

# Another representation of hand written English alphabets by a sequence of fuzzy sets

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## Abstract

In this paper, we describe how to represent lower case hand-written English alphabets by a sequence of two to seven fuzzy sets. Each fuzzy set represents an arc segment of the character and each arc segment is assumed to be a part of an ellipse. The part of an ellipse is defined by five quantities; its short and long radii, its orientation angle, whether it is a part of the lower half or the upper half, and whether it is the full half or a part of a half. Hence, we use the Cartesian product of five fuzzy sets to represent each arc segment. We show that this representation is a translation, rotation, and scaling invariant and that it can be used to generate the hand-written English alphabets. The representation we describe is different from the one proposed earlier by the author and when compared with the previous representation, the one described in this paper simulates more closely the behavior of how one writes English characters.

**Key words** : Fuzzy Sets; English alphabets; Writing Sequence; Representation; Hand-writing;

## 1. Introduction

We have shown in our earlier work [1] that hand-written standalone lower case English alphabets can be represented by a sequence of fuzzy sets. Using the fuzzy sets, the deviations from one writer to another can be handled, the features that distinguish different characters can be identified, and the errors in the thinning or other image processing steps for the pattern recognition can be reduced. In this paper, we describe a revised version of the representation, which is translation, rotation, and scaling invariant.

There have been many studies on the recognition of handwritten characters, mainly on the applications of optical character recognition systems. L. Chen[2] proposes a method using a segment partition algorithm so that the thinning process can be avoided. He tried to partition characters into segments, horizontal or vertical, and synthesize them into curved strokes. C. Lee and B. Wu[3] studied stroke extractions for Chinese characters. They determine character segments by finding boundary points and applying a contour tracking algorithm. J. Zhou et al.[5] studied a verification scheme to correct some of the errors in the neural network based recognizers, where the errors seem to come mostly from neglecting the segment features.

In this paper, we break down character writing into segments by taking the extreme points in the vertical direction as the

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segment boundary points. This is based on C. Remi et al.[4] where they found through experiments that higher level children tend to break up graphs into segments like adults would normally do while drawing graphs. In the paper by S. Djeziri et al.[6], we also find that a handwriting sequence can be broken into segments where the writer makes a small or large break during the course of writing. They use a kinematic theory of handwriting generation to analyze the handwriting kinematically and show that there are large or small breaks in the writing sequence. We consider that these breaks are at extreme points of the curve in the vertical direction and take them to be the segment boundaries in our segmentation.

We will consider each segment of a character by a part of an ellipse and represent it by the Cartesian product of the five fuzzy sets; the first of which is for the half length of the longer axis, the second is the ratio of the shorter axis length over the longer axis length, the third is for the angle change of the arc direction from the previous segment, the fourth is to indicate whether the arc is half of the half ellipse or the full half, and the last is to indicate whether it is curved toward the right or left from the moving direction.

## 2. Representation of arc segments

In this section, we describe how the five fuzzy set components described above can be defined. For all of the five component fuzzy sets, we will be using the triangular sets. The first component fuzzy set is to represent the half-length of the

long axis of the ellipse as shown in fig.1. The shortest length indicated by the fuzzy set number 1 in fig.1 is for the dots in the letters 'i' and 'j'. The lengths 0.02, 0.07, 0.12, 0.17, 0.22, 0.27 each corresponding to the fuzzy set numbers 1,2,...,7 are based on the assumption that each of the single characters are written in a square of 1.0 by 1.0 with margins.

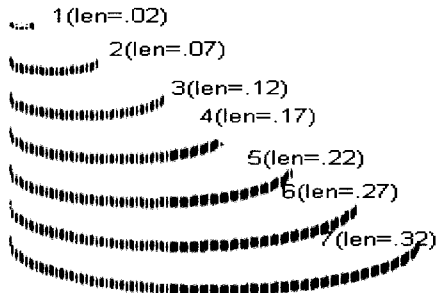


fig.1. Fuzzy Set Numbers for Arc Segment Length

The second component is to represent the curvature of the arc segment by using the ratio of the short axis length vs the long axis length of the ellipse. As shown in fig.2, the ratio is granulated into five categories, i.e.  $r=0.25, 0.50, 0.75, 1.25$  and they are used as the center of the supports for the five triangular fuzzy sets.

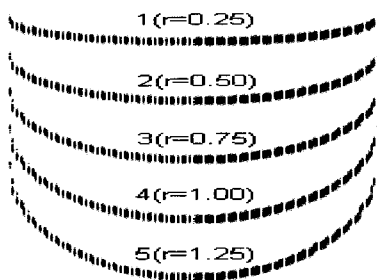


fig.2. Fuzzy Sets for Arc Shape  
(Ratio of the short axis vs the long axis length)

The third component is the arc direction for the first arc segment and the direction change for the other segments from the previous moving direction. Consider an ellipse whose longer axis lies on the x-axis with the left end point at the origin. The ellipse will be rotated and translated to form the arc segment. We will always assume that the writing stroke starts from the origin when viewed before the rotation. The arc direction is the direction of the long axis of the ellipse, i.e. the x-axis before it was rotated. We will be using 11 fuzzy sets for the directions shown in fig.3.

The fourth component is used to distinguish the arc segments curved towards the left of the moving direction from those curved towards the right of the moving direction as shown in fig.4. For the fuzzy sets, we use triangular sets with centers of supports at  $-0.25$  and  $0.25$  which are the minimum ratio of the

shorter axis length relative to the longer axis length. To determine the membership values of an arc segment for these

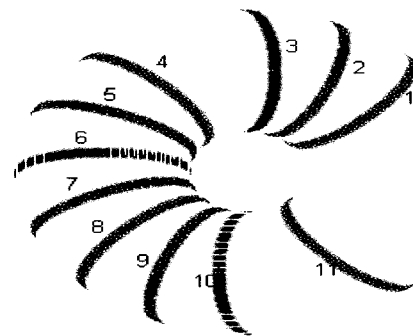


fig.3. Fuzzy Sets for Arc Direction

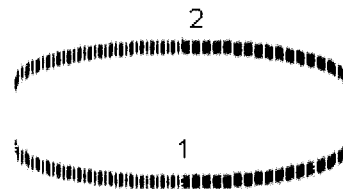


fig.4. Fuzzy sets for curvature of arc segments  
(curved toward right or left)

two fuzzy sets, first we translate and rotate the arc segment so that the starting end point lies at the origin and the other end point lies on the positive x-axis. Then we compute the maximum of the absolute values of the y-coordinate for all of the points in the arc segment. The ratio of this value relative to the maximum x-coordinate value is used for the fuzzification.

The last component is used to distinguish whether the arc segment is a full half ellipse, the first half part of a half ellipse, or the second half part of a half ellipse. Recall that the writing always starts from the left end point of the ellipse when it is located on the positive x-axis after the rotation and an example of the first fuzzy set with number 1 in fig.5 is the second arc segment of letter 'h'.

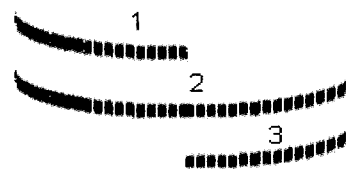


fig.5. Fuzzy Sets for Full or Half Ellipse

### 3. Representation of hand-written lower case standalone English letters

We have shown in the previous section that each arc segment of hand-written standalone English alphabets can be

represented by a fuzzy set obtained by a Cartesian product of five component fuzzy sets. As shown in Table 1-1 through Table 1-5, we denote these fuzzy sets by five digit hexadecimal numbers. Recall that we use 11 fuzzy sets for the arc direction and we must use a hexadecimal digit for the third component. The letters 'i', 'j', and 't' have two connected components and they are separated by an artificial arc segment of '00000'.

Table 1-1. Fuzzy set representation of lower case Hand-written alphabets - 1

A	b	c	d	E	f
63912	62112	22412	53812	61112	62112
61612	71812	52412	71712	63812	71812
61812	42612	31412	71712	22412	53712
22312	22A12		31412		42A12

Table 1-2. Fuzzy set representation of lower case Hand-written alphabets - 2

g	h	i	j	K	l
42912	62112	41211	34211	62112	72212
42612	71811	42912	62823	71811	72812
62823	31623	00000	52422	52422	
52412	31611	14012	00000	33422	
	31412	14612	14012	41412	
			14612		

Table 1-3. Fuzzy set representation of lower case Hand-written alphabets-3

m	N	o	p	q	r
23212	23212	61A12	32112	42912	31112
61721	61721	61612	71822	42612	31B12
61622	61622	32612	71522	71712	51912
61621	61621		44622	41712	31412
61622	23412		21312	31A12	
61621					
23412					

Table 1-4. Fuzzy set representation of lower case Hand-written alphabets - 4

S	T	U	v	w.	x
61112	62311	33222	33122	23122	33122
45622	62613	51713	51813	51813	51813
52412	00000	51611	51611	51611	32412
	51012	31012	31012	51613	00000
				51611	71812
			23A13		

Table 1-5. Fuzzy set representation of lower case Hand-written alphabets - 5

Y	z
33122	33122
31813	34912
31611	52B12
71623	62412
62422	

Fig.6 shows the images drawn by using the fuzzy set numbers in the above tables. The five digit fuzzy set number is interpreted as a part of an ellipse with a long axis length, short axis length, and the direction of the central axis. An equation for the ellipse is generated and is used to set the pixels on a 320 × 320 pixel image with a thickness band of 4 pixels.

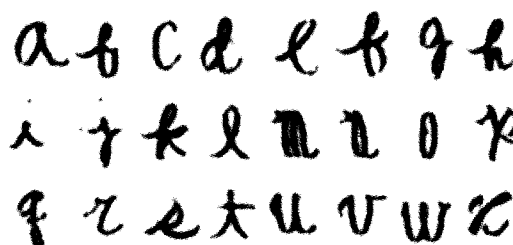


fig.6. Characters generated by the fuzzy sets



fig.7. Characters translated by changing only the starting point

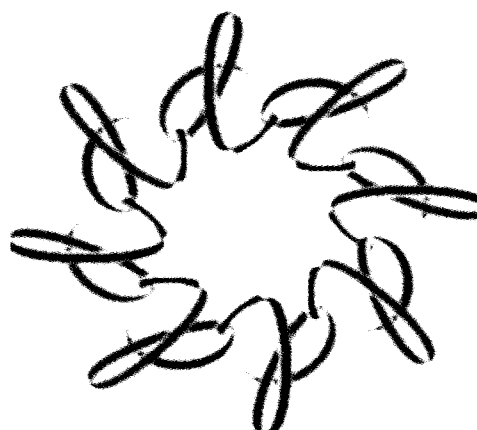


Fig.8. Rotated character images generated by only the direction of the first arc



fig.9 Scaled character images

Fig.7 shows five images of 'f' drawn by changing only the starting point of the character and they show that our representation is indeed a translation invariant. Similarly, we drew the eight figures of 'd' in fig.8 by simply changing the direction angle of the first arc segments incrementally by  $\frac{\pi}{4}$ . These figures show that our fuzzy set representation is a rotation invariant. The scaling invariant property is shown in fig.9 where the letter q's are drawn by scaling the support boundaries of the first component fuzzy sets.



fig.10. Connected characters

#### 4. Conclusion

We have shown that a sequence of two to seven fuzzy sets can be used to represent handwritten lower case standalone English alphabets and that the representation is a translation, rotation, and scaling invariant. One can use the representation to generate graphic images of the characters by using parts of

an ellipse. Our representation using a sequence of fuzzy sets is obtained by looking at how a man writes the character and no training is necessary for the recognition of the characters. We only need to take the fuzzy set intersections with predefined sequences of fuzzy sets for the 26 characters. When finer images are necessary, one can simply increase the number of fuzzy sets for each of the component fuzzy sets, which can be done fairly easily.

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