpha_1 t_2 - \alpha_1 t_1\} - \alpha_1 t_1\right] \end{aligned} \quad (18)$$

When t_1 is assumed as zero initial time, the inner terms of [$\cdots$] of Eq. (18) are eliminated all. Thus, $y_1(t)$ can be used to calculate the transformed displacement response. When the initial times are adequately chosen, the transformed response is assessed as the case that the actual response is applied.

2.3 Revise of transformed response

The transformed response can be calculated without requiring the data of the initial velocity and displacement measurements. However, the transformed curve differs from the actual response. The transformed displacement response $Y(t)$ has two types of error as compared with the actual displacement response $y(t)$. These are $t/t_2[y(t_2) - y(t_1)]$ and $y(t_1)$ as shown in Eq. (14). The former is calculated as a time function and the latter error is constant. When the second initial time t_2 of the transformed displacement response is adequately chosen in order that $y(t_2)$ may be equal to $y(t_1)$, the former error being changed in proportion to time can be eliminated. Then, the actual response and the transformed response contain a difference as much as the unknown initial value $y(t_1)$ at any time t. In order for the response conversion algorithm to present accurately, the free vibrated region has to be included in the dynamic behavior of a tested structure. The initial time t_1 is the first recorded time of the free vibrated region which compose the prior part of the measured acceleration signal. When there is the behavior of structure within the limits of the free vibrated region, the initial time t_2 fit to the above condition of the transformed response can be determined within the first natural period. If the length of acceleration signal is changed, another time t_1 can be used to calculating the transformed response. And if the signal length of free vibration is not less than the first natural period, there are several number of adequately chosen initial time t_2 corresponding to each t_1. It follows that the recursive analysis is necessary using the various initial time since the value of $y(t_2)$ is not exactly equal to $y(t_1)$ in estimating the transformed displacement from the discrete acceleration data. In this algorithm for each t_1, the optimal initial time t_2 of the transformed response is estimated by a standard of judgment that a wavy pattern for the posterior part of the transformed response curve is converged into a uniform value. Thus, the dynamic behavior of a tested structure has to include the unloading state at the posterior part for the purpose of successful application. Suppose that the actual displacement of the unloading region is zero, a converged value of the transformed displacement response is the estimated value of the displacement that is the equivalent for the first initial time t_1.

The procedure is iterated until the above-mentioned conditions are satisfied to the desired precision. An algorithm for the response conversion is outlined below.

step 1 : Assume the time intervals corresponding to the free vibration response and the unloading state respectively.

step 2 : Decide the varying ranges of the initial time t_1 and t_2.

step 3 : Time domain analysis ; Compute the transformed displacement for selected unloading time interval from Eq. (13). Frequency domain analysis ; Compute $y_1(t)$ by the inverse Fourier transformation of $-1/\omega^2 \cdot FT\{\ddot{y}(t)\}$. Then, compute the transformed displacement for selected unloading time interval from Eq. (18).

step 4 : Find optimal t_1 and t_2 using the above mentioned standard of judgment that the standard deviation for the curve of previous step is minimized.

step 5 : Construct the transformed displacement response using the two initial times of previous step. Then, correct the constant error $y(t_1)$ in the transformed response.

step 6 : Calculate the standard deviation corresponding to the free vibrated region of the transformed response curve.

step 7 : Redefine the time interval for the unloading state. Then, carry out the procedures recursively from step 3 to step 6.

step 8 : Find the optimal transformed response by the second standard of judgment that the standard deviation of step 6 is minimized. The resulting response curve is assessed as the identified displacement response.

3. Experimental Results

The applicability of the technique is verified by the example problems using the real bridge's superstructures under several cases of moving load. The structure is referred to as the Mangmi bridge with a total length of 123m in the Busan metropolitan area of Korea. This tested bridge consists of four-span continuous pre-stressed concrete girders. The width of the deck is about 10.5m. In order to obtain the forced vibration response of the bridge, a moving load, which was generated by a loaded truck weighing about 30tons, was imposed on the bridge. The test vehicle ran at speed of 30km/h, 40km/h, 60km/h sequentially. The moving load was applied at the first lane of the bridge deck of the third span with a clear length of 30m. Two type of sensors were attached to the G1-girder and G3-girder at the center point of the tested span as shown Fig. 1. High-sensitivity and low-frequency accelerometers of the force-balance type were used to measure the acceleration response of the bridge subjected to the moving load.

In order to compare with the results of the response conversion algorithm, the actual displacements were measured by the direct measuring devices, LVDTs, which had a fixed reference to work properly. The analogue sensor signals were converted to digital data and recorded in a PC-based portable data acquisition system. In each test, the acceleration data were recorded for from duration 5.8sec to duration 7.0sec with a sampling rate of 200Hz as shown Fig. 2. The recording times were chosen to assure that

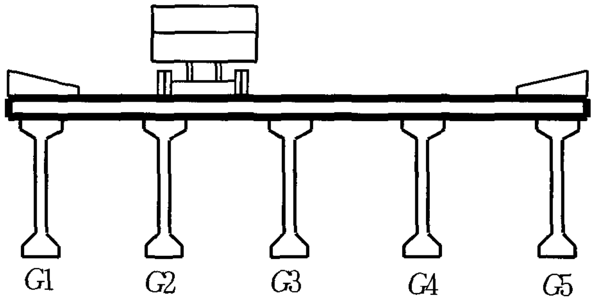
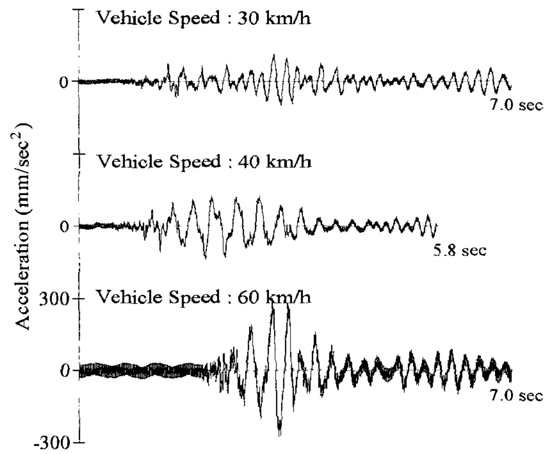
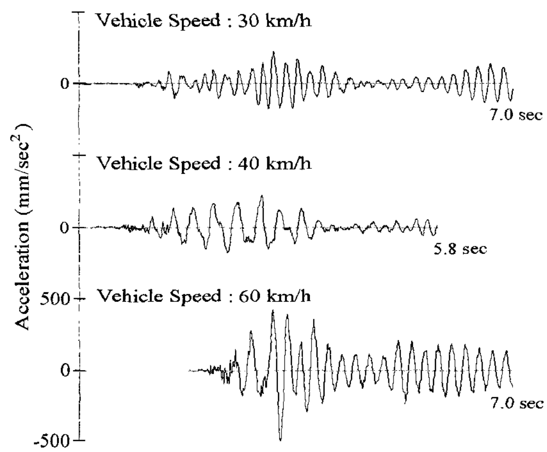


Fig. 1 Moving load test of the Mangmi bridge



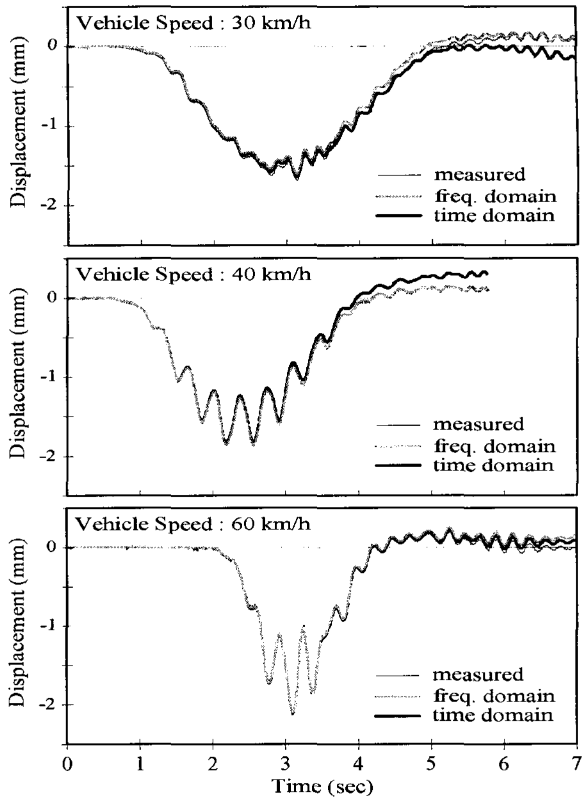
(a) Girder 1



(b) Girder 3

Fig. 2 Acceleration responses of the Mangmi bridge within these durations the shape of vibration would have free vibration response sufficiently. In this test, the prior part of the measured acceleration records was constitute of the unloading signals, and the free vibrated response was recorded on the posterior part. The unloading signals are affected on the noise more than the structural characteristics relatively. Then, the initial time t_2 was determined using the acceleration signals being reversely arranged against a time. The varying range of the initial time t_1 was assumed as 0.15sec. The procedure was repeatedly performed for 30 times since the sampling interval was 0.005sec. As the varying range of t_1 is extended, the more precise identified responses are expected. In contrast with this, the run time will be increased. According to the author's experiences, the varying range was determined to be more than a half of the first natural period 0.22sec of the tested bridge. In order that the theoretical optimal points of t_2 exist more than three, the varying range of t_2 was assumed as 0.35sec which was determined to be more than 1.5 times of the first natural period. The standard deviation of the unloading region in the transformed response curve is the first converge criteria of the recursive technique. In this example, the more exact experimental results were induced by changing the start time of the unloading state till 0.25sec. The corrected transformed responses corresponding to each unloading region are calculated, and these are compared mutually in the free vibrated time region. Then, the optimum curve is assessed as the final identified displacement response. The free vibrated regions being used the second converge criteria of the recursive technique were assumed as two times of the first natural period in this example.

Fig. 3 shows the response conversion results of the each acceleration data. From these results, it has been observed that the proposed algorithm generally can assess the displacement response successfully. However, there exists the possibility that a little error may be generated in the free vibration as shown Fig. 3(a). This error can be found out when the measured acceleration signals include the noise of the specified frequency characteristics, or the unknown actual displacement $y(t_2)$ is estimated as the value containing a wide difference with $y(t_1)$. Table 1 shows the absolute maximum displacement and the identified response error corresponding to the each curve of Fig. 3. In the assessment of the absolute maximum displacement which is one of the most important factor, the mean values of percent error are calculated as 1.8% and 1.7% in the frequency and the time domain method respectively. The identified response error(IRE) is defined as follows



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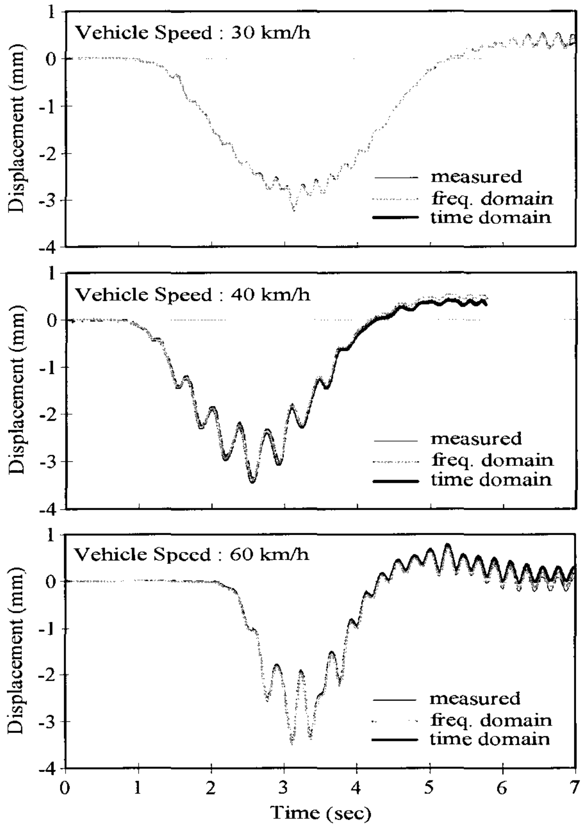


(b) Girder 3

Fig. 3 Identified displacement responses of the Mangmi bridge

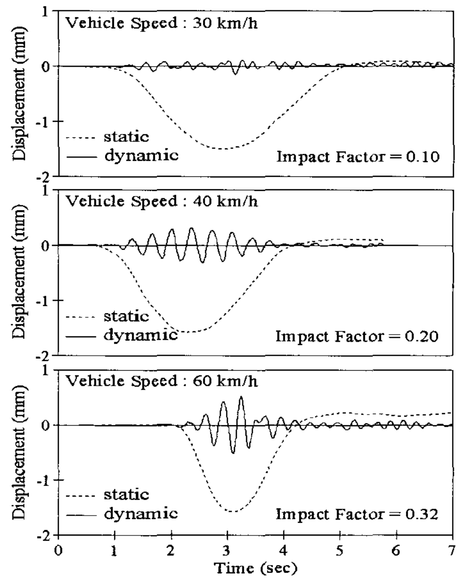
Table 1 Response conversion results of the Mangmi bridge

	Girder	Vehicle Speed (km/h)	IRE (%)	Absolute Maximum Displacement		
				Measure (mm)	Identify (mm)	Percent Error(%)
Frequency Domain Method	G1	30	7.1	1.68	1.61	4.2
		40	1.6	1.87	1.86	0.5
		60	9.6	2.11	2.10	0.5
	G3	30	2.2	3.17	3.22	1.6
		40	4.3	3.35	3.41	1.8
		60	6.0	3.41	3.49	2.3
Time Domain Method	G1	30	10.6	1.68	1.65	1.8
		40	11.3	1.87	1.83	2.1
		60	6.4	2.11	2.11	0.0
	G3	30	2.3	3.17	3.22	1.6
		40	3.2	3.35	3.44	2.7
		60	9.5	3.41	3.47	1.8

$$IRE = \sqrt{\frac{\sum[S_i(t) - S_t(t)]^2}{\sum S_t^2(t)}} \times 100\ (\%) \tag{19}$$

where S_i is the identified response conversion signal of the acceleration response, and S_t is the true signal. Although the measurement error of the displacement records can not be removed, the identified response error can be estimated by assuming that the measured signal is considered as the true signal S_t. In the assessment of the identified response error, the mean value is calculated as 5.1% in the frequency domain method. As shown the case of the time domain method in Table 1, the mean value is calculated as 7.2%.

In the response prediction method of the frequency domain, the response conversion according to the frequency can be available. It is not possible in the time domain method. Not only the static component can be extracted, but also the dynamic displacement component can be separated by the structural mode from the identified displacement response. Since the clear length of test span is 30m, the test vehicle which run at speed of 60km/h operates on the tested bridge deck during 1.8sec. Suppose that the operating time of 1.8sec is a half period, it can be assumed that the low frequency component corresponding to the static deflection of 0.3Hz may be included in the measured acceleration signal. Since this frequency is not coupled with the first natural frequency of 4.5Hz of the structure, the identified response according to the frequency can be estimated by changing over the limits of integration in the response conversion procedure of the frequency domain. In this example, the identified response of the low frequency component by the moving load is separated from the displacement response. Fig. 4 shows the identified displacement curves to be separated by the static and



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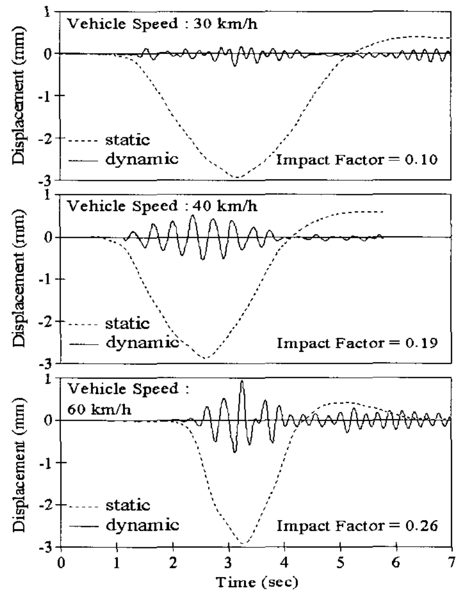

(b) Girder 3

Fig. 4 Static and dynamic components of the Mangmi bridge

dynamic components of the Mangmi bridge. Where the dynamic displacement component of the full line and the static displacement component of the dotted line are the results of the limit frequency of integration, 1.5Hz. The impact factor of a bridge to be estimated from the dynamic test is calculated by comparing the dynamic displacement with the static displacement. It may include the uncertainty of more or less since the dynamic tested vehicle can not pass the location of the static test exactly. However, in case that the result of Fig. 4 is applied, this type of error can be removed. If the reliability of the identified responses are ensured, it is expected that the proposed method for estimating the impact factor can be useful in the bridge's dynamic test.

4. Conclusions

An improved formulation using the transformed response is proposed for the dynamic response conversion algorithm in this paper. This algorithm is based on the iterative method. In the formulation of the indirect integration scheme, the change amounts and the transformed responses about three kinds of physical responses are defined. The transformed response can be obtained from the measured acceleration records without requiring the knowledge of the initial velocity and displacement information. The relationship between the transformed response and the actual response is derived using the structural system matrix that is induced by the process of a time domain modal vibration test technique.

In this study, in order to apply the proposed response conversion algorithm to the integration scheme of the frequency domain, the Fourier transform of the displacement response is separated into the frequency component of the measured acceleration signal and the frequency component of the initial values. Then, the method of estimating the transformed displacement response is proposed by the inverse Fourier transformation of the former component only. In the response prediction method of the frequency domain, the response conversion according to the frequency can be available. It is not possible in the time domain method. Not only the static component can be extracted, but also the dynamic displacement component can be separated by the structural mode from the identified displacement response. In the response conversion results of the frequency domain, the identified response according to the frequency can be estimated by changing over the limits of integration. If the reliability of the identified responses is ensured, it is expected that the proposed method for estimating the impact factor can be useful in the bridge's dynamic test.

The feasibility for physical application of the proposed technique is tested by the example problems using the real bridge's superstructures under several cases of moving load and the results are compared with the actually measured displacements using LVDTs. It has been observed that the proposed technique can assess the displacement responses successfully when the measured acceleration signals include the unloading state and the free vibration behavior.

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