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Fuzzy sets for fuzzy context model

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Abstract.

In the first part an overview on fuzzy sets and fuzzy numbers is given. A detailed treatment of these notions is introduced in [1, 2, 3]. This sintetically presentation is useful in understanding and in developping the applications in context problems. In the second part, fuzzy context model is given as an application of fuzzy sets and the fuzzy equilibrium equation is solved [4,5].

Key words : fuzzy numbers, Linguistic variables.

I. Preliminaries

The theory of fuzzy sets is considered among the necessity of extending the area of applicability of classical mathematics in the meaning of enforcing the possibilities of mathematical modeling the real world systems.

One can appreciate that the nature of reality, as well as our thinking manner and the symbolism of inter-human communication languages represent sources of uncertainty, imprecision, vagueness and ambiguity (Linguistic variables). The necessity of maneuvering of certain not necessarily "perfectly determinable" features, i.e., "imprecisely defined" (but with a measurable degree of imprecision) resulted in imposing the theory of fuzzy sets; it is proved to be the systematic framework fit to the management of imprecision and ambiguity [6]. Here a sintetically presentation of fuzzy sets and fuzzy numbers is given.

I.1. Fuzzy sets

Let A be an nonempty set. An application $\mu : A \to [0,1]$ is termed as a fuzzy subset of A. The couple (A, μ) is also named fuzzy set. We also remark that, for $\mu : A \to [0,1]$ and $x \in A$, $\mu(x)$ signifies "the degree of membership" of the element x at the subset determined by μ. Within a matched context (A - set of sentences), $\mu(x)$ may mean the degree of truth of the sentence x.

An example (seemingly anecdotal) which may appear in the personnel policy in a company can bring about clarifying in the process of assimilation of the notion of fuzzy set; the subset of the employees who are "vigorous for work" (as unique criterion) can be designed in the following manner [cf. 2]:

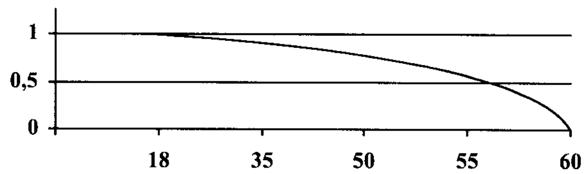

Fig. 1

therefore, we can say, for example, that a 50 (or 55) years old person corresponds to the established criterion for the measure (or degree) 0.8 (0.6, respectively).

From another point of view the estimation of certain economic data may be done in the following way: the price of product A would be between a and c and most probable it would be b, where a,b,c, are real numbers and $a \le b \le c$.

In the first example (Fig. 1) one remarks both the usefulness of the individual degrees of membership and the representation of the set as an entity, in the case of global reasons.

In both cases, the settlement of the concrete (numerical) elements may be suspicion of subjectivism - a fact which may be avoided by an appeal to experts opinions and criteria.

With fuzzy subsets (fuzzy sets) operations similar with those with subsets (or sets) can be carried out.

Be it $\mu, \eta : A \to [0,1]$ - the intersection of the fuzzy subsets μ and η is defined through $\mu \cap \eta: A \to [0,1]$, $(\mu \cap \eta)(x) = \min\{\mu(x), \eta(x)\}$;

- the reunion of the fuzzy subsets μ and η is defined through $\mu \cup \eta: A \to [0,1]$, $(\mu \cup \eta)(x) = \max\{\mu(x), \eta(x)\}$;
- the complementary of the fuzzy set μ is defined through μ': $A \to [0,1]$, $\mu'(x) = 1-\mu(x)$.

The arithmetic structure of the interval [0.1] (that is the operations "+" and "·" permits to define other numerous operations with fuzzy subsets). We remind:- the probabilistic sum of two fuzzy subsets $\mu, \eta : A \to [0,1]$ given by $\mu + \eta$: $A \to [0,1]$, $(\mu + \eta)(x) = \mu(x) + \eta(x) - \mu(x) \cdot \eta(x)$;

- <u>the product</u> of two fuzzy subsets $\mu,\eta: A \to [0,1]$, given by

$$(\mu \circ \eta): A \to [0,1], (\mu \circ \eta)(x) = \mu(x) \cdot \eta(x)$$

Out of the non-standard operations that have as a starting point the association of certain magnitudes or features to linguistic degrees ("less","very"etc.) we mention: - the operator of the association of the degree "very" when A is in correspondence with a subset of real number set (through the significance of the elements of A can be represented on the real axis), a case in which the operations with real numbers are transferred to A (obtaining operations with elements from A): if $\mu : A \to [0,1]$ represents a property, then we shall say that $\mu_k: A \to [0,1]$, $\mu_k(x) = \mu(x+k)$, $k \in A$ represents the satisfying of the property obtained from the property given by associating of the linguistic degree "very" of k level. Similarly $\mu_{-k}: A \to [0,1]$, $\mu_{-k}(x) = \mu(x-k)$ is obtained by associating the linguistic degree "less" of k level as in fig. 2.

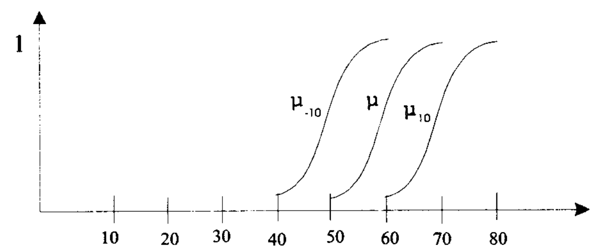

Fig. 2

where $\mu: [0,\infty] \to [0,1]$ represents the property to "be aged", μ_{10} and μ_{-10} represents the property to "be very aged" and, respectivelly, to "be less aged".

The notion of "level subset" proves to be important for the study of fuzzy subsets: for $\mu: A \to [0,1]$, the subsets (classical meaning) $_{\mu}A_{\alpha} = \{x \in A \mid \mu(\alpha) \geq \alpha\}$ where $\alpha \in [0,1]$ are termed <u>level subsets</u> of fuzzy sets (A, μ). One can immediately say that there occur :

i) $_{\mu}A_0 = A$;
ii) $\alpha_1 \leq \alpha_2$, $\alpha_1, \alpha_2 \in [0,1] \Rightarrow {}_{\mu}A_{\alpha_2} \subseteq {}_{\mu}A_{\alpha_1}$

The following result is also known: if (X_α), $\alpha \in [0.1]$ is a family of subsets of a set X so that:

i) $X_0 = X$;
ii) $\alpha \leq \beta \Rightarrow X_\beta \subseteq X_\alpha$;
iii) for any sequence $(\alpha_i)_{i \in N}$ so that $\alpha_i \in [0,1]$, ;

$\forall i \in N, \alpha^1 \leq \alpha^2 \leq \ldots; \lim \alpha^n = \alpha$ we have $X_\alpha = \bigcup X_{\alpha^n}$

then a function $\mu: X \to [0,1]$ exists and is unique so that $X_\alpha = {}_{\mu}X_\alpha, \forall \alpha \in [0,1]$.

We shall close the paragraph with an outcome that binds the fuzzy set to the theory of possibility. We remind that for a nonempty set (set of reference) Ω, an application pos: $P(\Omega) \to [0,1]$ where $P(\Omega)$ represents the set of all subsets of Ω, is named <u>application of possibility</u> if it satisfies the conditions:

i) pos (Φ) = 0, pos (Ω) = 1;
ii) $A \subseteq B \Rightarrow$ pos (A) ≤ pos (B)
iii)pos(AUB)= max {pos(A), pos(B)}.

We obtain a fuzzy subset $\mu: \Omega \to [0,1]$ through μ(x) = pos ({x}).

Reciprocally, having $\mu = \Omega \to [0,1]$, pos: $P(\Omega) \to [0,1]$, pos (A) = $\sup_{x \in A} \{\mu(x)\}$ is an "application of possibility".

I.2. Fuzzy numbers

Generally, by <u>fuzzy number</u> we mean any, continous convex application $\mu: R \to [0,1]$ so that there is $x_\mu \in R$ with $\mu(x_\mu) = 1$. The example given in the previous paragraph regarding the estimation of the price may be considered as a particular case of a fuzzy number.

Usually, the following form (suggested by H.Hellendorn) is considered:

$$\mu(x) \begin{cases} 0 & x \leq a; \\ \Pi_1(x) & x \in (a,b); \\ 1 & x \in (b,c); \\ \Pi_2(x) & x \in (c,d); \\ 0 & x \geq d; \end{cases}$$

where $\Pi_1(\Pi_2)$ is a continuous increasing (decreasing) function,

$$\lim_{\substack{x \to a \\ x \succ a}} \Pi_1(x) = 0, \quad \lim_{\substack{x \to d \\ x \prec d}} \Pi_2(x) = 0, \quad \lim_{\substack{x \to b \\ x \prec b}} \Pi_1(x) = 1,$$

$$\lim_{\substack{x \to c \\ x \succ c}} \Pi_2(x) = 1, \ \forall a,b,c,d \in R$$

When $\Pi_1(x) = \dfrac{x-a}{b-a}$, $\Pi_2(x) = \dfrac{d-x}{d-c}$, $a \neq b$, $c \neq d$

trapezoidal fuzzy numbers are obtained (that become, for b=c, triangular fuzzy numbers) as in fig. 3.

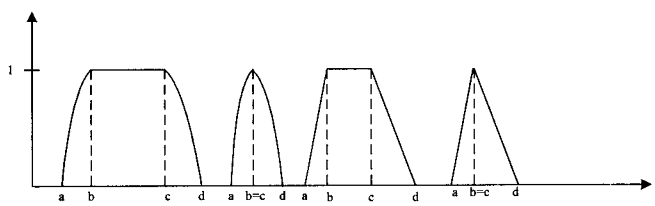

Fig. 3

where the last two situations represent trapezoidal and triangular fuzzy numbers, respectively.

It is worth remarking that a usual number may be interpreted as a fuzzy number (by convention it is considered in this way), i.e., $r \in R$ leads to :

$$\tilde{r}: R \to [0,1],\ \tilde{r}(x) = \begin{cases} 1 & x = r; \\ 0 & x \neq r. \end{cases}$$

Generally, using the principle of extension given by L. Zadeh, the operations with fuzzy numbers are done according to the formula:

- for $\mu,\eta: R \to [0,1]$, $\mu \div \eta: R \to [0,1]$ is given by $(\mu \div \eta)(x) = \sup_{x=y*z} \{\min\{\mu(y), \eta(z)\}\}$, where "*" might be any of the usual applications with real numbers "+", ".", "-", ":" obtaining, $\oplus, \otimes, \Theta, \div$ respectively. It is easy now to conclude that :

$$(\tilde{r} \oplus \mu)(x) = \mu(x - r),$$

$$(\tilde{r} \otimes \mu)(x) = \begin{cases} \mu(\frac{x}{r}), r \neq 0; \\ x = 0; r = 0; \\ x \neq 0; r = 0. \end{cases}$$

We remark here that the previous operations with the fuzzy numbers of $\tilde{r}$ form return to the usual arithmetical operations, in other words, in the case of the operations with numbers of $\tilde{r}$ form, numbers of the same form are obtained.

However, this property is not fulfilled if we refer to the set of the trapezoidal (triangular) fuzzy numbers. This fact, together with the fact that, usually, in applications trapezoidal or triangular fuzzy numbers are used, made it necessary to find out new practical modalities to define the operations so that the result should be of the same type with the numbers under consideration. The properties of associativity, commutativity and the existence of the neutral element ($\tilde{0}$ for $\oplus$ and $\tilde{1}$ for $\otimes$) will be maintained; on the other hand the equalities regarding the inverse of an element will change into equivalents. To be more specific, we shall have in this later case $\mu \oplus (-\mu)$ symmetric ($-\mu$: R→ [0,1] vs. $(-\mu)(x) = \mu(-x)$) relatively to the ordinate's axis (in this case we say that $\mu \oplus (-\mu)$ is equivalent with $\tilde{0}$); similarly, (as a principle) $\mu \otimes \mu^{-1}$ equivalent with $\tilde{1}$, $\mu^{-1}: R \to [0,1]$, $\mu^{-1}(x) = \mu(\frac{1}{x}), x \neq 0$.

Turning back to the necessity of finding out optimum variants to define the operations, we remark, first of all, that a fuzzy number (in general) μ: R → [0,1] may be characterized by a family of intervals (named intervals of confidence), i.e., by a family of intervals ($\alpha \in [0,1]$) where:

$$\mu_\alpha = \left[\underline{x_\alpha}, \overline{x_\alpha}\right],$$

$$\underline{x_\alpha} = \inf\{x \mid \mu(x) = \alpha\},\ \overline{x_\alpha} = \sup\{x \mid \mu(x) = \alpha\}$$

If,for example, μ: R→[0,1] is represented by fig. 4 then $\mu_\alpha = \left[\underline{x_\alpha}, \overline{x_\alpha}\right]$.

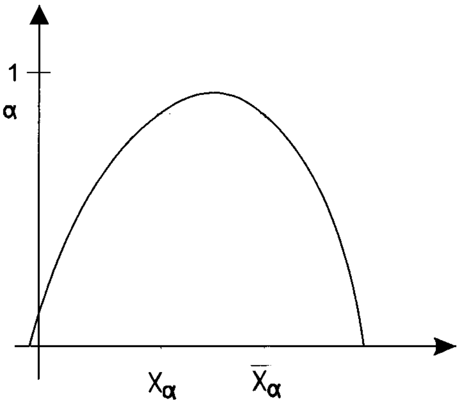

Fig. 4 Interval of confidence

In the case of the trapezoidal fuzzy numbers μ = (a,b,c,d):

$$\mu_\alpha = [(b-a)\,\alpha+a,\ -(d-c)\,\alpha+d]$$

and in the case of the triangular fuzzy numbers μ = (a,b,c):

$$\mu_\alpha = [(b-a)\,\alpha+a,\ -(c-b)\,\alpha+c]$$

Using the fact that fuzzy numbers are characterized by the family of these intervals, the definition of the operations is performed by means of this family and thus :

- be it μ,η: R→ [0,1], we define the fuzzy number $\mu \oplus \eta$: R→ [0,1] as being the fuzzy number determined by the family of intervals:

$(\mu \oplus \eta)_\alpha = [\underline{x_\alpha} + \underline{y_\alpha}, \overline{x_\alpha} + \overline{y_\alpha}]$, where $\alpha \in [0,1]$,

$\mu_\alpha = [\underline{x_\alpha}, \overline{x_\alpha}]$, $\eta_\alpha = [\underline{y_\alpha}, \overline{y_\alpha}]$;

-similarly $\mu \otimes \eta$ is defined (in the case $\underline{x_\alpha} > 0, \underline{y_\alpha} > 0$) by : $(\mu \otimes \eta) = [\min\{\underline{x_\alpha} \cdot \underline{y_\alpha}, \underline{x_\alpha} \cdot \overline{y_\alpha}, \overline{x_\alpha} \cdot \underline{y_\alpha}, \overline{x_\alpha} \cdot \overline{y_\alpha}\}$, $\max\{\underline{x_\alpha} \cdot \underline{y_\alpha}, \underline{x_\alpha} \cdot \overline{y_\alpha}, \overline{x_\alpha} \cdot \underline{y_\alpha}, \overline{x_\alpha} \cdot \overline{y_\alpha}\}]$;

- $\mu\ \Theta\ \eta$ is defined by :

- $(\mu \Theta \eta)_\alpha = [\underline{x_\alpha} - \overline{y_\alpha}, \overline{x_\alpha} - \underline{y_\alpha}]$;

- $\mu \div \eta$ is defined by: $(\mu \div \eta)_\alpha = [\frac{\underline{x_\alpha}}{\overline{y_\alpha}}, \frac{\overline{x_\alpha}}{\underline{y_\alpha}}]$,

$(\overline{y_\alpha} \neq 0, \underline{y_\alpha} \neq 0)$.

When μ and η are triangular fuzzy numbers we obtain that $\mu \oplus \eta$ and $\mu \Theta \eta$ are triangular fuzzy numbers. The numbers $\mu \otimes \eta$ and $(\mu \div \eta)$ are not triangular but they may give triangular approximations.

Remarking that in the triangular fuzzy numbers μ=(a,b,c), μ_0=[a,c] and μ_1={b}, or, in other words, μ_0 and μ_1 determine the fuzzy number μ, one should accept that in the case of operations $\otimes$ and $\div$ the triangular numbers are determined by $(\mu \div \eta)_0$, $(\mu \div \eta)_1$ and $(\mu \div \eta)_0$, $(\mu \div \eta)_1$ respectively.

Reviewing, we shall have, for μ=(a,b,c), η=(m,n,p):

$\mu \oplus \eta$ = (a + m, b + n, c + p); $\mu \Theta \eta$ = (a - m, b - n, c - p);

$\mu \otimes \eta$ = (am, bn, cp); $\mu \div \eta = (\frac{a}{p}, \frac{b}{n}, \frac{c}{m})$; in case a ≥ 0, m ≥ 0.

We have also:

$-\mu$ = (- c, - b, - a) ;

$\mu^{-1} = (\frac{1}{c}, \frac{1}{b}, \frac{1}{a})$, for the case a > 0;

ln μ = (ln a, ln b, ln c);

$e^{\mu} = (e^{a}, e^{b}, e^{c})$, for the case a > 0.

The maximum error for the approximation given to the operation $\otimes$ is:

$$\frac{p-\sqrt{np}}{p-n} \; if \; \frac{a}{p} < \frac{b}{n} (p \neq n); \quad \frac{\sqrt{mn}-m}{n-m} \; if \; \frac{c}{m} < \frac{b}{n} (n \neq m)$$

For μ^{-1} , $ln\, \mu$ and e^{μ} we have respectively

$$\max \{ -(\frac{1}{\sqrt{b}} - \frac{1}{\sqrt{c}})^2, -(\frac{1}{\sqrt{a}} - \frac{1}{\sqrt{b}})^2 \}.$$

$$\max \{ \frac{1}{\ln b - \ln a} - \frac{a}{b-a}, \frac{c}{c-b} - \frac{1}{\ln c - \ln b} \}, (a \neq b \neq c).$$

$$\max \{ \frac{-a}{b-a} + \frac{1}{b-a} \ln \frac{e^b - e^a}{b-a}, \frac{c}{c-b} + \frac{1}{c-b} \ln \frac{e^c - e^b}{c-b} \},$$

(a ≠ b ≠ c).

We also remind that, for a function of two real variables z=f(x,y), interpreting *x* and *y* as fuzzy numbers, it results that *z* is a fuzzy number, that is f(μ, η) = τ,

$$\text{where } \tau(c) = \sup_{f(a,b)=c} \{\min | \mu(a), \eta(b)\}.$$

In the language of confidence intervals we have (in the case where *f* continuous function):

$$\underline{\tau_\alpha} = \min \{ c \mid c = f(a,b), a \in \mu_\alpha, b \in \eta_\alpha \}.$$

$$\overline{\tau_\alpha} = \max \{ c \mid c = f(a,b), a \in \mu_\alpha, b \in \eta_\alpha \}.$$

In a relation of the form F(A,X) = B (of interest for the study of _reciprocal determination of the economic indicators situated in relations of functional dependence), where A, B, X can be interpreted as fuzzy numbers, be X = f(A,B) the implicite function defined by F(A,X)=B.

In the case of fuzzy numbers it is considered that *X* is given by the confidence intervals, in a similar manner to the previous case.

In connection with the ordering of the fuzzy (triangular) numbers, the following variants are used:

a) the distance between the triangular fuzzy numbers (Minkovski distance) is defined by :

- for μ = (a_1, b_1, c_1), η = (a_2, b_2, c_2)

$\rho(\mu,\eta) = \frac{1}{3}\left((a_1 - a_2)^p + (b_1 - b_2)^p + (c_1 - c_2)^p\right)^{\frac{1}{p}}$, (usually *p=2* is considered);

- the distance up to a fuzzy (fixed) number is estimated and according to this distance the given fuzzy numbers are ordered;

b) To a triangular fuzzy number μ=(a,b,c), $\hat{\mu} = \frac{a+2b+c}{4}$ is associated. In a first stage, the numbers (e.g. μ, η, τ, ...) are ordered after $\hat{\mu}$, $\hat{\eta}$, $\hat{\tau}$, ... It is obvious that we can have $\hat{\mu} = \hat{\tau}$ ...; in this case we shall say that μ=(a, b, c) precedes τ=(m, n, p) if b < n or b = n and a < m (if also *a=m*, then from $\hat{\mu} = \hat{\tau}$, *p=c*, that is μ = τ).

II. Fuzzy context model

In the following lines we shall introduce a model of transfer in a fuzzy context of the economic indicators.

It is known that formulae of the type:

$$\Gamma_n = A_0 + \frac{A_1}{1+r} + \ldots\ldots\ldots\ldots + \frac{A_n}{(1+r)^n}$$

are used, for example, for the calculation of the economic value of an asset, admitting that the life span of the asset is n, the flow of income along the life span is A_0= 0, A_1 = V_1,..., A_n = V_n, and r represents the rate of interest - or - in the calculation of the price of a share (a case in which r represents the efficiency of the share A_1,.... A_n represents the flow of dividends, etc.).

In fact, r (in problems of previsions, amortizing) cannot be estimated but only by fuzzy numbers. Moreover, we shall give fuzzy variants for the previously considered formula.

Be r_k = (a_k, b_k, c_k), k = 1,2, ...,n the estimated value of r during the year k. The confidence intervals are given - for this case - by

$$r_k(\alpha) = [a_k + (b_k - a_k) \times \alpha, c_k - (c_k - b_k) \times \alpha], \alpha \in [0,1], k = 1,2,\ldots,n \cdot$$

We denote $i_k(\alpha) = a_k + (b_k - a_k) \times \alpha; \; s_k(\alpha) = c_k - (c_k - b_k) \times \alpha$

In the context of the confidence intervals, we then obtain:

$$\Gamma_n(\alpha) = \left[A_0 + \frac{A_1}{1+s_1(\alpha)} + \ldots + \frac{A_n}{\prod_{k=1}^{n}(1+s_k(\alpha))}, A_0 + \frac{A_1}{1+i_1(\alpha)} + \ldots + \frac{A_n}{\prod_{k=1}^{n}(1+i_k(\alpha))} \right]$$

$\alpha \in [0,1]$

In the particular case when A_0= 0, A_1 = A_2 =.......= A_n (=A) and r is globally appreciated by r = (a,b,c), a < b < c then:

$$\Gamma_n(\alpha) = \left[\frac{1-(1+c-(c-b)\times\alpha)^n}{c-(c-b)\times\alpha}, \frac{1-(1+(a+(b-a)\times\alpha))^n}{a+(b-a)\times\alpha} \right], \quad \alpha \in [0,1]$$

Having the confidence intervals given before, Γ_n may be represented using the membership function:

$\mu_{\Gamma_n}: R \to [0,1],\quad \mu_{\Gamma_n}(x) = \sup_{x=y_1+\ldots+y_n} \{\min\{\mu_1(y_1),\ldots,\mu_n(y_n)\}\}$, where:

$$\mu_i : R \to [0,1],\quad \mu_i(x) \begin{cases} 0,\ x \le (1+c)^{-i}; \\ \dfrac{1+c-x^{\frac{-1}{i}}}{c-b},\ (1+c)^{-i} \le x \le (1+b)^{-i}; \\ \dfrac{x^{\frac{-1}{i}}-1-a}{b-a},\ (1+b)^{-i} \le x \le (1+a)^{-i}; \\ 0,\ x \ge (1+a)^{-i}. \end{cases}$$

$i = 1, 2, \ldots, n$.

The formula for $\Gamma_n(\alpha)$ may still be generalized considering $A_1, \ldots A_n$ to be fuzzy numbers, that is:

Be $A_k = (i_k, m_k, t_k)$, $k=1,2,\ldots,n$, (in order to simplify the calculations) the condition $i_k > 0$ is useful - a fact that does not restrict the generality of the problem.Then

$$\Gamma_n(\alpha) = \left[A_0 + \sum_{e=1}^{n} \frac{i_e + (m_e - i_e)\alpha}{\prod_{j=1}^{e}\left(1+s_j(\alpha)\right)},\quad A_0 + \sum_{e=1}^{n} \frac{t_e - (t_e - m_e)\alpha}{\prod_{j=1}^{e}\left(1+i_j(\alpha)\right)} \right],$$

are obtained, where $\alpha \in [0,1]$.

Turning back to the case $A_1 = \ldots = A_n$ (=C), $A_0=0$, that is $\Gamma_n = \sum_{i=1}^{n} A(1+r)^{-i} = A\sum_{i=1}^{n}(1+r)^{-i}$ we also obtain the equilibrium equation:

$$A = \Gamma_n \left(\sum_{i=1}^{n} (1+r)^{-i} \right)^{-1}.$$

The former equality may be taken as a fuzzy equation (in the unknown A) and approached as such with the specific methods of solving.

In the context of fuzzy numbers the value of A that satisfies the former relation (knowing Γ_n and r) differs from the value given by the latter relation (this is a consequence of the relaxation of the relative equalities to the reversibility of elements).

In this case, the accepted variant for A is obtained from the latter relation. In the case $r = (a,b,c)$ ($\Gamma_n \in R$), is obtained:

$$A(\alpha) = \left[\frac{\Gamma_n(a+(b-a)\alpha)}{1-\left[1+(a+(b-a)\alpha)\right]^{-n}},\ \frac{\Gamma_n(c-(c-b)\alpha)}{1-\left[1+(c-(c-b)\alpha)\right]^{-n}} \right],$$

where $\alpha \in [0,1]$.

Conclusion

The paper presents a development to use fuzzy numbers for interest rates. The elementary mathematics of finance may be extended to handle fuzzy interest rates and fuzzy time periods. Thus, the results in this paper indicate a good application of fuzzy set approach to finance.

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Vita

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