퍼지 및 지능시스템학회 논문지 2003, Vol. 13, No. 6, pp. 749-753

The Properties on Fuzzy Submachines of a Fuzzy Finite State Machine

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Abstract

In this paper we introduce the concepts on retrievability, separability and connectedness of fuzzy submachines, which generalize those of crisp submachines. And also we generalize crisp primary submachines to those with fuzziness, from which we obtain the decomposition theorem of fuzzy submachines.

Key words : Retrievable fuzzy submachines, Strongly connected fuzzy submachines, Primary fuzzy submachines,

1. Introduction

Fuzzy automata theory has been developed by many researchers, since Wee [1] introduced the concept of fuzzy automata following Zadeh [2,3]. Algebraic techniques to study fuzzy automata were given by D.S. Malik, J.N. Mordeson and M.K. Sen [4,5,6]. In particular, in [5] they introduced the notion of subsystems of a fuzzy finite state machine in order to consider states as fuzzy.

The fuzzification of state sets is required for applications in many areas. For example, S.Y. Hwang et al. [7] and K. Peeva [8] constructed fuzzy acceptors with state sets as fuzzy, and applied them to syntactic pattern recognition.

In this paper we give the structure of fuzzy submachines with states as fuzzy, and generalize the concepts of retrievability and connectedness of crisp submachines to those of fuzzy submachines and investigate the corresponding properties. And also we give the primary fuzzy submachines, from which we obtain the decomposition theorem of fuzzy submachines.

Before going further, we introduce the following definitions and notations. Let A be a fuzzy subset of Q, with the membership $\mu_A : Q \to [0,1]$. The set

$suppA = \{x \in Q \mid \mu_A(x) > 0\}$ is called the support of A. If B is also a fuzzy subset of Q, then the fuzzy sets $A \cup B, A \cap B$ are defined as

$$\mu_{A \cup B}(x) = \mu_A(x) \vee \mu_B(x),\ \forall x \in Q,$$

$$\mu_{A \cap B}(x) = \mu_A(x) \wedge \mu_B(x),\ \forall x \in Q.$$

접수일자 : 2002년 7월 19일
완료일자 : 2003년 10월 16일
The author would like to thank the reviewers for their helpful suggestions on revising this paper. This research was supported by the Daegu University Research Grant, 2003.

When we want to exhibit an element $x \in Q$ that typically belongs to a fuzzy set A, we may demand its membership value to be greater than some threshold $\alpha \in (0,1]$.

The ordinary set of such elements is the α-cut A_α of A,

$A_\alpha = \{x \in Q \mid \mu_A(x) \gg \alpha\}$. It is easily checked that the following properties hold:

$$(A \cup B)_\alpha = A_\alpha \cup B_\alpha,\ (A \cap B)_\alpha = A_\alpha \cap B_\alpha.$$

A is said to be included in B, denoted by $A \subset B$, if for each x, $\mu_A(x) \leq \mu_B(x)$.

If A_α and B_α are disjoint for all α, then we say that fuzzy sets A and B are disjoint.

And if α-cuts $A_\alpha \setminus B_\alpha$ define a fuzzy set, the it is denoted by $A \setminus B$.

A fuzzy finite state machine is a triple $M = (Q, X, \mu)$, where Q and X are finite nonempty sets and μ is a membership of some fuzzy subsets of $Q \times X \times Q$, i.e., $\mu : Q \times X \times Q \to [0,1]$. Let X^* denote the set of all words of elements of X of finite length. Q is called the set of state and X is called the set of input symbols. Let λ denote the empty word in X^* and $|x|$ denote the length of $|x|$, $\forall x \in X^*$.

2. Properties on fuzzy submachines

Let Q and X be finite nonempty sets, and let X^* denote the set of all words of elements of X of finite length. If μ is a membership of some fuzzy subset of $Q \times X \times Q$, i.e., $\mu : Q \times X \times Q \to [0,1]$, a triple $M = (Q, X, \mu)$ is a fuzzy finite state machine. Define $\mu^* : Q \times X^* \times Q \to [0,1]$ by

$$\mu^*(q, \lambda, p) := \begin{cases} 1 & \text{if } q = p \\ 0 & \text{if } q \neq p \end{cases}$$

and

$$\mu^*(q, xa, p) = \bigvee\{\mu^*(q, x, r)\wedge\mu(r, a, p) \mid r\in Q\},$$
$$\forall p, q\in Q, \forall x\in X^*, \forall a\in X.$$

Then

$$\mu^*(q, xy, p) = \bigvee\{\mu^*(q, x, r)\wedge\mu^*(r, y, p) \mid r\in Q\},$$
$$\forall p, q\in Q, \forall x, y\in X^*.$$

This means that a fuzzy subset μ of $Q\times X\times Q$ can be naturally extended to a fuzzy subset μ^* of $Q\times X^*\times Q$ under max-min operation.

To simplify the notation, we write "$x\gg\alpha$", which means that x is greater than or equal to α if $\alpha>0$, and x is positive if $\alpha=0$.

Definition 2.1. Let $X=(Q, X, \mu)$ be a fuzzy finite state machine. Let $p, q\in Q$. p is called an immediate α-successor of q if there exists $a\in X$ such that $\mu(q, a, p)\gg\alpha$, $0\le\alpha\le 1$. And p is called an α-successor of q if there exists $x\in X^*$ such that $\mu^*(q, x, p)\gg\alpha$, $0\le\alpha\le 1$.

If $M=(Q, X, \mu)$ is a fuzzy finite state machine, μ is a fuzzy subset of $Q\times X\times Q$. From the definition 2.1, we note that "p is an immediate α-successor of q" means that there exists $a\in X$ such that $(q, a, p)\in Q\times X\times Q$ is an element of α-cut of μ, $0\le\alpha\le 1$. In fuzzy set theory, we usually consider the support of μ as 0-cut. From the meaning, we naturally define 0-successors as follows:

p is said to be an immediate 0-successor of q if there exists $a\in X$ such that $\mu(q, a, p)>0$. In the same ways, we have the definition of 0-successors, which are simply called successors.

Let $M=(Q, X, \mu)$ be a fuzzy finite state machine, and let $q\in Q$. We denote by $S^\alpha(q)$ the set of all α-successor of q. And if $T\subset Q$, the set of all α-successors of T in Q, denoted by $S_Q^\alpha(T)$, is defined to be the set $S_Q^\alpha(T) = \bigcup\{S^\alpha(q) \mid q\in T\}$. This is in fact a mapping of the set of all subsets of Q into itself. If no confusion arises, then we write $S^\alpha(T)$ for $S_Q^\alpha(T)$.

The structure of fuzzy finite state machine is applied in many areas, for example in fuzzy automata [9,10] and syntactic pattern recognition [7,8]. But the crisp state set restricts the areas of application. S.Y. Hwang *et al.* [7] and K. Peeva [8] defined fuzzy acceptors with the structure of fuzzy finite state machine, and applied to syntactic pattern recognitions. But the states and transitions are modified as fuzzy. So we need to define fuzzy submachines, which is submachines with states as fuzzy.

Now we give the definition of fuzzy submachines, which generalize the crisp submachines of a fuzzy finite state machine given by D.S. Malik, J.N. Mordeson and M.K. Sen [6].

Definition 2.2. Let $M=(Q, X, \mu)$ be a fuzzy finite state machine, and let $T_\alpha\subset Q$. Let $N_\alpha=(T_\alpha, X, \mu^{T_\alpha})$, where μ^{T_α} is a fuzzy subset of $T_\alpha\times X\times T_\alpha$. The fuzzy finite state machine N_α is called an α-submachine of M if

$$(1)\ \mu\mid_{T_\alpha\times X\times T_\alpha}=\mu^{T_\alpha} \text{ and } (2)\ S_Q^{1-\alpha}(T_\alpha)\subset T_\alpha.$$

Moreover, if T is a fuzzy subset of Q with T_α as α-cuts, then $N=(Q, T, X, \mu^T)$ is called a fuzzy submachine of M if for each α, N_α is an α-submachine of M.

To show that the definition 2.2 is the natural generalization of crisp submachine, let $M=(Q, X, \mu)$ be a fuzzy finite state machine, and let T be a crisp subset of Q. We consider the submachine $N=(T, X, \mu^T)$ of M defined by D.S. Malik, J.N. Mordeson and M.K. Sen [6], i.e., $\mu\mid_{T\times X\times T}=\mu^T$, and $S_Q^0(T)\subset T$. We note that if each α-cut T_α of T is itself, then

$$\mu^T=\mu\mid_{T\times X\times T}=\mu\mid_{T_\alpha\times X\times T_\alpha}=\mu^{T_\alpha}.$$

and

$$S_Q^{1-\alpha}(T_\alpha)=S_Q^{1-\alpha}(T)\subset S_Q^0(T)\subset T=T_\alpha.$$

Thus the fuzzy submachine (Q, T, X, μ^T) can be considered as the generalized form of submachine of $M=(Q, X, \mu)$. S_Q^0 will be simply denoted by S_Q.

Let $N=(Q, F, X, \mu^F)$, $P=(Q, T, X, \mu^T)$ be two fuzzy submachines of $M=(Q, X, \mu)$, and let T, F be fuzzy subsets of states Q with $T\subset F$. Then

$$\mu^{T_\alpha}=\mu\mid_{T_\alpha\times X\times T_\alpha}=(\mu\mid_{F_\alpha\times X\times F_\alpha})\mid_{T_\alpha\times X\times T_\alpha}=\mu^{F_\alpha}\mid_{T_\alpha\times X\times T_\alpha},$$

and

$$S_{F_\alpha}^{1-\alpha}(T_\alpha)=S_{T_\alpha}^{1-\alpha}(T_\alpha)\subset T_\alpha, \forall\alpha.$$

Thus P is also called a fuzzy submachine of N.

Remark 2.3. In the definition 2.2, we defined a fuzzy submachine $N=(Q, T, X, \mu^T)$, which satisfies the conditions related to the α-cuts T_α. If we note that the state set Q is finite, then the image $Im(T)$ of fuzzy set T is finite. Thus it is natural to restrict our interests only to the α-cuts T_α, $\alpha\in Im(T)$. Throughout this paper, the α-cuts of fuzzy sets will be considered only for $\alpha\in Im(T)$, if no confusion arises.

The fuzzy finite state machine with no proper submachine is exactly strongly connected [6]. This property can be extended to fuzzy submachines.

Definition 2.4. Let $N=(Q, F, X, \mu^F)$ be a fuzzy submachine of $M=(Q, X, \mu)$, and let $P=(Q, G, X, \mu^G)$ be a fuzzy submachine of N. P is called a proper fuzzy submachine of N if $G\neq\phi$ and $G\neq F$.

If F,G are crisp sets, then P is exactly a proper crisp submachine of M defined in [6].

Definition 2.5. Let $N=(Q,F,X,\mu^F)$ be a fuzzy submachine of M. Then N is called strongly connected if $p\in S^{1-F(q)}(q)$ for $\forall p,q\in suppF$.

From this definition we can easily check that if N is strongly connected, then it must be of the form (Q,aI_A,X,μ^{aI_A}), where I_A is the indicator function of $A\subset Q$.

Theorem 2.6. Let $N=(Q,F,X,\mu^F)$ be a fuzzy sub machine of $M=(Q,X,\mu)$. Then N is strongly connected if and only if N has no proper fuzzy submachines.

Proof. Suppose N has a proper fuzzy submachine $P=(Q,G,X,\mu^G)$. Then there exists some α such that G_α is a nonempty proper subset of F_α. Let $p\in F_\alpha\setminus G_\alpha$ and $q\in G_\alpha$. Since P is a fuzzy submachine of N, $S^{1-F(q)}(q)\subset G_{F(q)}\subset G_\alpha$, which implies $p\notin S^{1-F(q)}(q)$, that is, N is not strongly connected. Conversely, we assume that N is not strongly connected. Then there exist $p,q\in suppF$ such that $p\notin S^{1-F(q)}(q)$. Define a fuzzy subset G of Q by $\mu_G=F(q)I_{S^{1-F(q)}(q)}$, where I_A is the indicator function of A. Then it can be easily checked that $P=(Q,G,X,\mu^G)$ is a fuzzy submachine of N, and is proper. This completes the proof.

Corollary 2.7 [6]. Let $M=(Q,X,\mu)$ be a fuzzy finite state machine. Then M is strongly connected if and only if M has no proper submachines.

The retrievable fuzzy finite state machine can be decomposed to strongly connected submachines [6]. This decomposition property can be extended to fuzzy submachines.

Definition 2.8. Let $N=(Q,F,X,\mu^F)$ be a fuzzy sub machine of $M=(Q,X,\mu)$. N is said to be retrievable if $\forall p,q\in suppF$, $p\in S^{1-F(q)}(q)$ if and only if $q\in S^{1-F(p)}(p)$.

Theorem 2.9. Let $N=(Q,F,X,\mu^F)$ be a fuzzy sub machine of $M=(Q,X,\mu)$. Then N is retrievable if and only if N is the disjoint union of strongly connected fuzzy submachines.

Proof. Suppose N is retrievable, and let $q\in suppF$. If we let $F_q=F(q)I_{S^{1-F(q)}(q)}$, then (Q,F_q,X,μ^{F_q}) is clearly a fuzzy submachine of N. If $r,s\in S^{1-F(q)}(q)$, then by the retrievability of N, $q\in S^{1-F(r)}(r)\cap S^{1-F(s)}(s)$. Moreover, $F(q)=F(r)=F(s)$. Thus $r\in S^{1-F(s)}(s)$, that is, (Q,F_q,X,μ^{F_q}) is a strongly connected fuzzy submachine. And $F=\cup_{q\in suppF}F_q$. Now to show that N can be expressed as the disjoint union of F_q's, let any two fuzzy sets $F_q=F(q)I_{S^{1-F(q)}(q)}$, $F_{q'}=F(q')I_{S^{1-F(q')}(q')}$ be given, and assume that $F_q\cap F_{q'}\neq\phi$. Then there exists an element

$$a\in S^{1-F(q)}(q)\cap S^{1-F(q')}(q').$$

By the retrievability of N $q,q'\in S^{1-F(a)}(a)$, and $F(q)=F(q')=F(a)$. Since (Q,F_a,X,μ^{F_a}) is strongly connected, $F_q=F_{q'}$. Hence N is the disjoint union of distinct, strongly connected fuzzy submachines (Q,F_q,X,μ^{F_q}).

Conversely, let $N=\cup_{i=1}^n P_i, P_i=(Q,G_i,X,\mu^{G_i})$ be the disjoint union of strongly connected fuzzy submachines P_i. Let $p,q\in supp\cup_{i=1}^n G_i$, and assume that $p\in S^{1-(\cup_{i=1}^n G_i)(q)}(q)$. Then there exists some j such that $(\cup_{i=1}^n G_i)(q)=G_j(q)$, thus $p\in S^{1-G_j(q)}(q)\subset (G_j)_{G_j(q)}$. And also $p,q\in suppG_j$. Since P_j is strongly connected, $q\in S^{1-G_j(p)}(p)$. Hence $q\in S^{1-(\cup_{i=1}^n G_i)(p)}(p)$, which implies that $N=\cup_{i=1}^n P_i$ is retrievable.

Corollary 2.10 [6]. Let $M=(Q,X,\mu)$ be a fuzzy finite state machine. Then M is retrievable if and only if M is the disjoint union of strongly connected submachines.

Let $N=(Q,F,X,\mu^F)$ be a fuzzy submachine of $M=(Q,X,\mu)$. And let $q\in Q$. In the proof of Theorem 2.9, a fuzzy subset F_q of F is denoted by $F_q=F(q)I_{S^{1-F(q)}(q)}$. (Q,F_q,X,μ^{F_q}) is called a singly generated fuzzy submachine of N.

Definition 2.11. Let $N=(Q,F,X,\mu^F)$ be a fuzzy submachine of $M=(Q,X,\mu)$. And let $P=(Q,A,X,\mu^A)$ be a fuzzy submachine of N. Then P is called a primary fuzzy submachine of N if

(1) there exists $q\in Q$ such that $A=F_q\neq\phi$,

(2) if $A\subset F_s$, $s\in Q$, then $A=F_s$.

Every fuzzy finite state machine can be decomposed to primary submachines [6]. This decomposition property can be extended to fuzzy submachines as follows.

Theorem 2.12. Let $N=(Q,F,X,\mu^F)$ be a fuzzy submachine of M. And Let $P=\{P_1,P_2,\cdots,P_n\}$ be the set of all distinct primary fuzzy submachines $P_i=(Q,A_i,X,\mu^{A_i})$ of N. Then

(1) $N=\bigcup_{i=1}^n P_i$,

(2) $N\neq\bigcup_{i=1,i\neq j}^n P_i$ for any $j\in\{1,2,\cdots,n\}$.

Proof. (1). Let $q\in suppF$. Then either $F_q\in P$ or there exists $q'\in suppF\setminus S^{1-F(q)}(q)$ such that $F_q\subset F_{q'}=A_i$ for some i, which implies that $F_q\subset\bigcup_{i=1}^n A_i$. If we note that $F=\bigcup_{q\in suppF}F(q)I_{\{q\}}$ and $F(q)I_{\{q\}}\subset F_q$, then

$F \subset \bigcup_{i=1}^{n} A_i$. Hence $N = \bigcup_{i=1}^{n} P_i$. (2). Let $A_i = F_{q_i}$, and let $G = \bigcup_{i=1, i\neq j}^{n} A_i$, $j \in \{1, 2, \cdots, n\}$. It suffices to show that $F(q_j) \neq G(q_j)$. If we assume that $G(q_j) = A_i(q_j) = F_{q_i}(q_j)$ for some i, then

$$G(q_j) = \begin{cases} F(q_i) & \text{if } q_j \in S^{1-F(q_i)}(q_i), \\ 0 & \text{otherwise}. \end{cases}$$

If $G(q_j) = 0$, then $G(q_j) \neq F(q_j)$ since $F(q_j) \neq 0$, which proves the theorem. Suppose that $q_j \in S^{1-F(q_i)}(q_i)$, then $S^{1-F(q_i)}(q_j) \subset S^{1-F(q_i)}(q_i)$. If $F(q_i) = F(q_j)$, then $F_{q_j} \subset F_{q_i}$. However this contradicts the maximality of $A_j = F_{q_j}$ since $F_{q_j} \neq F_{q_i}$. Thus $F(q_i) \neq F(q_j)$, which implies that $G(q_j) = F(q_i) \neq F(q_j)$. Hence $N \neq \bigcup_{i=1, i\neq j}^{n} P_i$ for any $j \in \{1, 2, \cdots, n\}$.

Theorem 2.13. Let $N = (Q, F, X, \mu^F)$ be a fuzzy submachine of M. Then the following assertions are equivalent.

(1) N is retrievable.
(2) Every primary fuzzy submachine of N is strongly connected.

Proof. (1)⇒(2) : Let $P = (Q, A, X, \mu^A)$ be a primary fuzzy submachine of N. Then $A = F_q$ for some $q \in suppF$. Let $p_1, p_2 \in S^{1-F(q)}(q) = suppA$. By retrievability of N, $q \in S^{1-F(p_1)}(p_1)$ and $F(p_1) = F(p_2) = F(q)$. Thus $p_2 \in S^{1-F(p_1)}(p_1)$, which implies that P is strongly connected.

(2)⇒(1) : By Theorem 2.12, $N = \bigcup_{i=1}^{n} P_i$, where $P_i = (Q, A_i, X, \mu^{A_i})$ are primary fuzzy submachines of N. Suppose that each P_i is strongly connected. From the proof of Theorem 2.9, N is retrievable.

If we take the fuzzy set F as I_Q, then we obtain the following corollaries.

Corollary 2.14 [6]. Let $M = (Q, X, \mu)$ be a fuzzy finite state machine. Let $P = \{P_i, P_2, \cdots, P_n\}$ be the set of all distinct primary submachines of M. Then (1) $M = \bigcup_{i=1}^{n} P_i$, and (2) $M \neq \bigcup_{i=1, i\neq j}^{n} P_i$ for any $j \in \{1, 2, \cdots, n\}$.

Corollary 2.15 [6]. Let $M = (Q, X, \mu)$ be a fuzzy finite state machine. Then the following assertions are equivalent.

(1) M is retrievable.
(2) Every primary submachine of M is strongly connected.

The connectedness of fuzzy finite state machine can be defined as fuzzy in the following.

Definition 2.16. Let $N = (Q, F, X, \mu^F)$ be a fuzzy submachine of $M = (Q, X, \mu)$. And let $P = (Q, T, X, \mu^T)$ be a fuzzy submachine of N. Then P is said to be separated if there exists a proper submachine $P' = (Q, T', X, \mu^{T'})$ such that $P \setminus P' = (Q, T \setminus T', X, \mu^{T \setminus T'})$ is a fuzzy submachine of P. And if N has no separated proper submachine, it is said to be connected.

Theorem 2.17. Let $N = (Q, F, X, \mu^F)$ be a fuzzy submachine of M. Then the following assertions are equivalent.

(1) N is retrievable.
(2) Every singly generated fuzzy submachine of N is primary.
(3) Every nonempty connected fuzzy submachine of N is primary.

Proof. (1)⇒(2) : Let $N = (Q, F, X, \mu^F)$ be retrievable. By Theorem 2.12 $N = \bigcup_{i=1}^{n} P_i$, where the P_i are primary fuzzy submachines of M. And by Theorem 2.13, the P_i are strongly connected. Let F_q be a singly generated fuzzy submachine of N. Then F_q is included in some primary fuzzy submachine P_j. Since P_j is strongly connected, by Theorem 2.6 $F_q = P_j$. Hence F_q is primary.

(2)⇒(1) : By Theorem 2.13, it suffices to show that every primary fuzzy submachine of N is strongly connected. Suppose that F_q is primary. Since every singly generated fuzzy submachine of N is primary, obviously F_q has no proper submachines. Thus by Theorem 2.6, F_q is strongly connected.

(3)⇒(2) : Suppose that (2) does not hold. Then there exists some singly generated fuzzy submachine F_q of N which is not primary. Clearly, F_q is connected, but not primary. Thus (3) does not hold.

submachines. Thus by Theorem 2.6, F_q is strongly connected.

(1)⇒(3) : Let N be retrievable. By Theorem 2.9, N can be expressed as the disjoint union of strongly connected fuzzy submachines $P_i = (Q, T_i, X, \mu^{T_i}), i = 1, 2 \cdots, n$. Let $R = (Q, V, X, \mu^V)$ be any nonempty connected submachine of N. Then

$$R = R \cap N = R \cap (\cup_{i=1}^{n} P_i) = \cup_{i=1}^{n} (R \cap P_i).$$

Each $R \cap P_i$ is a fuzzy submachine of P_i. Since P_i is strongly connected, Theorem 2.6 implies that $R \cap P_i$ is not proper, that is, either $R \cap P_i = \phi$ or $R \cap P_i = P_i$.

Thus $R = \bigcup_{j=1}^{k} P_{i_j}, 1 \leq i_1, \cdots, i_k \leq n$. But the connectedness of R implies that $R = P_{i_j}$ for some i_j. Since P_{i_j} is strongly connected, R must be primary.

As a corollary of Theorem 2.17, we obtain the following.

Corollary 2.18 [6]. Let $M=(Q,X,\mu)$ be a fuzzy finite state machine. Then the following assertions are equivalent.

(1) M is retrievable.
(2) Every singly generated fuzzy submachine of M is primary.
(3) Every nonempty connected fuzzy submachine of M is primary.

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